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AN EDDY DRAG MODEL OF TURBULENCE

D.R.H. BEATTIE

Australian Nuclear Science & Technology Organisation
Private Mail Bag 1, Menai, NSW 2234, AUSTRALIA

ABSTRACT

Eddies are assumed to lose energy through fluid flow drag. The concept leads to an inverse dependence of mixing length, and hence also of eddy viscosity, on drag coefficient in the log-layer where a constant drag coefficient may be assumed. This eddy viscosity dependence is assumed to also apply to the near wall region with appropriately modified drag coefficient, leading to an eddy viscosity expression of the form $\mu_t/\mu=\kappa y^+/[1+b/(y^+u^+)]$, which correctly varies cubically with y near the wall and linearly with y away from the wall. Choices $\kappa=0.407$ and b=275 lead to an accurate "universal" velocity profile. Many unorthodox flows not conforming to the "universal" profile may be fitted by altering coefficient values. The phenomenological basis of the eddy viscosity form permits both interpretation and correlation of these data.

INTRODUCTION

Several types of turbulent flow have velocity profiles which disagree with the "universal" profile, even for developed pipe flow. For these flows, hereafter referred to as "anomalous" or "unorthodox" turbulent flows, velocity profiles away from the wall vary logarithmically with wall distance, as with conventional flows, but with slope and/or intercept parameters disagreeing with the "universal" values. In some cases, velocity profiles do not approach $u^+=y^+$ as the wall is approached. Examples of unorthodox turbulent flows are the simultaneous flow of gas and liquid with moderate to high gas content; flows with drag-reducing additives; flows affected by magneto-hydrodynamic influences; and flows with turbulent-modifying flow boundaries (eg "riblets", or flexible "compliant surfaces").

Although current models of turbulence can often (but not always) be made to fit unorthodox velocity profiles with suitably modified empirical coefficients, they are unable to a priori predict data trends, and in most cases are unable to offer explanations for the anomalous behaviour. Phenomenological models such as the various mixing length theories do not have sufficient insight into turbulence to be able to predict or explain the anomalous behaviour. Current models emphasise application to complex flow geometries. These models also cannot handle anomalous turbulence since they use empirical coefficients or functions to implicitly or explicitly invoke the "universal" profile in the near-wall constant shear stress region of the flow.

As a first step towards understanding unorthodox turbulent flows, this work returns to the simple mixing length concepts of Prandtl, as described in standard texts, and extends them to provide a phenomenological basis for assessing how coefficients of the model might be affected in unorthodox flow circumstances. As will be demonstrated, Prandtl's model so extended also describes simple turbulent pipe flow better than other simple alternatives.

It is widely considered that Prandtl conceived his mixing length ℓ, the distance an eddy (considered as a sphere) moves before losing its identity, as an analogue of the mean free path of the kinetic theory of gases. However, having formulated the concept, he further developed the mixing length form by simple empirical methods. This work retains the concept of an eddy as a simple solid during its lifetime, but considers the drag on the eddy in order to provide an improved description of the mixing length.

DEVELOPMENT OF THE MODEL

From the definition of viscosity by

$$\tau=(\mu+\mu_t)\ du/dy,$$

the near-wall velocity profile in pipe flow is given in wall units by

$$u^+=\int_0^{y^+}(1+\mu_t/\mu)^{-1}dy^+. \tag{1}$$

As discussed in standard texts, Prandtl's mixing length theory predicts that, away from the viscous sublayer,

$$\mu_t/\mu=\ell^+=\kappa y^+. \tag{2}$$

It is assumed here that an eddy is created from dissipating eddies; acts initially as a solid object; loses energy through drag acting on it as it moves; and loses its identity after a particular fraction of its initial energy is dissipated. Its remaining energy then contributes to the formation of new eddies. As with Prandtl's model, the distance travelled while it retains its identity is the mixing length ℓ.

