11th Australasian Fluid Mechanics Conference
University of Tasmania, Hobart, Australia
14-18 December 1992

HELICAL FLOW OF NON-NEWTONIAN FLUIDS IN A CO-AXIAL RHEOMETER

S.N. BHATTACHARYA, S.H. JAVADPOUR and A.G. CHRYSS

Rheology & Materials Processing Centre, Dept of Chemical & Metallurgical Engineering
Royal Melbourne Institute of Technology, GPO Box 2476V
Melbourne, VIC 3001, AUSTRALIA

SUMMARY

A recirculatory flow has been introduced in a coaxial rheometer to prevent particles of multiphase mixtures from settling during the rheological measurements. The effect of axial flow of Newtonian and non-Newtonian fluids superimposed on the rotational flow within a co-axial (annular) geometry has been studied to establish the relationship between shear stress, shear rate and axial flow. The flow pattern generated in such a system becomes of helical nature.

A mathematical analysis has been carried out for the flow of a Herschel Buckley fluid assuming a co-axial set up in which the rotating inner cylinder (bob) is of infinite length. The analysis indicates that the radial and axial shear stress components, which constitute the total shear stress profoundly alters the viscosity profile in the annular region. The shear stress measured on the surface of the inner cylinder is found to decrease with increase in the axial flow rate within the annulus.

The mathematical analysis has also been reduced to that for a Bingham fluid and the result has been compared with experimental measurements, giving good agreements. The experiments also confirm that for low axial velocity and high ratio of bob length to annular gap the end effects could be neglected.

INTRODUCTION

Helical flow is generated in the annular region of a co-axial rotational rheometer when one or both of the cylinders are rotated and an axial flow is super-imposed. An on-line rotational co-axial rheometer used for continuous viscosity measurements and drilling mud in a rotating drilling machine encounter this kind of flow.

Following the analysis of a superposition of annular and Couette flow by Rivlin (1956). several authors (Noll, 1958; Coleman and Noll, 1959; Bird and Curtiss, 1959) carried out formulations and solutions of an unsteady state, laminar, tangential flow of an incompressible fluid in the annular space between two cylinders of which one or both were rotating. Fredrickson (1959) solved for the combined axial and tangential flow in an annulus for an inelastic non-Newtonian fluid. Fredrickson and also Coleman and Noll (1959b) concluded that the helical flow of any fluid may be characterised by two parameters, the angular velocity and the shear stress - shear rate which are dependent upon the axial pressure gradient.

Diercks and Schowalter (1966) presented an analytical solution for a power law fluid in a system with the outer cylinder rotating. Rea and Schowalter (1967) experimentally measured the velocity profile of a power law fluid in a helical flow system and compared their findings with predictions made from tube viscometer data. Tanner (1963) and Savins and Wallick (1966) solved the helical flow problem using a model due to Oldroyd (1958).

Huilgol (1990) proposed a trial and error method to solve the helical flow problem for general fluids in terms of four parameters including one related to the axial pressure gradient. Detailed numerical analysis of the helical flow of power law fluids for any cylinder radii ratio and experimental verification was carried out by Sestak et al. (1990). In all the papers mentioned so far the emphasis was to investigate the flow enhancement, if any, due to the presence of helical flow.

Bhattacharya et al.(1990) solved the problem of axial flow in a rotational rheometer for power law fluids. In their final equations the axial pressure gradient term was eliminated, thereby reducing the number of variable parameters to three. Following this study, Javadpour and Bhattacharya (in press) extended the analysis to that for helical flow of a Bingham plastic fluid in an annular space. They obtained the shear rate, shear stress and viscosity distribution for different values of flow rate, torque and yield stress. This study is now extended to a more general type of shear thinning fluid with a yield stress. In this paper an analysis is presented of the helical flow of Herschel Bulkley fluid in an annulus. An examination is done to check the suitability of the solutions when reduced to the model of Bingham plastic or power law fluid by appropriately modifying parameters of the Herschel Bulkley equation.

Experimental measurement has been carried out to determine the effect of axial velocity on shear stress - shear rate relationship.

