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MULTIPLE SOURCES OF BUOYANCY IN A NATURALLY VENTILATED ENCLOSURE

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ABSTRACT

This paper outlines experimental and theoretical modelling of a naturally ventilated enclosure containing two or more point sources of buoyancy. The work follows previous research at the University of Cambridge on natural ventilation of enclosures where the density stratification in a space produced by a single source was predicted by utilizing the theory of turbulent buoyant plumes. This work has been extended to cover multiple sources in an attempt to use realistic models of practical problems. Examples include naturally ventilated buildings and industrial/medical processes involving dispersal of contaminants by air currents.

The experimental technique involved the use of a water-filled enclosure with salt solution injected at appropriate points. These sources of negative buoyancy then formed turbulent plumes within the enclosure. Two sources of unequal strength were found to produce a vertical density profile consisting of three distinct, fully mixed layers. The paper describes a theoretical model that successfully predicts the depths and densities of the layers. The positions of the interfaces between the three layers were found to be a function only of the effective area of the enclosure openings, A*, the height of the enclosure, H, and the ratio of the strengths of the two sources of buoyancy, B_1/B_2.

The behaviour of a system with one source of positive buoyancy at the lower boundary and one of the negative buoyancy at the upper boundary of the enclosure is also reported. Two distinct types of flow were found depending on the values of A^*/H^2 and B_1/B_2. This latter case has particular relevance to the prediction of thermal stratification in rooms with both cold and warm surfaces present.

NOMENCLATURE

A*	effective area of vents (m^2):	**B**	buoyancy flux (m^4s^{-3})
b	plume radius (m):	**c**	pressure loss coefficient
C	constant:		
g',G'	reduced density = $g(\rho - \rho_0)/\rho_0$ where ρ_0 is a reference density (m^2/s)		
H	height of enclosure (m):	**h**	height of interface (m)
Q	volume flux (m^3/s):	**v**	fluid velocity (m/s)
α	entrainment constant		
ξ,η	non-dimensional interface heights		
γ	reduced momentum of forced plume source (21)		
ψ	B_1/B_2		

INTRODUCTION

Natural ventilation of enclosures is an important topic in several areas of engineering, particularly in the building industry.The research findings reported here form a part of the wider effort to develop relatively simple models of the complex fluid flow within enclosures with internal sources of heat or buoyancy.

Previous research by Linden et al (1990) examined the ventilation of enclosures containing a single point or line source of buoyancy on the floor of the enclosure. Using the pioneering work of Morton et al (1956) and assuming the buoyancy source to have zero initial volume and momentum fluxes, it was found that the plume arising from such an enclosed source led to the formation of two layers of mixed fluid in the enclosure. The first layer from the base of the enclosure to a height, h, being of the same density as the ambient fluid and the second layer of reduced density above. The interface height, h, was found to be simply a function of the geometry of the enclosure and its openings. The density of the upper layer of fluid also depended on the strength of the buoyancy source.

Thermal stratification (or stratification of contaminant concentration) in practical situations does not generally exhibit a sharp change in density between two internally well-mixed layers as described in the simple model above. A more linear change is observed from ambient conditions at the bottom of the enclosure to a maximum temperature at the top (e.g. see Cooper and Mak (1991), Jacobsen (1988), Gorton and Sassi (1982)). In the work described below, the approach of Linden et al (1990) has been extended to cover the case of two sources of buoyancy. The fluid mechanics are similar to the single source case but the flow analysis is complicated by the fact the stronger of the two plumes develops through a region of two distinct values of density.

MATHEMATICAL MODEL

Two-plumes of positive buoyancy.