In the turbulent region outside the viscous sublayer, a constant drag coefficient C_o may be assigned to the eddy. It is easy to show that the above model leads to

$$\ell\sim d/C_o, \tag{3}$$

where d is a characteristic eddy dimension. To maintain consistency with Prandtl's model at large distances from the wall, d is assumed to be

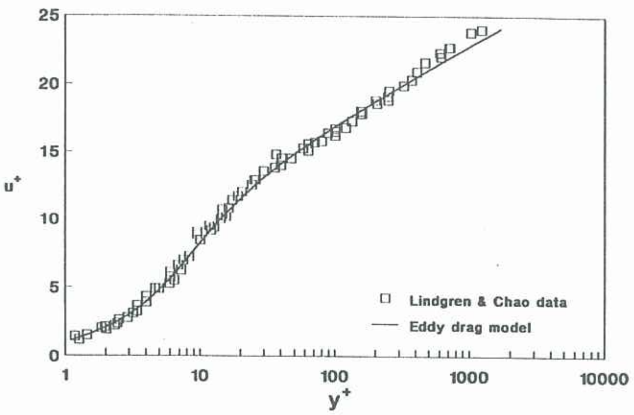

Fig. 1 Comparison of predicted and experimental non-dimensional velocity profile

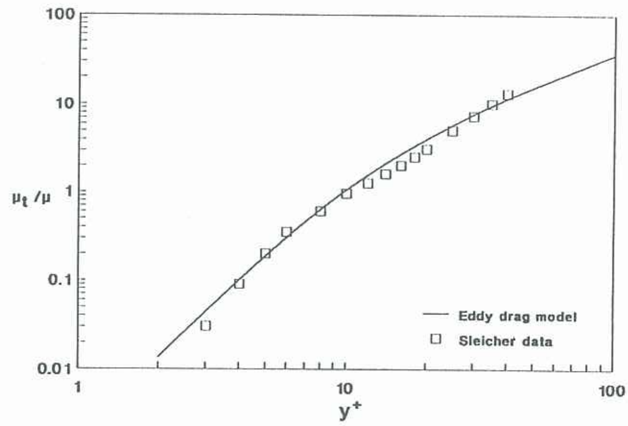

Fig. 2 Comparison of predicted and experimental eddy viscosity distribution

proportional to the distance from the wall:

$$d \sim y.$$

Equation (3) then becomes

$$\mu_t/\mu=\ell^+=k_1 d^+/C_o=k_1 k_2 y^+/C_o \equiv \kappa y^+ \qquad (4)$$

where k_1 and k_2 are proportionality constants.

This μ_t relation is now assumed to apply also to the near-wall region provided the constant far-from-the-wall drag coefficient C_o is replaced by C, the appropriate near-wall value. Noting that C is expected to approach an inverse relation with the eddy Reynolds Re_E number at lower velocities, the loss coefficient is assumed to be of the form

$$C=C_o+a/Re_E \qquad (5)$$

where a is a constant. This approaches the correct high and low Re limits.

The third term of equation (4) then becomes $k_1 d^+/(C_o+a/Re_E)$. To maintain consistency of the equation, a corresponding change must also be made to the fifth term. Equation (4), conceived for the turbulent region of the flow, becomes

$$\mu_t/\mu=\kappa y^+/(1+a/(C_o Re_E))=k_1 d^+/(C_o+a/Re_E). \qquad (6)$$

As developed, this should now also apply to the viscous sublayer. From the second and third terms of this equation,

$$d^+=\kappa\ y^+ C_o/k_1.$$

Re-expressing the eddy Reynolds number in terms of wall parameters,

$$Re_E=\rho u d/\mu=d^+ u^+$$

and substituting into the third term of equation (6) leads to an expression for turbulent viscosity:

$$\mu_t/\mu=\kappa y^+/[1+ak_1/(\kappa y^+ u^+ C_o^2)], \qquad (7)$$

which should apply both within the viscous sublayer and beyond it.

The velocity profile is found by substituting this, with appropriate choices of the coefficients κ and the group $b=ak_1/(\kappa C_o^2)$, in equation (1) and integrating over the flow area.

EVALUATION OF THE MODEL

A preliminary examination of the form of equation (7) confirms that trends predicted by the model are qualitatively correct for the viscous sublayer, "buffer" region, and log layer. μ_t approaches zero as the wall is approached, so equation (2) correctly approaches $u^+=y^+$. Substitution of this into equation (7) confirms the theoretical requirement, first demonstrated by Reichardt (1951), that μ_t varies as y^3 as y approaches zero. (To the authors's knowledge, the present model is the first phenomenological model with this consequence. Other models use specially structured empirical functions to achieve an eddy viscosity dependence on y^3 as the wall is approached.) For large wall distances, equation (7) approaches $\mu_t/\mu=\kappa y^+$, thus yielding the correct logarithmic profile here. The transition between these limits is smooth, implying a "buffer" region which merges smoothly with the viscous sublayer and log-region limits.

