

RAINDROPS IN THE SEA II - EXPERIMENTAL STUDIES OF VORTEX RING GENERATION

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ABSTRACT

When a drop of (dye) water falls from a small height (say, 20mm) onto a water surface, a laminar vortex ring can be seen to travel down from the surface into the body of the water for some considerable distance. This remarkable phenomenon is not observed if the fall height is too great, or if detergent is added to the water. The penetration depth also appears to be related to the state of oscillation of the drop at impact. The present work offers experimental evidence coupled with a theoretical explanation of the physics of the process in an attempt to explain the factors governing ring formation. Acceleration of the free surface relative to the body of the fluid during the drop-surface coalescence process is observed with high speed photography and this is shown to be consistent with a model for vorticity generation based on free surface stress relaxation. It is shown that this model explains many of the observations which have appeared without explanation in the literature over the past century.

INTRODUCTION

For over a hundred years the phenomenon of vortex ring production by a drop hitting a water surface has been studied from the point of view of determining the factors which affect the 'goodness' of the ring and hence its penetration distance. Factors such as drop shape on impact, surface tension and impact velocity have all been studied usually without reference to the mechanism which gives rise to the generation of the vortex ring. Without a satisfactory mechanism for the generation of the vortex ring, explaining the effect of the above factors on ring production is clearly impossible.

We have already seen that the explanations proposed by Thomson and Newall (1885) and Chapman and Critchlow (1967) cannot be correct (Morton and Cresswell 1992) and that there must be generation of vorticity during the coalescence process implying surface acceleration relative to the underlying fluid.

Before proceeding with an explanation of the physics involved in vortex ring production, it would be beneficial to survey briefly the work which has been done over the past century or so with a view to identifying the observations with which any proposed model must be consistent.

Thomson and Newall (1885) first reported that vortex rings form when drops fall into miscible liquid, such as drops of ink from a fountain pen into a glass of water, and that the depth of penetration varies cyclically with the height of fall. Thus the penetration, and presumably therefore the total amount of generated vorticity, is linked to the oscillation of the falling drop and hence to its shape on impact. Thomson and Newall saw the need to explain why the ring forms and

assumed a sheet of vorticity encompassing the drop after it has penetrated the water surface, an explanation which we have seen to be incorrect.

Chapman and Critchlow (1967) concluded that maximum penetration was achieved for drops which were spherical on impact and oscillating from oblate to prolate so as to offset flattening of the drop on impact. Their model for vortex ring formation was based on pressure gradients due to surface tension, but did not identify the source of vorticity nor explain why higher velocity impacts fail to produce vortex rings. Keedy (1967) found a relationship between fall height and penetration depth similar to that of Chapman and Critchlow, but did not attempt to explain ring formation.

Rodriguez and Mesler (1985, 1988) observed 'bouncing' and 'floating' drops, and identified drops which coalesced with the surface (low velocity, vortex rings formed) and drops which splashed (high velocity, no vortex rings). They reported that maximum penetration of an oscillating drop occurs when the drop is prolate at impact, in conflict with Chapman and Critchlow (1967) and Keedy (1967).

One interesting study was the numerical simulation of the drop impact problem carried out by Harlow and Shannon (1967a,b). A marker-and-cell (MAC) technique was used to solve the Navier-Stokes equations on a finite difference grid, with 'massless marker particles' being advected with the flow in order to follow the evolution of boundary geometry with time. Surface tension was neglected and the free surface condition of zero tangential stress was very crudely modelled. The modelled equations were inviscid although significant numerical diffusion would have played a similar role to viscosity. The resultant flow showed no vorticity at all, and was in fact criticized by Carroll and Mesler (1981) who pointed out that vortex rings should have been formed although no vorticity was seen in the numerical solution.

It has been stated that drops which impact with too great a velocity fail to produce vortex rings. Also Keedy (1967) found that the addition of surfactants (detergent) to the drop greatly reduced the penetration distance of the ring. Hsiao, Lichter and Quintero (1988) found that these two observations are, in fact, related. Drops can be characterized by their surface tension and kinetic energy on impact, through the Weber number, the square root of the ratio of drop kinetic energy to surface energy, where $We = U(\rho D/T)^{1/2}$ with fall speed U , diameter D , and surface tension T . Hsiao et al. (1988) gave a critical value $We_c \approx 8$ below which vortex rings are seen to form on impact. This critical Weber number points to the importance of surface tension, but more importantly also shows that it is not the impact velocity which is important, but the ratio of the kinetic to surface energies.