A schematic of two plumes of positive buoyancy in a ventilated enclosure is shown in Figure 1. Three layers of fluid of different densities are formed. Both sources (B_1 and B_2) are assumed to be "virtual" in that they each release a finite quantity of buoyancy (equivalent to heat in the thermal situation) but zero mass and momentum; i.e. they produce "unforced plumes". These plumes develop through Layer 0 of the same density as the ambient fluid. A distinct interface then occurs at $z = h_1$ where the reduced gravity in the weaker plume, G_1', is equal to that of the fluid in Layer 1, g_1'. The stronger plume from source B_2 passes through Layer 1 and mixes with Layer 2 at height $z = h_2$, where $G_2' = G_2'$. The following analysis has much in common with that by Linden et al (1990) for a single source in a ventilated enclosure. The volume flux through the top and bottom openings is given by:

$$Q_t = Q_b = A^* \left(G_2' \left(H - h_2\right) + G_1' \left(h_2 - h_1\right)\right)^{1/2}, \qquad (1)$$

where A* is the "effective opening area" defined as:

$$A^* = \frac{a_t \; a_b}{\left(\frac{1}{2c} \; (a_t^2 + a_b^2)\right)^{\frac{1}{2}}} \qquad (2)$$

where a_t, a_b and c are the top and bottom opening areas and the associated pressure loss coefficient, respectively.

Following the approach of Morton et al (1956) and Morton (1959), in the present study the underlying assumptions used to model fluid flow in the plumes include: i) the ratio of the mean speed of inflow at the edge of a plume to the mean vertical velocity on the plume axis is a constant, α, and ii) profiles of mean vertical velocity and buoyancy are each of similar form at all heights. A number of volume flux, buoyancy flux and density relations may then be identified.

$$Q_t = Q_b = Q_{22} = Q_{11} + Q_{21}\,, \tag{3}$$

$$B_1 + B_2 = Q_{11} G'_{11} + Q_{21} G'_{21} = Q_{22} G'_{22} \tag{4}$$

For the weak plume:

$$B_1 = G'_1 \; Q_1 = \text{constant} \tag{5}$$

$$Q_{11} = C\,(B_1 h_1^5)^{1/3}\,, \tag{6}$$

$$G'_{11} = \frac{1}{C}(B_1^2 h_1^{-5})^{1/3} = g'_1\,, \tag{7}$$

Here $C = \frac{6}{5}\alpha(\frac{9}{10}\alpha)^{1/3}\pi^{2/3}$, α being the entrainment constant for the plume.

For the strong plume: for $0 < z < h_1$

$$B_2 = G'_2 Q_2 = \text{constant}, \tag{8}$$

$$Q_{21} = C\,(B_2 h_1^5)^{1/3}\,, \tag{9}$$

$$G'_{21} = \frac{1}{C}(B_2^2 h_1^{-5})^{1/3}\,. \tag{10}$$

[Note: the first term of double subscripts refers first to the plume and the second term to the interface, e.g. Q_{21} is the volume flux in Plume 2 passing through Interface 1 (between Layers 1 and 2)].

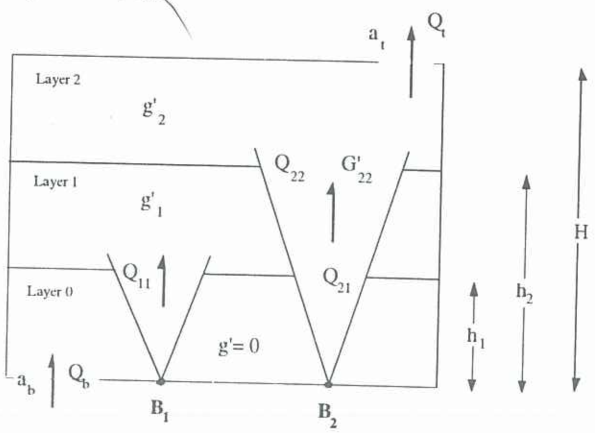

Figure 1 Ventilated enclosure with two positive buoyancy sources

The behaviour of the stronger plume in Layer 1 is influenced by the fact that it experiences a step change in the surrounding liquid density at $z = h_1$. The authors have modelled this by considering the plume to consist of two parts. The first, an unforced plume in Layer 0. The second a "distributed" plume of finite mass flux and momentum originating at $z = h_1$ and developing in an environment of uniform density. This latter plume has a reduced buoyancy flux, B'_2, relative to its surroundings in Layer 1 as compared to that in Layer 0, B_2.