Quantitative evaluation of the model, in which model predictions, with suitable choices of κ and the group $b\equiv(ak_1/\kappa C_o^2)$, are compared with experimental data, is performed below.

Conventional turbulent flow

The model describes conventional flow reasonably well with κ and b choices of 0.407 and 275. The conventional Karman coefficient value 0.407 is required to ensure the correct slope of the logarithm region of the velocity profile. Interestingly, the result b=275 is obtained if the proportionality coefficient k_1 is assumed to be 1.0, and the eddy drag coefficient equation is assumed to be C=24/Re + 0.46, a form which adequately describes the drag coefficient for solid spheres. Figures 1-4 compare predictions of the model using these coefficients and published data for velocity profile, turbulent viscosity, near-wall turbulent shear stress

$$\overline{u'v'}/(u^*)^2|_{r=R}=(1-\mu/\mu_t)^{-1},$$

near-wall turbulent energy production

$$(\mu/\rho)\overline{u'v'}/(u^*)^4(du/dy)|_{r=R}=(2+\mu_t/\mu+\mu/\mu_t)^{-1},$$

and near-wall dissipation

$$[(\mu/\rho\ u^{*2})(du/dy)]^2|_{r=R}=(1+\mu_t/\mu)^{-2}.$$

As can be seen, the agreement is good, although

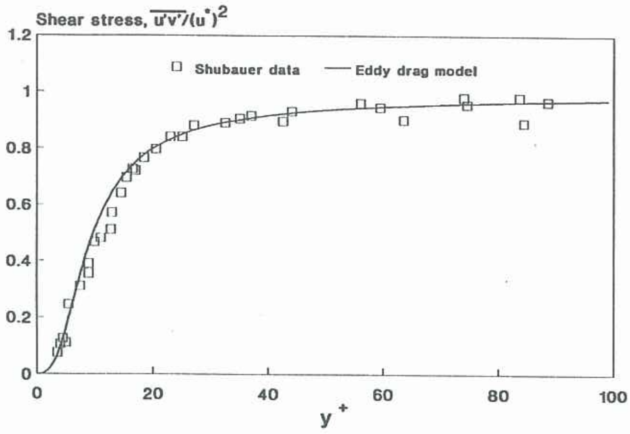

Fig. 3 Comparison of predicted and experimental turbulent shear stress distribution

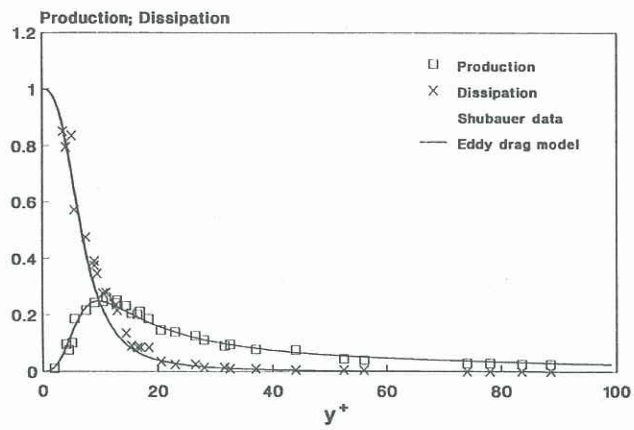

Fig.4 Comparison of predicted and experimental non-dimensional turbulence production and dissipation profiles

some discrepancies can be seen in the buffer region of the curves.

Unorthodox Turbulent Flows

The model has been further evaluated in terms of its compatibility with unorthodox friction factor and velocity profile data. A number of such data have been examined for this purpose. Many, but not all, of them can be described by the model with suitable choices of coefficients b and κ. The connection between these coefficients and the coefficients of the eddy drag equation of the model allows data to be interpreted in a way which may lead to improved understanding of the causes of unorthodox turbulent flow, and/or may provide a means of developing improved predictive equations for such flows. An example of each type of application of the model is given below.

In the first example, the effect of a magnetic field on a conducting fluid is considered. The present model allows the existing simple qualitative explanation of turbulence reduction due to the field to be quantified, so leading to simpler and more accurate predictions than those resulting from other published models of such flows. The second example considers the less-understood effect of drag-reducing additives. In this case, the model suggests a means by which the effect occurs.

Conducting fluid flowing parallel to a magnetic field. Liquid metal coolants proposed for fusion reactors are designed as far as possible to follow paths parallel to the reactor's magnetic field to reduce large magnetic pressure drops induced by the coolant conductor crossing field lines. Even so, the field opposes turbulent fluctuations normal to it, so turbulence is reduced.