THEORETICAL ANALYSIS

A helical flow is generated within the annular region of two co-axial cylinders of radii R_1, and R_2 ($R_2 > R_1$) by rotating the inner cylinder with angular velocity, Ω and superimposing an axial flow on this rotating fluid. In a cylindrical co-ordinate system the velocity field may be written as

$$\upsilon = (\upsilon_r = 0, \upsilon_\theta = r\omega(r), \upsilon_z = \upsilon(r)) \quad (1)$$

which automatically satisfies the equation of continuity. Also from the postulated velocity field we obtain

$$\dot{\gamma} = \sqrt{(r\omega'(r))^2 + (\upsilon'(r))^2} \quad (2)$$

where $\dot{\gamma}$ is defined as total shear rate. Equation (2) serves to emphasise that the term "total shear rate" and "velocity gradient" are not necessarily synonymous.

The non-slip boundary condition at the inner and outer cylinder is

$$\upsilon(R_1) = \upsilon(R_2) = 0 \quad (3)$$

$$\upsilon_\theta(R_1) = R_1\Omega, \upsilon_\theta(R_2) = 0 \quad (4)$$

The equations of motion can be written as

$$\frac{1}{r}\frac{d}{dr}(rt_{(rr)}) - \frac{1}{r}t_{(\theta\theta)} - \frac{\delta\Phi}{\delta r} = -\rho\omega^2 \quad (5)$$

$$\frac{1}{r}\frac{d}{dr}(rt_{(rz)}) - \frac{\delta\Phi}{\delta z} = 0 \quad (6)$$

$$\frac{1}{r}\frac{d}{dr}(r^2 t_{(r\theta)}) - \frac{\delta\Phi}{\delta\Theta} = 0 \quad (7)$$

where ρ is the fluid density (constant), P is the fluid pressure, $t_{(rz)}$, $t_{(r\theta)}$, are the usual components of the total shear stress τ, and ϕ is a measure of the gravitational force and fluid pressure.

The non-zero components of the stress tensor are

$$t_{(rz)} = \frac{\upsilon'(r)}{\dot{\gamma}}\tau(\dot{\gamma}); t_{(r\theta)} = \frac{r\omega'(r)}{\dot{\gamma}}\tau(\dot{\gamma}) \quad (8)$$

The equations of motion (5) to (7) require

$$\Phi = f(r) + 2\alpha z + b\theta \quad (9)$$

$$t_{(rz)} = \alpha r + \frac{1}{r}\beta \quad (10)$$

$$t_{(r\theta)} = \frac{1}{2}b + \frac{1}{r^2}c \quad (11)$$

where α, β, b and c are constants and determined by application of boundary condition.

From a combination of (8) with (10) and (11) there results a pair of differential equations for the velocity components

$$\upsilon'(r) = \left(\alpha r + \frac{\beta}{r}\right)\frac{\dot{\gamma}}{\tau(\dot{\gamma})} \quad (12)$$

$$\omega'(r) = \left(\frac{b}{2r} + \frac{c}{r^3}\right)\frac{\dot{\gamma}}{\tau(\dot{\gamma})} \quad (13)$$

where from (2) one may write

$$\tau(\dot{\gamma}) = \sqrt{\left(\alpha r + \frac{\beta}{r}\right)^2 + \left(\frac{b}{2} + \frac{c}{r^2}\right)^2} \quad (14)$$

These relations are well-documented in the literature, (Schowalter, 1978).

The expression for the three constant Herschel - Bulkley model is

$$\tau(\dot{\gamma}) = \tau_0 + \kappa\dot{\gamma}^n$$
$$\text{for.}\, \tau > \tau_0 \quad (15)$$
$$\text{and.}\, \eta(\dot{\gamma}) = \kappa\gamma^{n-1} + \tau_0\dot{\gamma}^{-1}$$

where τ_θ is the yield stress , κ is the consistency index, n is the power law exponent and $\eta(\gamma)$ is the apparent viscosity.

It can be shown form the equation of motion that the stress field regardless of the fluid rheology is given by

$$t_{r\theta} = -\frac{M}{2\pi r^2} \quad (16)$$

where M is the torque per unit length , with $t_{r\theta}$ being negative, because the inner cylinder rotates, $\omega'(r) < 0$.