$$B'_2 = B_2 - g'_1 Q_{21}\,, \tag{11}$$

$$G'_{22} = g'_2\,. \tag{12}$$

It is convenient to define the ratio of the two interface heights as:

$$\frac{h_2}{h_1} = 1 + f\,(B_1/B_2)\,. \tag{13}$$

where $f(B_1/B_2)$ is some arbitrary function of the ratio of source strengths, B_1/B_2, to be determined. Equations (2), (6) and (9) may be rearranged to obtain:

$$Q_{22} = Q_{11}\left(1 + \left(\frac{B_1}{B_2}\right)^{1/3}\right). \tag{14}$$

Substituting (4), (12) and (14) into (1) we obtain

$$Q_{11}\left(1+\left(\frac{B_1}{B_2}\right)^{1/3}\right) = A^*\left[\frac{(B_1+B_2)\;(H-h_2)}{Q_{11}\left(1+\left(\frac{B_1}{B_2}\right)^{1/3}\right)} + G'_{11}\,(h_2-h_1)\right]^{1/2}, \tag{15}$$

then using (4) and (6) and defining $\xi = h_1/H$:

$$\frac{A^*}{H^2 C^{3/2}} = \frac{(1+(B_1/B_2)^{1/3})^{3/2}}{(1+B_1/B_2)^{1/2}} \times \left[\frac{\xi^5}{\left(1-\xi-\frac{(1-(B_1/B_2)^{2/3})}{(1+B_1/B_2)}\,f\left(\frac{B_1}{B_2}\right)\xi\right)}\right]^{1/2}. \tag{16}$$

This result is equivalent to that of previous research (Linden et al. (1990) for a single plume (ie when $B_1/B_2 = 0$) and for two plumes of equal strength (ie $B1/B2 = 1$) providing $f\,(B_1/B_2) = 0$ in both cases.

Analysis of "distributed" plume

The behaviour of Plume 2 in Layer 1 is that of a plume from a source of finite volume and momentum flux, developing in an environment of uniform density. The model used for this analysis is shown in Figure 2.

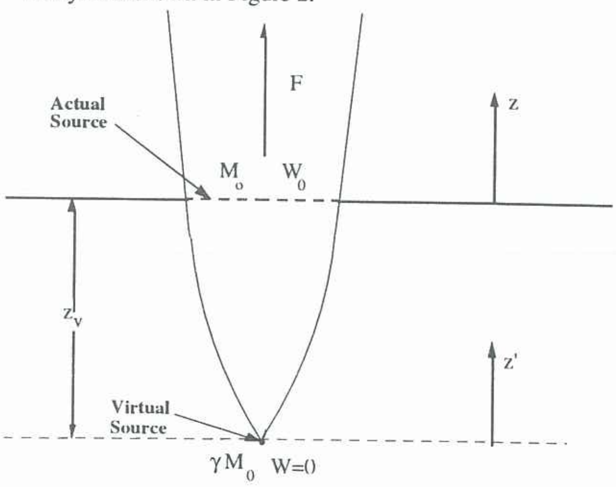

Figure 2 Analysis of plume originating from a "forced" source

A plume originating from a virtual source may be treated as having uniform velocity and density across any horizontal cross section (ie "top hat" profiles) the governing differential equations fluid flow in any buoyant plume are then (Caulfield, 1991):

$$\frac{dW}{dz} = 2\,\alpha\,M^{1/2} \;(17a): \quad \frac{dM^2}{dz} = 2\,FW\,, \tag{17b}$$

where b = plume radius, v = fluid velocity, $W = (vb^2)$, $M = (vb)^2$ and $F = g'W\;(=B/\pi)$. Eliminating z from (17) gives

$$M^{3/2}\,dM = \frac{B}{2\alpha}\,W\,dW, \tag{18}$$

and thus,

$$\frac{2}{5}\left(M^{5/2}(z) - M^{5/2}(0)\right) = \frac{F}{4\alpha}\left(W^2(z) - W^2(0)\right). \tag{19}$$

A source of finite volume flux, W_o, and momentum flux, M_o, may be modelled as originating from a "virtual source" of finite momentum flux, γM_o, but zero volume flux situated some distance z_v above the actual source. Using a second coordinate $z' = z - z_v$ we then have:

$$M(z') = \left(\frac{5F}{8\alpha}\,W^2(z') + (\gamma M_o)^{5/2}\right)^{2/5}. \tag{20}$$