Empirically (see eg the review of Lielausis 1975), magnetohydrodynamic effects increase solid object drag coefficients by a factor $1+gRe^hHa^i$, where Ha, the Hartmann number, is a dimensionless magnetic field and coefficients g, h and i depend on the shape of the solid body.

If a global rather than local view is taken of the above Reynolds number effect if applied to eddy drag, equation (6) implies that a magnetic field parallel to the flow can be allowed for by dividing the Karman coefficient by a factor of the form $1+gRe^hHa^i$. Empirically, the present model with b=275, as for the "universal" profile, and with $\kappa=0.407/(1+Re^{2.1}Ha^{-1.3})$, in line with the above suggestion, describes "fully turbulent" data from various sources for liquid metals flowing parallel to a magnetic field. Figures 5 and 6 compare predictions of the above with representative velocity profile and friction factor data. The agreement is better than that achieved by other published models of these flows (see eg Genin and Krashnoshchekova 1982).

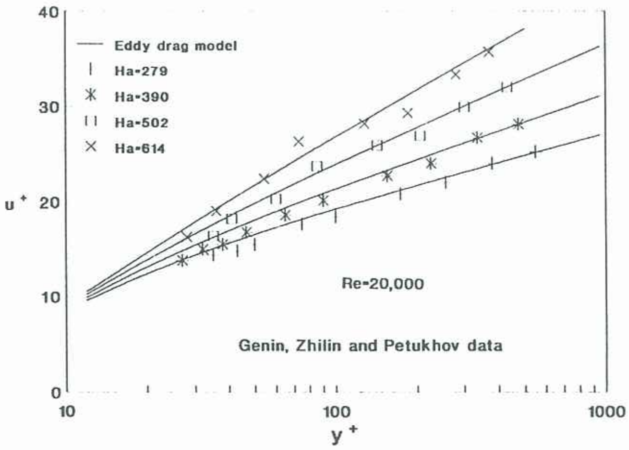

Fig.5 Comparison of predicted and experimental non-dimensional velocity profiles for mercury flowing parallel to a magnetic field

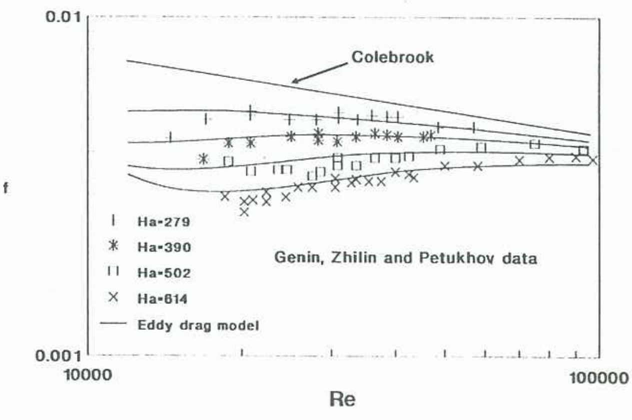

Fig. 6 Comparison of predicted and experimental friction factors for mercury flowing parallel to a magnetic field

Flows affected by drag reducing additives. In many cases, drag reducing additives shift velocity profiles in the log region above but parallel to the "universal" profile, extending the sublayer region in doing so (Virk 1975). The effect can be simulated with the present model by increasing the coefficient of the laminar component of the eddy drag coefficient. This is consistent with the empirical finding (Virk 1975) that the drag reducing mechanism, currently not understood, occurs in the sublayer region, and suggests additives increase the eddy viscous drag but not the eddy turbulent drag.

CONCLUDING DISCUSSION

Prandtl's mixing length theory has been simply extended so that a single form of velocity profile covers the viscous, buffer, and log region of velocity profiles. The derived form of eddy viscosity leading to the velocity profile equation behaves correctly at small and large wall distances, and is the only phenomenologically-developed form known to the author to achieve this. The model is compatible with conventional turbulent pipe flow data, and with many unorthodox turbulent pipe flow data. The model implies that the slope and intercept parameters of the log-layer are not decoupled, as assumed in conventional models. This result is consistent with conclusions of an examination of unorthodox gas-liquid flow velocity profile and friction factor data (Beattie 1977). The phenomenological basis of the model allows it to be used as a tool leading to improved prediction and/or interpretation of unorthodox turbulent pipe flows.

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