EXPERIMENTAL STUDIES

In order to establish that the ideas concerning vorticity generation were correct, an experimental study was initiated with the aim of observing the surface acceleration which must accompany generation of vorticity. High speed photographs were taken of drop entry and subsequent vortex ring formation using a Hycam 16mm rotating prism camera running at 2500fps. The optical arrangement used to obtain simultaneous views from of the fluid from just above and just below the surface without an obscuring meniscus is shown in Figure 1. A fresnel biprism with a 4° acute angle was used to obtain views looking slightly down onto the surface from above and slightly up onto the surface from beneath. The receiving tank was filled so that the surface was exactly level with the top of the front glass sheet, thus eliminating any meniscus which would have partially obscured the view. The half-lens is necessary to correct for the different refractive indices and hence different focal depths in the air and the water, and the filter simply corrects for the different exposures in the water and in the air due to light absorption in water. The dropper was capable of delivering one drop at a time on demand. Fine talcum powder was dusted onto the surface over a small area where the impact would take place. Drops were then filmed as they entered the water. Figure 2 (next page) shows a series of frames taken from one such film. Time $t = 0$ corresponds to the instant of contact between drop and surface. Frames 2 and 3 show that the boundary between the tank surface and drop surface (marked by talcum powder) rises up the outside of the drop, in the opposite direction to all of the initial momentum in the problem. With some thought it becomes clear that this surface acceleration does indeed generate vorticity of the correct sign for downwards propagation of the resultant ring.

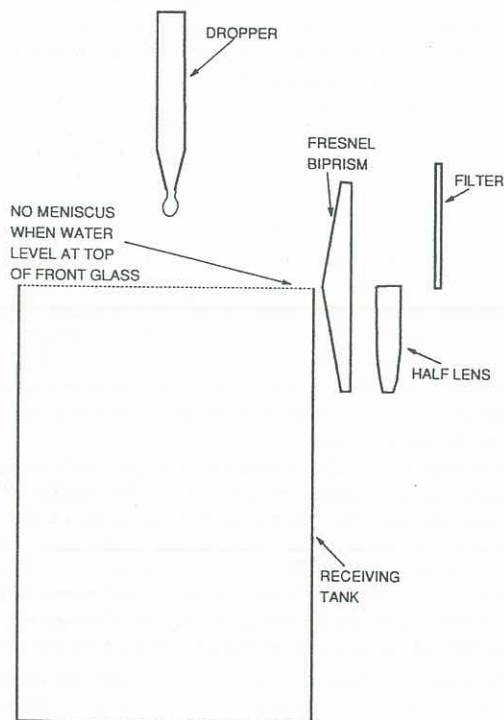


Figure 1: Optical arrangement used for photographing drop-surface coalescence process

PHYSICS OF VORTICITY GENERATION

If a fluid element is to describe a curved path, then it must undergo some combination of rotation and deformation (see Figure 3). Either the fluid element rotates, or it must deform due to the tangential velocity gradient along a radius line. Pure rotation involves finite vorticity and a zero rate of strain whereas irrotational motion requires zero rotation (vorticity) and a finite rate of strain. This rate of strain implies the existence of a stress given by the product of the rate of strain and the viscosity, which is not a problem in the interior of a fluid, or at the edge of a fluid bounded by a rigid wall. However, a free surface cannot support such a stress and thus at the free surface itself the fluid element must undergo pure rotation. Any finite rate of strain leads immediately to a surface stress which accelerates the surface tangentially in such a manner as to bring the surface stress back to zero. Thus the level of vorticity at the surface is determined by the curvature of the streamlines and any departure from this value results in a surface stress which generates vorticity of the correct sign to return the vorticity level back to the prescribed amount. A mathematical treatment of this principle in generalized orthogonal curvilinear coordinates may be found in Batchelor (1967), pp 364-366.