Substituting equation (15) into (12) and (13) and using (2) and the expression for the volume rate of flow given by

$$Q = 2\pi\int_{R_1}^{R_2} r\upsilon(r)dr \quad (17)$$

one obtains

$$\int_{R_1}^{R_2}(\alpha r^2 + \beta)r^2\Gamma(r,\alpha,\beta)\left[\frac{Mr^{-3}}{2\pi\sqrt[n]{\kappa}\Gamma(r,\alpha,\beta)} + \tau_0\right]^{\frac{1}{n}} dr = 0 \quad (18)$$

and

$$\int_{R_1}^{R_2}(\alpha r^2 + \beta)r^2\Gamma(r,\alpha,\beta)\left[\frac{Mr^{-3}}{2\pi\sqrt[n]{\kappa}\Gamma(r,\alpha,\beta)} + \tau_0\right]^{\frac{1}{n}} dr + \frac{MQ}{2\pi^2} = 0 \quad (19)$$

where

$$\Gamma(r,\alpha,\beta) = \frac{1}{\sqrt[n]{\kappa}}\left[r\sqrt{1 + \frac{4\pi^2 r^2}{M^2}(\alpha r^2 + \beta)^2}\right]^{-1} \quad (20)$$

We have found the values of α and β from the two simultaneous nonlinear equations (18) and (19) by a numerical procedure as described previously (Javadpour and Bhattacharya, in press). The values of α and β obtained from computations allow us to calculate the velocity profile. Also, viscosity is obtained as total shear - stress divided by the total shear - rate, i.e.

$$\eta = \frac{\sqrt{t_{(r\theta)}^2 + t_{(rz)}^2}}{\sqrt{\upsilon'^2 + (r\omega')^2}} \quad (21)$$

EXPERIMENTAL

An axial flow coaxial rotational rheometer has been developed in the Rheology and Material Processing Centre to measure rheological properties of settling type multiphase mixtures. An illustration of the rheometer is given in Figure 1. The recirculation of the fluid provides an axial flow superimposed on the rotational flow created by the rotating bob within the annular section of the rheometer. The rheometer measures the torque for a given angular velocity of the bob and an axial flow rate through the annulus. The torque was measured by the measuring head of the contraves RM 115 rheometer. The axial flow was obtained by direct measurement. The radii of the cup and bob for the rheometer were respectively 0.0215 m (R_1) and 0.0242 m (R_2). A liquid height of approximately 50 mm above the tip of the bob was maintained to minimise any end effect during measurement.

The test fluids used in this experiment were solutions of carbopol in a mixture of glycerol - water. A 0.3 wt% carbopol in a 50 wt% glycerol - water was used as a Bingham plastic test fluid. The Herschel - Buckley fluid was produced by dissolving 5 wt% carbopol in a 50 wt% glycerol solution. Although it is arguable whether carbopol solution exhibits a true yield stress, it was considered satisfactory for this study.

RESULT AND DISCUSSIONS

Theoretical results of shear stress, shear rate and viscosity profile for Herschel Bulkley and Bingham plastic fluids flowing through the annular region of a rotational rheometer are presented here. The theoretical analysis for the Bingham plastic fluid has been achieved by substituting for the power law exponent, n in equation (15) equal to 1. The assumed values of the radii of the inner and outer cylinders of the rheometer have been taken as 0.0215m and 0.0242m respectively; they correspond to the dimension of the rheometer used in this laboratory for the experimental study. Values of n, κ and τ_o have been arbitrarily chosen for the sake of theoretical analysis.

The stress component, $t_{r\theta}$ termed as shear stress for the Herschel - Buckley fluid is plotted against the shear rate in Figure 3. The axial flow rate produces a reduction in shear stress for a given shear rate. This is the stress component which is measured by the rotating bob of a rotational cup and bob rheometer. This reduction in stress is similar to the result reported by Savins and Wallick (1966). They predicted that the imposition of a relative rotation increased the axial discharge rate of annular flow in the z-direction. This prediction is then expected since Figure 3 shows that in a helical flow the viscosity is reduced for a shear dependent fluid meaning that the axial flow will be enhanced with increased relative rotation. Further examination of Figure 3 indicates that the shear reduction is somewhat suppressed at higher shear rate. For example, the reduction in shear stress at $10s^{-1}$ shear rate for a change in flow rate from 10^{-5} to 10^{-4} m^3/s is 19% while for the same flow difference at 100 s^{-1} shear rate the reduction in shear stress is 1%. This is not unusual considering that the shear dependent characteristics of a Herschel Bulkley fluid is higher at low shear rates and decreases steadily with increase in shear rate.

The solution for Herschel Bulkley fluids could be easily reduced to that of a Bingham plastic fluid by substituting for n equal to 1 in the fluid model in Equation (15). The theoretical prediction of shear stress versus shear rate at various axial flow rates for a Bingham plastic fluid is presented in Figure 2. The parameters of the Bingham plastic model used in this analysis as those obtained for the 0.3% carbopol solution. The solid lines in Figure 2 represent the theoretical prediction for different axial flow rates. The experimental data in Figure 2 show some difference with the theoretical curve.

