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EXPERIMENTAL INVESTIGATION OF TWO-PHASE FLOWS AT LOW REYNOLDS NUMBERS

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Abstract

Two phase flow ,liquid-solid, analysis in glass capillary tubes was performed using a dual beam, frequency shifted, Laser-Doppler Velocimetry (LDV) system. The study was directed towards measuring the flow rate in low Reynolds number flows by optical (non invasive) measurements, and verifying the ability of the maximum velocity to represent the average cross sectional velocity. A modular experimental system was erected including a rotating diffraction grating based LDV optics and fast digitising electronics. It was shown that the effects of the non-parallelism and nonlinearity of the fringe spacing contributes about 5% uncertainty in the measured doppler frequency.

Measurements were performed in single phase and two phase flows for a range of Reynolds numbers and particulate concentrations . The results indicated two main conclusions: a. The classical parabolic velocity distribution in laminar flows is not always preserved and is influenced by the Reynolds number and the concentration of the particulate phase. b. The amount of deviation from the parabolic profile is small. Consequently, for practical engineering applications, the maximum velocity at the tube's centre can represent the average cross sectional velocity over large range of Reynolds number ($30 \leq Re \leq 600$) and particulate concentration ($0.005\% \leq \phi \leq 0.05\%$) with an accuracy of better than 95%.

Introduction

The applications of multi-phase flows and flow rate measurement is of great importance in several engineering and bio-medical fields. Pneumatic transport carries reactants in chemical plants, slurries of coal suspended in water, Latex particles in emulsion paints and blood flow are just a few examples (Happel, 1986).Laser-Doppler Velocimetry (LDV) is a potential non-invasive method which could be appropriate for such applications. Laser-Doppler measurements, in two-phase flows in tubes of small dimensions (2-5 mm in diameter) over wide range of Reynolds numbers (Re=30-200) and various particulate volume fraction ($\phi = 0.5\% - 45\%$), were intensively reported in literature. The accuracy of the measured flow rate, compared with independent methods, is reported to decrease as the Reynolds number increases with a relative error between 5% and 30% (Kawata et al., 1974, Tanaka et al., 1974, Feke and Riva, 1978).

It is also reported that as the particulate concentration increases the signal processing become more difficult, due to low signal-to-noise ratio. The problem is more pronounced near the tube walls, particularly at high Reynolds numbers. This leads to less accuracy in determining the velocity distribution, and as a consequence less accurate flow rate evaluation (Riva et al., 1979,1981,1982, Brain and Riva, 1982, Highman et al., 1979, Koyama, 1982, Vlachos and Withelaw, 1974).

We can conclude that most of the previously published work have all indicated some kind of difficulty in determining the flow rate accurately and in performing near wall measurements. This paper is concerned to pursue the development of an accurate, non invasive measurement technique for two phase flows in capillaries. It relates to longer term objective to develop an instrument suitable for velocity measurement in flows with high particulate concentration and low Reynolds numbers.

Poiseuille Flow

For a laminar symmetric flow, in tube with circular cross section of radius R, the velocity profile is given as (Kjaer and Enni, 1987):

$$\overline{U} = \overline{U}_{f_{max}} \left\{ 1 - \left(\frac{r}{R}\right)^{\frac{n+1}{n}} \right\} \tag{1}$$

where $\overline{U}_{f_{max}}$, is the fluid maximum (mean) velocity at the specific cross section, $n = 1$ for Newtonian fluids, $n < 1$ for non-Newtonian fluids. The particles local velocity, $\overline{U}_p$, in a dilute mixture with a constant and uniform particles distribution, can be modelled as (Drew, 1974, Cox and Mason, 1971)

$$\overline{U}_p = \overline{U}_{f_{max}} \left\{ \left[1 - \left(\frac{r}{R}\right)^2 \right] + \frac{8}{9}\lambda^2 \frac{(0.25 + 2.5\phi)}{(1 + 9.05\phi)} \right\} \tag{2}$$