Applying this solution at z' = z when $M(z') = M_o$ and $W(z') = W_o$ then:

$$\gamma = \left(1 - \frac{5F}{8\alpha}\,\frac{W_o^2}{M_o^{5/2}}\right)^{2/5}. \tag{21}$$

In the present situation for Plume 2 in Layer 1 the source buoyancy flux is B_2' and

$$\frac{5F}{8\alpha}\,\frac{W_o^2}{M_o^{5/2}} = \frac{B_2'}{B_2}. \tag{22}$$

From (7) (9) and (11)

$$\frac{B_2'}{B_2} = \left(1 - \left(\frac{B_1}{B_2}\right)^{2/3}\right), \tag{23}$$

$$\text{and so}\quad \gamma = \left(\frac{B_1}{B_2}\right)^{4/15}. \tag{24}$$

To close the problem Q_{22} is required in terms of h_2, h_1, B_1 and B_2. From (17a) it may be shown that:

$$f\left(\frac{B_1}{B_2}\right) = \frac{3}{5}\,\frac{\psi^{1/5}}{(1-\psi^{2/3})^{1/2}}\int_a^b (t^2+1)^{-1/5}\,dt \tag{25}$$

where $\psi = B_1 / B_2$, $b = (1-\psi^{2/3})^{1/2}\,(1+\psi^{1/3})/\psi^{1/3}$ and $a = (1-\psi^{2/3})^{1/2}/\psi^{1/3}$.

Hence the heights of the two interfaces h_2 and h_1 are determined by only two parameters A^*/H^2 and the ratio of the source strengths, B_1/B_2.

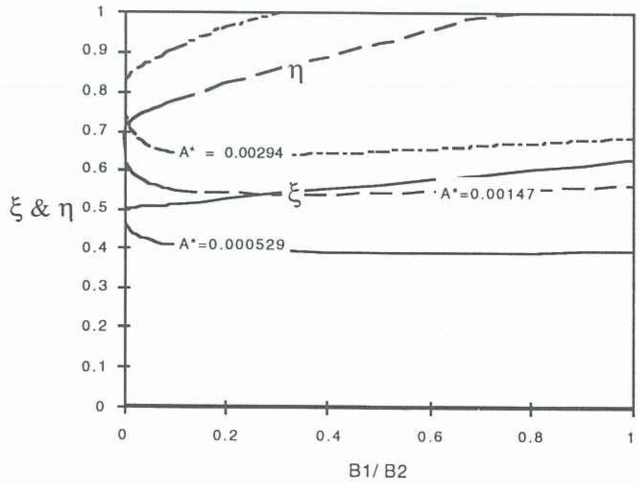

Figure 3 Theoretical prediction of interface heights (H = 0.25m)

The magnitudes of the non-dimensional interface heights, ξ and η ($=h_2/H$), are shown in Figure 3 for three values of the geometric parameter A^*/H^2. These results were obtained by simultaneous solution of (16) and (25) by iterative relaxation of initial estimates of ξ and η. For some cases of relatively large A^*/H^2, the theoretical model predicts the absence of Layer 2 and solution to the problem is obtained by modifying Equation (16) appropriately.

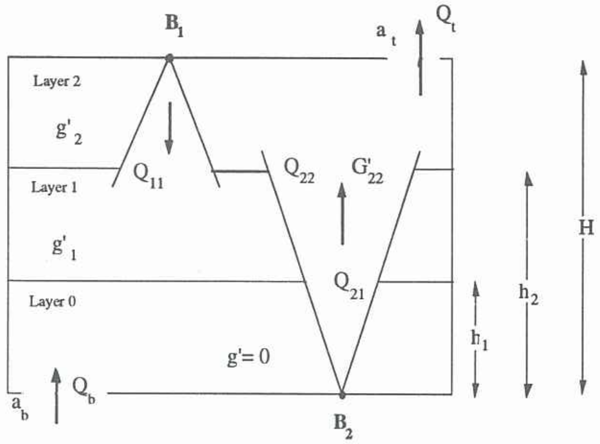

Figure 4 Ventilated enclosure with buoyancy sources of opposite sign

Two plumes with buoyancy of opposite sign

The second situation considered in this work is shown schematically in Figure 4 where an enclosure contains two plumes. The first and weaker plume (B_1) is of negative buoyancy the second (B_2) is of positive buoyancy. This situation is commonly encountered in building natural ventilation problems where there is both a source of heat (a convector, say) and a cold surface (an outside window). The governing equations for fluid flow, buoyancy, etc are similar to those given above and will be presented in full in a forthcoming publication. The pressure balance equation equivalent to (16) is