The shear dependent viscosity profile of a Herschel Bulkley fluid is found to be significantly affected by the imposition of the axial flow rate as shown in Figure 4. The radial position expressed as R_2/R_1 is equal to 1 at the bob surface. The viscosity is found to increase as it moves away from the boundaries of the concentric tubes reaching a maximum viscosity at an intermediate position. The other interesting point to note is that as the axial flow increased the difference between maximum and minimum viscosities reduced and the variation of viscosity in the radial direction was severely dampened at a high axial flow rate. The effect is, however, dependent on the applied shear rate. An alternative viscosity profile was obtained holding Q constant and varying total shear stress as shown in Figure 5. Here also the peak viscosity was observed at some intermediate radial position.

CONCLUSION

The theoretical analysis for the helical flow of a Herschel Bulkley fluid indicates that shear stress ($t_{r\theta}$)for a given shear rate ($r\omega'$) decreases with increase in axial flow rate. Experimental results for a Herschel Bulkley and a Bingham plastic fluid in a helical flow show some deviations with the theoretical prediction.

Prediction is also made of the radial viscosity profile in the annular region. These profiles are found to be affected by axial flow as well as shear stress.

REFERENCE

1. **Bhattacharya S.N., Chryss A., Connell H.J. & Shepherd** (1990)
5th National Conf.Rheo., Melbourne, 15

2. **Bird R.B., C.Curtiss D.**(1959)
Chem. Eng. Sci., 11,108

3. **Coleman B.D., Noll W.**(1959)
Arch.Rational Mech.Anal.,3,289

4. **Coleman B.D., Noll W.J.**(1959)
Applied Phys.,30,10,1508

5. **Dierckes A.C., Schowalter W.**(1966)
Ind.Eng.Chem.Fund.5,263

6. **Fredrickson A.G.**(1959)
Chem.Eng.Sci.,11,108

7. **Huilgol R.R.**(1990)
5th National Conf.Rheo., Melbourne,43

8. **Javadpour S.H., Bhattacharya S.N.**(in press)
Jr.Sci.I.R.Iran

9. **Noll W.**(1958)
Arch.Rational Mech.Anal.,2,197 3

10. **Oldroyd J.G.**(1958)
Proc.Roy.Soc.,A245278

11. **Rea D.R., Schowalter W.**(1967)
Trans.Soc.Rheol.11,125

12. **Rivlin R.S.**(1956)
J.Rational Mech.Anal.,5,179

13. **Savins J.G., Wallick G.C.**(1966)
A.I.Ch.E.J.,12,357,12

14. **Sestak J., Houska M., Zitny R. & Dostal M.**(1990)
3rd European Conf.Rheo.437

15. **Schowalter W.R.**(1978)
Mechanics of Non-Newtonian Fluids, Pergamon Press Inc.