where, $\lambda = a/R$, is the ratio of particle-to- tube diameter and ϕ is the particles volume fraction. Equation (2) indicates that for small values of λ, the particles velocity becomes $\overline{U}_{f_{max}}(1 - (r/R)^2)$, which is the local fluid velocity, as given by equation (1). It also indicate, that the particles lead the fluid by a quantity which depends on the values of ϕ and λ. For LDV measurements, a correction should be introduced into equation (1) to account for the effect of the finite control volume dimensions on the recorded velocity, particularly for cases of relatively large values of α, the ratio between the control volume length to inner tube diameter. Kreid (1974), suggested the following model to describe the velocity distribution, $\overline{U}$. His model seems to hold valid up to a ratio of $\alpha = 1$.

$$\overline{U} = \overline{U}_{f_{max}} \left\{ \left[1 - \left(\frac{r}{R}\right)^2 \right] + A_1 + A_2 \right\} \tag{3}$$

$$A_1 = -\alpha^2/2 \;,\; A_2 = \frac{\alpha[1 + (r/R)]e^{-\delta_y^2}}{\sqrt{\pi}(1 + erf|\delta|)} \tag{4}$$

$$\delta_y = S/b_y \;,\; \alpha = b_y/R \tag{5}$$

where S is the distance between the centre of the control volume and the inner tube's wall and b_y is the length of the control volume (see Fig. 1).

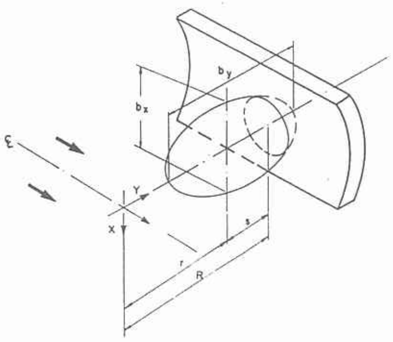


Figure 1: Schematic illustration of the interaction of the LDA control volume with the capillary tube.

Experimental System

The experimental system centres around a dual beam Laser-Doppler Velocimeter (LDV) and consists of three main subsystems: the optical system, the signal processing and data acquisition unit and the flow system.

The Optical System

The optical system, shown schematically in Fig.2, is a conventional forward-scatter LDV system with a rotating diffraction grating used for beam splitting and frequency shifting. All the optical components were mounted on micrometric tables which allowed movements with an uncertainty of about $\pm 5\mu m$. However, the need to perform measurements of low velocities in tubes of very small diameters created several difficulties, mainly the requirement for very low frequency shifting ($0.5KHz < F_s < 10KHz$) and for small dimensions of the control volume. The low value of frequency shift forced the use of very low diffraction grating rotational speed. Hence, a geared dc motor, with a tachogenerator mounted on the other end of the motor shaft, was used. The tachogenerator generates dc voltage linearly related to its rotational speed.The temporal instabilities, which are probably caused by discontinuities in the gear's tooth motion, were measured and quantified and found to be much more significant at the very low range, as can be seen from Fig. 3. The figure displays a very good fit of the mean frequency shift to a linear correlation with the tachogenerator voltage as measured by a digital voltmeter. The relatively large rms values are still much lower than those that would be obtained from non geared motor operating with a low driving voltage.

The need for small control volume dimensions required large beam diameter and short focal length for the focusing lens (L3, see Fig. 2). Such a combination is a potential source for deformation in the linearity of the fringes and in the symmetry of the ellipsoidal contour of the control volume (Durst and Stevenson, 1979). The alignment procedure concerned mainly lenses L2 and L3, shown in Fig. 2. As a result a calibration test was conducted to evaluate the nonlinearity and non-parallelism of the fringes. The test was performed by moving a glass fibre at constant velocity, V, through the length (along $Y-$ axis) of the control volume (rotating on a disk mounted on a dc motor). The results are displayed in Fig.4, where the fringe spacing,δ as measured by the LDV system ($\delta = V/F$), with its corresponding standard deviation are displayed against the longitudinal location in the control volume. The standard deviation is the result of nonuniform fringe spacing (at a certain location) and the variation in speed of the dc motor, as shown in Fig.3 .