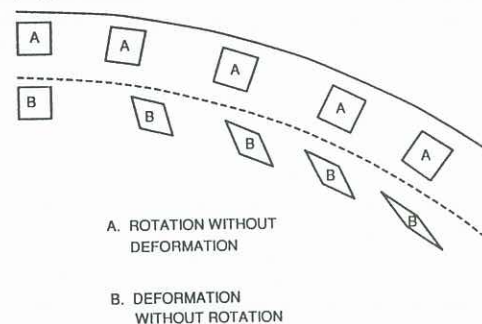


Figure 3: Rotation/deformation of a fluid element describing a curved path

The role of surface tension is to force the streamlines into curvature at the junction between the drop and the surface, and hence to provide the conditions necessary for vorticity generation during coalescence (Figure 4). Consider a drop problem in which the surface tension is set to zero. There is therefore nothing, in theory, to prevent the junction between the curved surface of the drop and the flat surface of the receiving liquid having an infinitesimal radius of curvature and for the streamlines to be straight. The existence of surface tension makes this situation energetically unstable and produces acceleration of the surface normal to the surface itself at the cusp between drop and receiving fluid. This acceleration causes the junction to become 'blunted' but also serves to curve the streamlines at this point. It is this streamline curvature which produces the necessary vorticity generation required for the phenomenon of vortex ring formation. The problem of surface tension effects in the contact of two liquid surfaces was studied numerically by Oguz and Prosperetti (1989) who studied the way in which the surface changed shape and the speed with which the 'liquid bridge' expanded due to surface tension effects. Later (1990) they studied the problem of droplet impact but were apparently both unaware of the boundary condition produced by streamline curvature at a free surface and also under the impression that the concept of the vortex sheet existing between the droplet fluid and the receiving fluid was a valid one. Their numerical calculations were explicitly irrotational, and no reference was made to vortex rings.

The observation of the existence of a critical Weber number for vortex ring production (Hsiao et al, 1988) can also be explained in terms of the streamline curvature boundary condition on vorticity.