$$\frac{A^*}{H^2 C^{3/2}} = \left[\frac{\xi^5}{\left(1-\frac{B_1}{B_2}\right)(1-\xi) - \left(\frac{B_1}{B_2}\right)^{2/3}\left(\frac{\xi}{1-\xi(1+f)}\right)^{5/3} f\xi}\right]^{1/2} \tag{26}$$

where f is defined such that $(h_2/h_1) = 1 + f$ and $f = f(B_1/B_2)$. As in the case of two plumes of positive buoyancy, Plume 2 develops through a region of non-uniform density. The effective buoyancy flux of this plume relative to Layer 1, B_2', is given by

$$\frac{B_2'}{B_2} = \left(\frac{B_1}{B_2}\right)^{2/3}\left(\frac{h_1}{H-h_2}\right)^{5/3} \text{ and}$$

$$f\left(\frac{B_1}{B_2}\right) = \frac{3}{5}\left(\frac{B_2}{B_2'}\right)^{1/5}\left(\frac{B_2}{B_2'} - 1\right)^{3/10}\int_a^b \frac{dt}{[t^2+1]^{1/5}}, \tag{27}$$

where:

$$b = \left(1 + \left(\frac{B_1}{B_2}\right)^{2/3}\left(\frac{H-h_2}{h_1}\right)^{5/3}\right)\left(\frac{B_2}{B_2'}\right)^{1/5}\left(\frac{B_2}{B_2'} - 1\right)^{-1/2} \text{ and}$$

$$a = \left(\frac{B_2}{B_2'} - 1\right)^{-1/2}.$$

Again we find that the height of the two interfaces is determined simply by the geometry of the enclosure and the ratio of the strengths of the buoyancy sources. The density of the two upper layers in the enclosure may be readily calculated from the magnitude of the buoyancy flux and the velocity of the plume entering the layer of the same density.

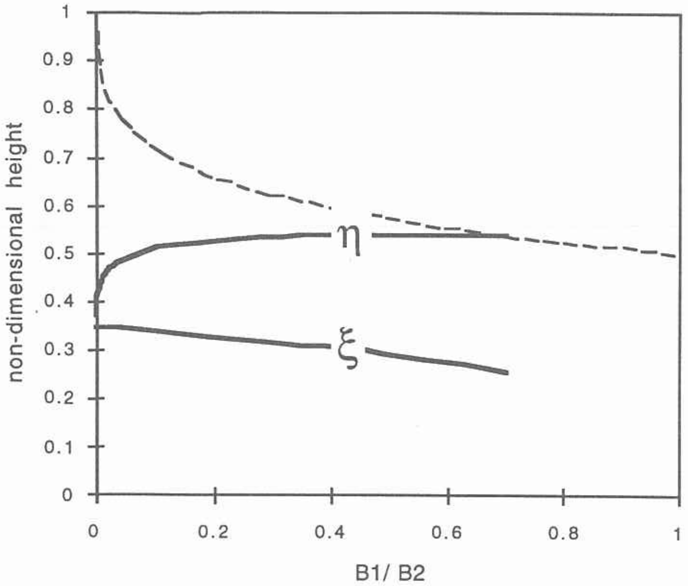

Figure 5 Theoretical prediction of interface heights for sources of opposite sign $(A^*/H^2 = 2.944 \times 10^{-3})$