The results obtained indicate a maximum variation of $\varepsilon_\delta = 3.33 \times 10^{-4} \mu m^{-1}$, in the average fringe spacing between extreme points within the control volume. The distortion parameter, $\varepsilon_\delta \equiv (\Delta\delta/\bar{\delta})/b_y$ ($b_y = 150\mu m$). The fact that Fig. 4 does not display a symmetric distribution along the control volume main axis ($Y-$ axis) also indicates the deviation of the contour from its symmetrical ellipsoidal shape. The main optical characteristics is given in table (1). As a result, the effective control volume length is $90\mu m$ and its diameter is $10\mu m$.

Table 1: Summary of main optical characteristics.

laser	HeNe 5 mW
beam spacing	17.7 mm
beam diameter (at $1/e^2$)	2.4 mm
focusing lens	$60mm$
collecting optics magnification	1.5
pinhole diameter	$15\mu m$
photomultiplier	Hamamatssu R1617

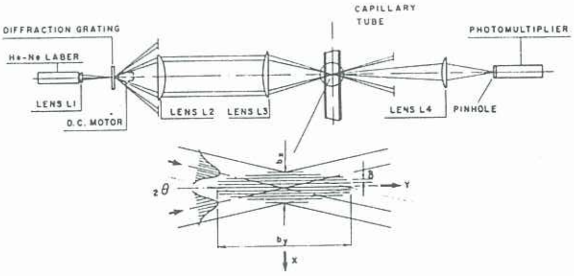


Figure 2: Schematic description of the optical system.

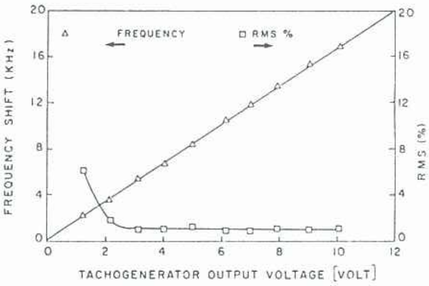


Figure 3: Calibration curve of the frequency shift and its rms with the tachogenerator voltage.

Signal Processing and Data Acquisition

Two types of signal processing systems were used, namely frequency counting and Fast Fourier Transformation (FFT), see Figs. 5. The first, which is much faster, is suitable for Doppler frequency measurements in single phase flows or at two phase flows of low particulate concentration where individual Doppler bursts are observed. The second option for processing is used in highly concentrated two phase flows where the quality of the Doppler bursts deteriorates and time-domain processing is no longer possible.This option is related to the long term objectives of this study. A comparison of the reference frequency with the frequency calculated by the FFT procedure (Rabiner and Gold, 1975)

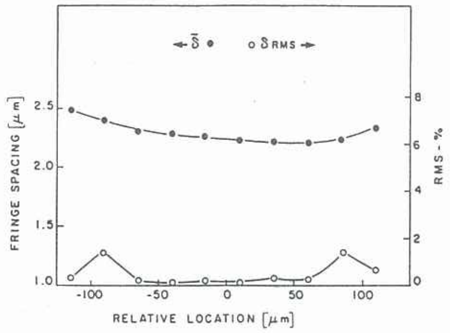
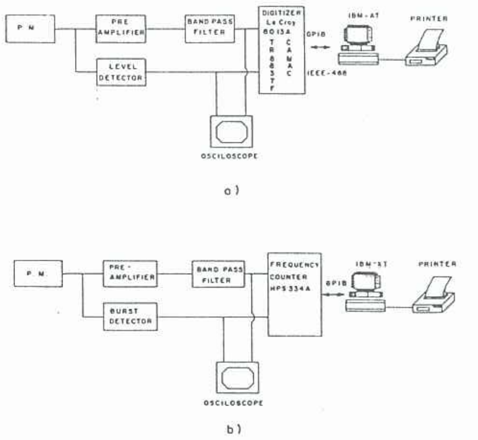