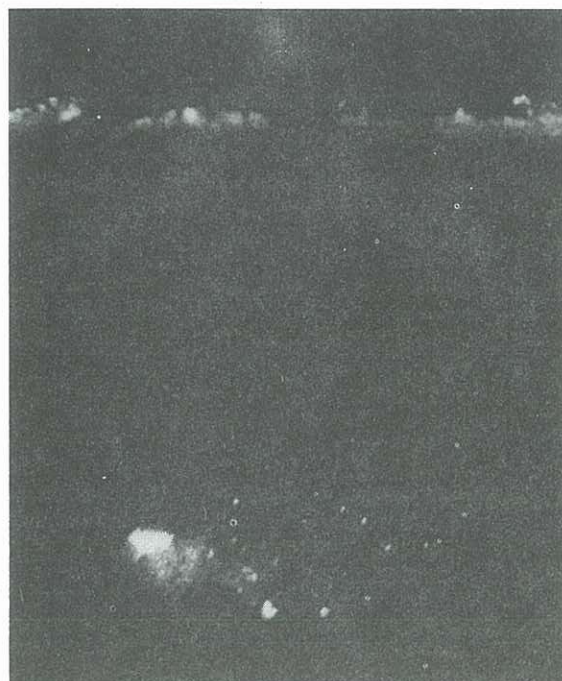
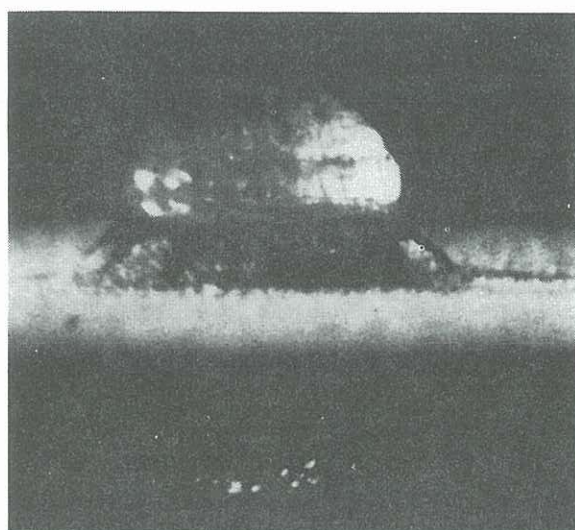
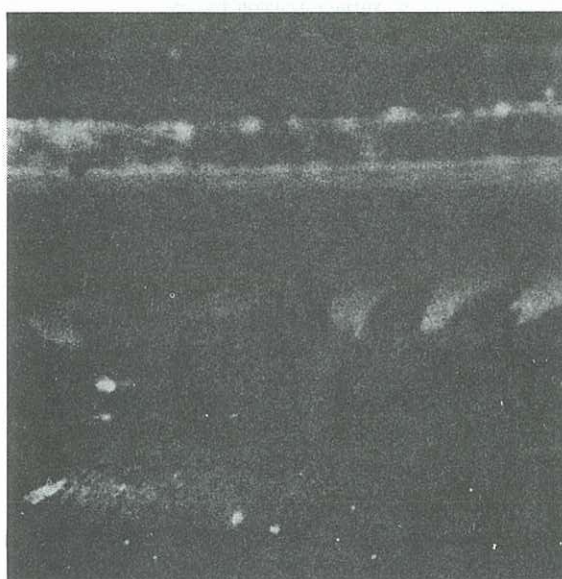
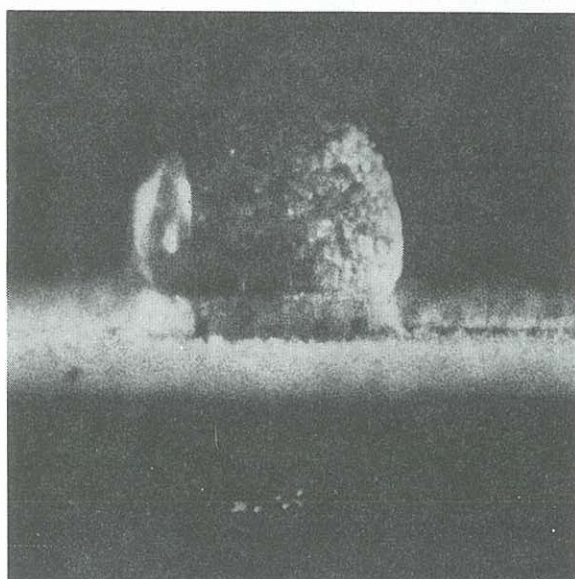
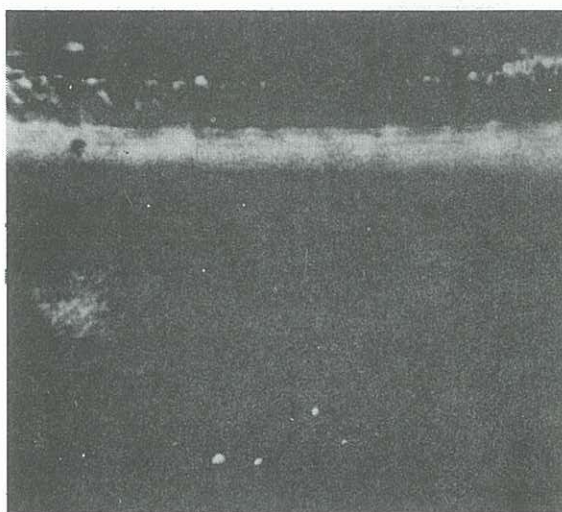
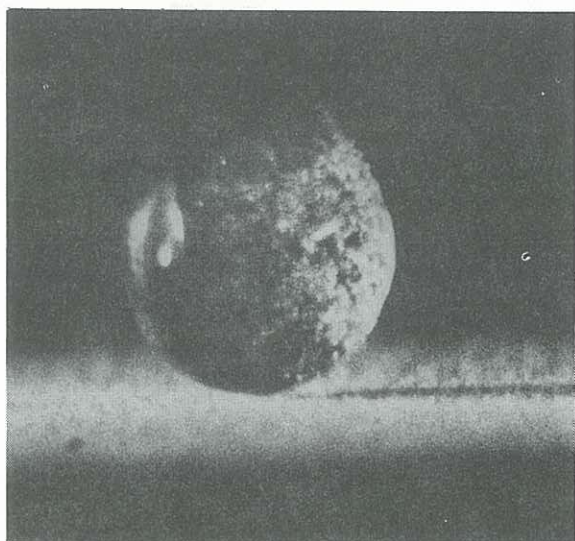


Figure 2: High speed photography of drop coalescence process. Frames are shown at $t = 0ms, 2ms, 4ms, 12.4ms, 26ms,$ and $56.4ms$ after contact.