The case of two plumes of opposite sign of buoyancy flux shows interesting behaviour not present in the case of positive sources only. Figure 5 shows the results from the solution of Equations (26) and (27) for a specific value of A^*/H^2. The three layers shown in Fig. 4 only exist in this configuration up to a limiting value of (B_1/B_2) at this point the density of Layer 1 becomes equal to that of the ambient fluid and only two layers are present. This condition is indicated by the dotted line in Fig. 5. The height of the upper layer in this limiting case is then:

$$\frac{H^4C^3}{A^{*2}}\left(1-\left(\frac{1-\eta}{\eta}\right)^{5/2}\right)^2=\left(\frac{1-\eta}{\eta^5}\right). \qquad (28)$$

For higher values of (B_1/B_2) Source 1 is sufficiently strong to form a layer of fluid less dense than ambient at the bottom of the enclosure. In this case there is a two-way flow through the bottom vent, a_b, and ambient fluid from this source rises as a plume to supply fluid for the middle layer.

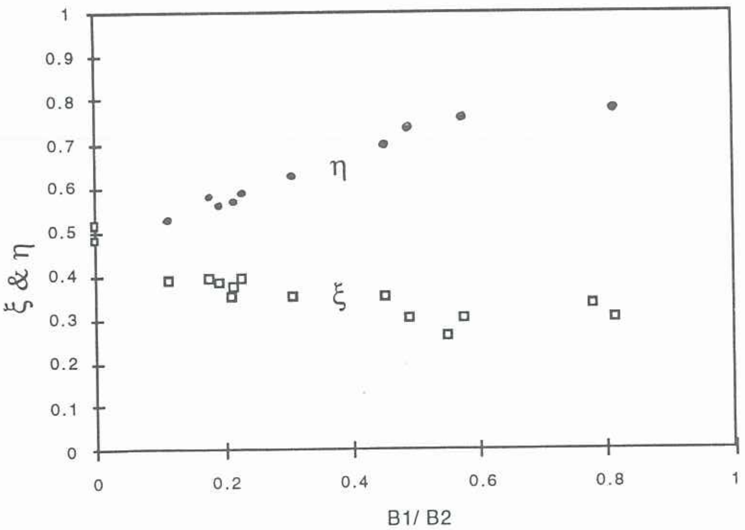

Figure 6 Experimental results for two negative sources $(A^*/H^2 = 0.0167)$

EXPERIMENTAL RESULTS

The mathematical models derived above has been extensively tested against experiments using a naturally ventilated perspex enclosure, 0.25m cubed, with water as the working fluid. Negative sources of bouyancy were generated using concentrated sodium chloride solution. Results for two sources of negative buoyancy (effectively two positive sources with the whole system inverted) at one particular value of A^*/H^2 are shown in Figure 6. The magnitudes of ξ and η show qualitative agreement with the theory given above. Some degree of mismatch is inevitable from two principle sources: a) the sources in the experiments are necessarily "forced" with finite volume and momentum fluxes and b) the pressure loss coefficient, c, for the enclosure vents will depend to some extent on the rate of flow Q_t. A full discussion of the correlation of all the available experimental data, taking account of the forced sources, will appear in a separate forthcoming publication.

Experiments were also carried for the sources of opposite sign. Positive buoyancy sources were implemented by injecting a mixture of water and methylated spirits into the enclosure. The experiments showed good agreement with the theory outlined above.

CONCLUSIONS

The behaviour of naturally ventilated enclosures containing multiple buoyancy sources has been investigated through analysis and experiment. A model to predict the thermal stratification within such enclosures has been developed based on the theory of flow in "forced" plumes. It has been found that when two buoyancy sources of the same sign were present within an enclosure stratification developed in the form of three layers of different densities. The position of the interfaces between these layers is determined by the geometrical parameter A^*/H^2 and the ratio of strengths of the sources of buoyancy (B_1/B_2). The density of the layers is a function of the absolute strength of the sources and the physical scale of the system.

An enclosure with two buoyancy sources of opposite sign shows similar but rather more complex flow characteristics. For low values of the paramenter (B_1/B_2) three layers are formed. However, there is a limiting condition for this configuration; ie at a given value of (B_1/B_2) the two layers of density differing from that of the ambient fluid merge to form a single layer. Higher values of (B_1/B_2) produce a more complex flow pattern.

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