Figure 4: Variation of the local average fringe spacing and its standard deviation along the main axis of the control volume.



a)

b)

Figure 5: Schematic illustration of the signal processing unit. (a) frequency counting based method. (b) FFT based method.

over the range of $0.5KHz - 200KHz$, showed very good agreements with errors of less than about 0.1%.

The Flow System

A closed-cycle flow system, with flow rate controlled by gravitation, is used to eliminate the need for constant consumption of the (expensive) Latex particles used for representing the discrete phase. The test tubes were all pyrex capillaries of $1.14 \pm 0.02mm$ diameter and $0.1 \pm 0.01mm$ wall thickness. For comparison to the LDV, flow rates were also measured by quantifying total volume flow during a period of time. These measurements were performed using accurate pipette and digital stopwatch with resulting maximum error of 0.01 millilitres/second.

Experimental Results and Discussion

Two velocity profiles of single phase flow at Reynolds numbers 63.4 and 567 were measured and the results are shown in Fig.6. Both profiles display a parabolic distribution as expected in laminar flows. The deviation of the measured profile from parabolic distribution with the zero velocity near the wall is associated with the large ratio between the dimensions of the control volume and the tube diameter (Kreid, 1974). The two lower curves in the figures show the distribution of the local rms velocity values. These values are low in the centre of the tube ($\approx 10\%$) increasing towards the wall ($\approx 50\%$). The tendency for lower rms values at the centre than near the wall is clear as it originates from corresponding lower absolute velocity differences within a finite radial distance. In order to evaluate the ability of Kreid's model to represent realistic measurements, a comparison with experimental data is shown in Fig.7. The figure shows very good agreement between the predicted (for $\alpha \approx 0.18$) and measured profiles (for Re=567). This is especially seen near the wall where the integration effect is at its maximum and, considering the large rms values and the spatial sensitivity, the average velocities recorded are relatively very accurate. Mean and rms velocity profiles of two phase flows (water and $20\mu m$ Latex particles) with particulate volume fraction of $\phi = 0.005\%$ and Reynolds number of 35.9, 94.8 and 264 are displayed in Fig.8. Although it was difficult to demonstrate smooth profiles, a tendency towards an increase in the relative velocities (U/U_{max}), near the wall with decrease in the Reynolds number, is clear. The same tendency can be seen in the previous figure of the single phase flow. Increasing the particulate concentration by five times ($\phi = 0.025\%$), see Fig. 9, seems to decrease the effect of the Reynolds number on the shape of the velocity profile. Further increase in the particle concentration to $\phi = 0.05\%$ indicates a change in the flow behaviour (see Fig. 10). The previous demonstrated results clearly display a change in the measured velocity profile with Reynolds number and particle concentration. Such a behaviour could originate mainly from the influence of the particulate phase on the effective viscosity. However, no quantitative physical explanation could yet be given. A general overview of the total volume flow measurement using LDV, when compared with a direct volume flow rate measurement, displayed very good agreement. Deviation of less than about 5% is observed over a flow range from 1 to 16 millilitres per minute.

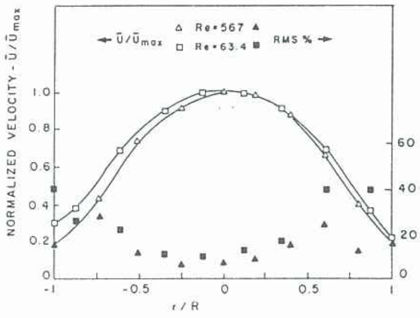


Figure 6: Velocity profile of single phase flow (water) in a capillary tube, $d = 1140\mu m, Re = 63.4, 567$.