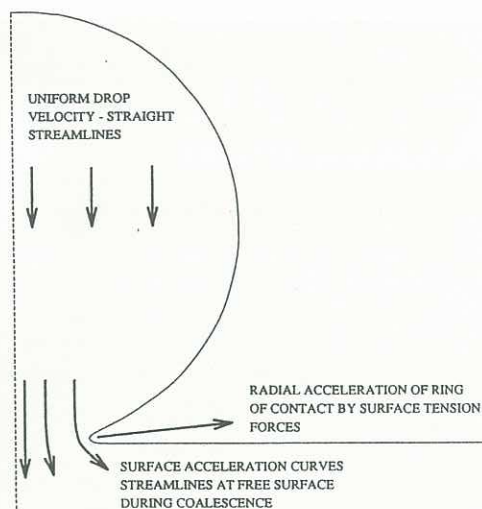


Figure 4: Curvature of streamlines by radial acceleration of ring of contact due to surface tension forces

The lateral acceleration of the annular ring of contact between droplet fluid and receiving fluid is directly related to the surface tension of the drop and the local surface curvature. Also, the time rate of change of the angle of the velocity vector following a fluid element, which is a function of the streamline curvature, is equal to the ratio of the lateral acceleration of the surface to the vertical (impact) velocity of the drop, which is in itself a function of the impact kinetic energy. Clearly if the ratio of the impact kinetic energy to the surface energy of the drop is too high then the curvature of the streamlines during the coalescence process will be too slight to produce significant vorticity. Also, as the vorticity is fixed locally on the free surface for a given streamline curvature and magnitude, then clearly the total amount of vorticity generated depends on the quantity which is diffused into the interior of the fluid and so a higher impact velocity will give rise to a shorter coalescence time and less opportunity for diffusion. Thus it is not surprising to find that there is a critical Weber number above which significant vorticity is not observed.

The effect of droplet shape at impact on the amount and distribution of vorticity is more complicated, but can be explained qualitatively in terms of the above arguments. Experimental observations (Rodriguez and Mesler, 1988) have shown that drops which are oblate on impact produce wider craters than those which are prolate for the same drop volume. The normal acceleration of the ring of contact by the surface tension forces may not be significantly altered, and the impact velocity remains unchanged. However for a lower vertical dimension, coalescence will take place over a correspondingly shorter time and therefore if diffusion is important a lower total amount of vorticity will be generated. Irrespective of whether this is the case or not, the vortical fluid will be spread over a larger diameter with a lower propagation velocity. Thus the vortex ring so formed will remain longer in the vicinity of the deformed surface and can then interact with that surface. Ohring and Lugt (1991) show numerical solutions for the interaction of a viscous vortex pair with a (deformable) free surface in which the free surface boundary conditions appear to be correctly modelled. These solutions show how a vortex pair can generate vorticity of opposite sign which can then cross-diffuse and annihilate, simply by being close to a curved free surface. Exactly what the effect of this is on the vortical fluid generated during coalescence depends on the distribution of vorticity and on the curvature of the surface around the impact crater. As this has not been calculated, we cannot say whether this provides a complete answer to the drop shape factor, but the above explanation is not inconsistent with the observed facts.

CONCLUSIONS

More than a century of work on vortex rings formed by falling drops has left a legacy of uncertainty in which many authors have stressed the importance of vortex rings with no apparent desire to think about the origins of the vorticity. The majority of the literature has concentrated on observations and empirical deductions based on those observations without leading to much understanding of the process involved. The generation of vorticity at a curved free surface due to the relaxation of the surface stress provides an answer to many of the discrepancies and unanswered questions which have arisen in work on this subject to date. We hope to follow this work up with a numerical study which will enable the vorticity generation during and after coalescence to be more fully understood.

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