16. **Tanner R.I.**(1963)
Rheo.Acta.3,Part I,21,PartII,26

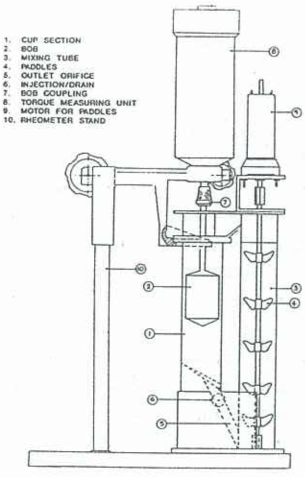

FIG. (1). AXIAL FLOW RHEOMETER

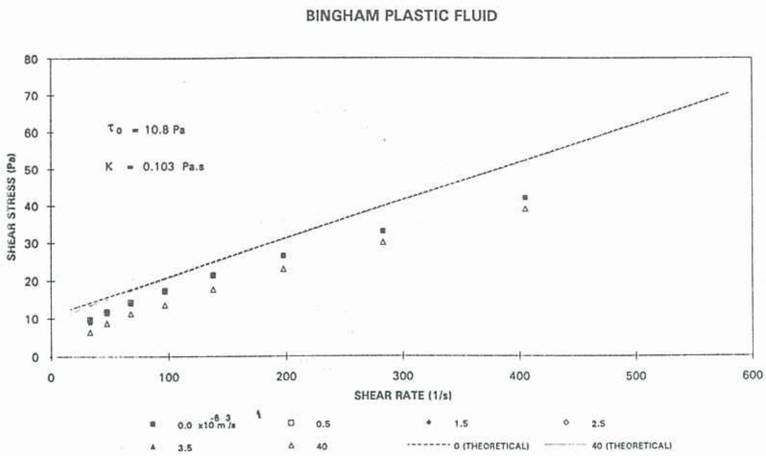

FIG 2. 0.3% CARBOPOL RESIN IN A 50% GLYCEROL SOLUTION

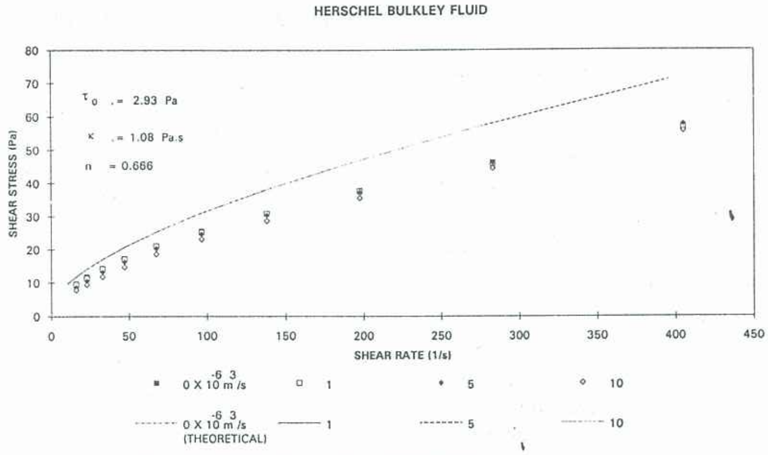

FIG 3. 0.6% CARBOPOL RESIN IN A 50% GLYCEROL SOLUTION

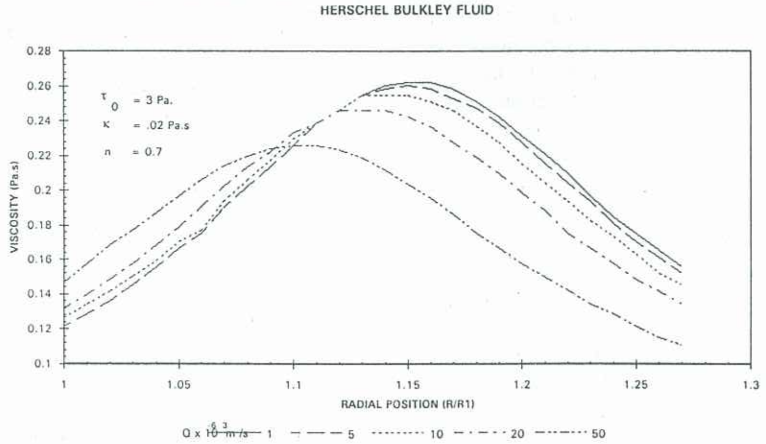

FIG 4. VISCOSITY PROFILE IN THE ANNULAR GAP OF THE RHEOMETER

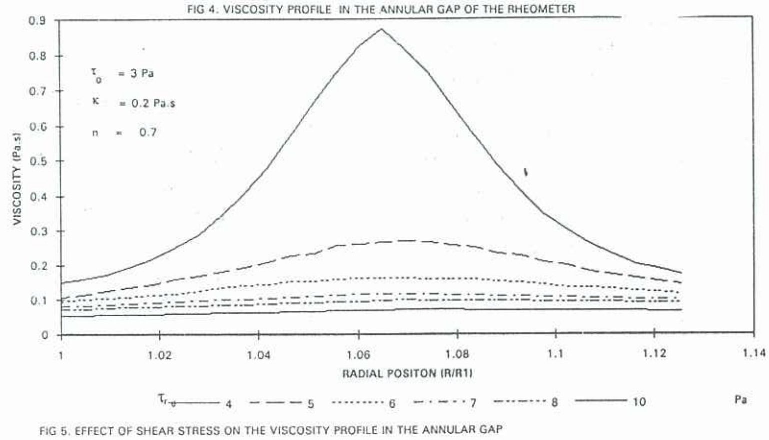

FIG 5. EFFECT OF SHEAR STRESS ON THE VISCOSITY PROFILE IN THE ANNULAR GAP