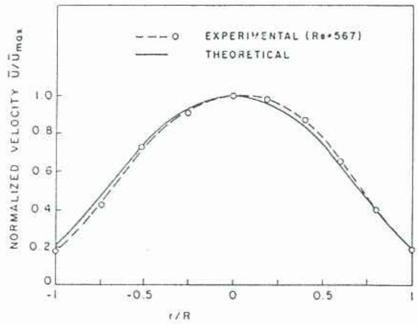


Figure 7: Comparison between measured, $d = 1140\mu m, Re = 567$ and predicted (Kreid, 1974) velocity profiles.

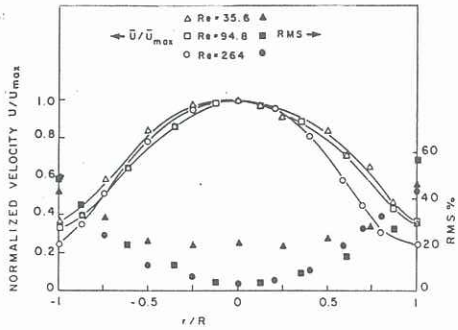


Figure 8: Mean and rms Latex particles velocity profile in two phase flow with volume fraction $\phi = 0.005\%, Re = 35.9, 94.8, 264$.

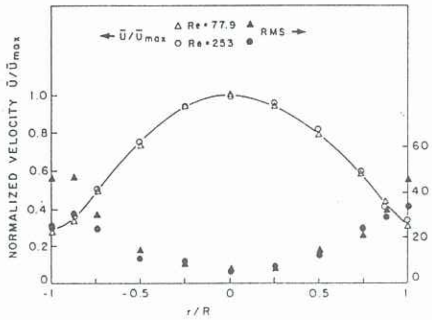


Figure 9: Mean and rms Latex particles velocity profile in two phase flow with volume fraction $\phi = 0.025\%, Re = 77.9, 253$.

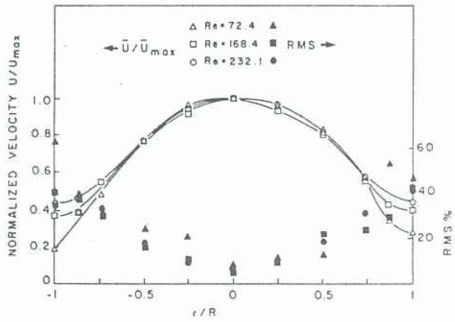


Figure 10: Mean and rms Latex particles velocity profile in two phase flow with volume fraction $\phi = 0.05\%, Re = 72.4, 168, 232$.

Conclusions

An experimental system for flow measurements in small glass capillaries ($d \approx 1mm$) is described. Velocity profiles across the capillary, of water (single phase flow) and water and $20\mu m$ latex particles (two phase flow) at different Reynolds numbers and volume fraction, are displayed. The recorded profiles are in good agreement with Kreid's (1974) predicted profile. As a result of the parabolic shape of the velocity distribution in laminar flows the integration effect, due to the finite dimensions of the control volume, is significant only near the wall and insignificant at the centre of the tube. It was observed that the measured velocity profiles, at various Reynolds numbers and at different particulate concentrations did not always preserve a parabolic distribution, and showed differences in the profiles shape. It is difficult to quantify the sources for the changes in the distributions of the normalised velocity. One probable cause could be the variation of the local effective viscosity with particle's concentration, which for the determination of the Hagen-Poiseuille distribution is considered constant. The sensitivity of the maximum (central) velocity to deviations of the velocity profile from a parabolic distribution is found to be small. Hence, it can represent with an accuracy of better than 95% the average cross sectional velocity and thus the total flow rate over a wide range of Reynolds number and particles concentration.

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