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# CONDUCTION BLOCKING EFFECTS IN STRATIFIED INTRUSION JETS

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## Abstract

When an initially isothermal and stationary fluid in a cavity is subjected impulsively to a horizontal temperature gradient hot and cold intrusions develop on the cavity ceiling and floor respectively. A divergence structure with an associated crest is observed to develop in the intrusions. It is suggested the divergence is a result of the changed temperature boundary conditions experienced by the intrusions as they travel onto the horizontal boundaries. In this paper the cold intrusion is simulated as a stratified jet in isolation from the remainder of the cavity flow. The aim is to test the hypothesis that it is the adjustment of the temperature distribution within the jet, via diffusion, that produces the divergence. For the cold jet, the transfer of heat away from the centre of the jet results in an adverse horizontal pressure gradient which acts to block the jet and produce the observed divergence.

## Introduction

When the walls of a cavity with insulated top and bottom, containing an initially stationary isothermal fluid, are instantaneously heated and cooled thermal boundary layers are generated traveling up and down the hot and cold walls respectively. This results in the ejection of intrusions into the cavity on the horizontal boundaries, hot on the upper boundary and cold on the lower boundary. It has been observed that regions of rapid divergence occur in the intrusions downstream of their corner of origin. Such features have been observed both experimentally and in numerical simulations (Patterson and Armfield, 1990). Figure 1 shows temperature contours in the cavity during the flow development, with the divergence features identified. As the flow continues to develop the core of the cavity fills with stratified fluid and the divergence reduces in size considerably. Although in the cavity flow the intrusion and associated divergence structures occur on both upper and lower boundaries, for the sake of clarity in the remainder of the paper, only the cold intrusion, that on the lower boundary, will be considered.

The flow travelling down the cold wall is in the form of a wall jet, generated by the entrainment of fluid from the interior of the cavity and the conductive transport of heat to the cold wall. As a result the wall boundary of the jet is maintained at the wall temperature, while the interior boundary is maintained at the cavity core temperature, resulting in an approximately exponential temperature profile in the region near to the wall. The corner at the bottom of the cold wall is a stagnation point and thus the wall jet is ejected into the cavity without any change in character, that is it retains the jet and thermal boundary layer features it has on the wall. After being ejected into the cavity the intrusion undergoes the divergence described above.

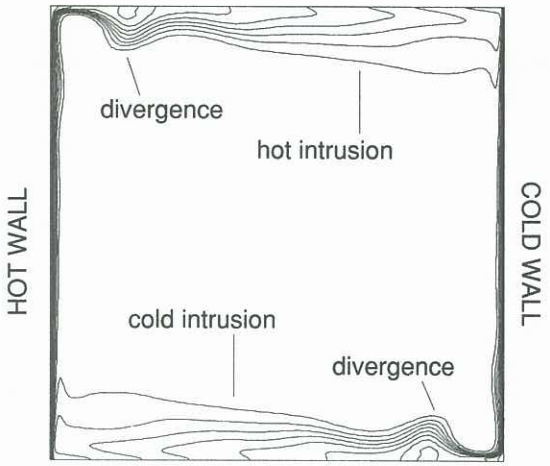


Fig. 1. Temperature contours in cavity.

A number of theories have been postulated to explain the occurrence of the crest or divergence region occurring in the intrusion. Patterson and Imberger (1980) followed by Schladow, Street and Patterson (1989) suggested that it resulted from the accumulation of vorticity in the corner region. Ivey (1984) followed by Paolucci and Chenoweth (1989) suggested that it was an internal hydraulic jump resulting from an internal densimetric Froude number of greater than one. Recently Patterson and Armfield (1990) suggested that it resulted from the change in boundary conditions resulting in a density blocking effect. The change in boundary condition would, it was suggested, result in a change in the internal the temperature field, via diffusion, resulting in the fluid in the centre of the jet downstream of the corner of origin being cooler then the fluid at the same height at the corner. This would result in a horizontal temperature gradient and resulting adverse pressure gradient blocking the fluid in the centre of the jet, which would

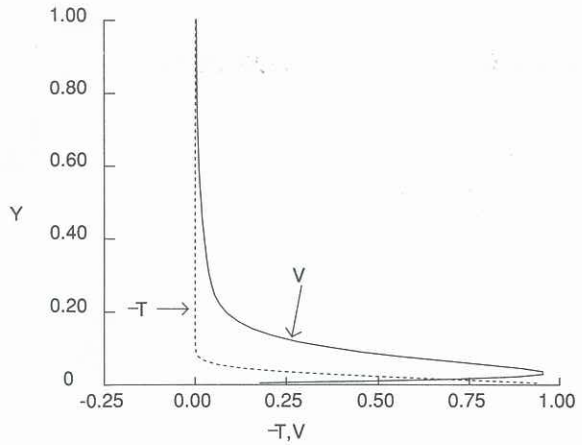


Fig. 2. Profiles of temperature $\times -1$ (dashed line) and velocity (solid line) at inlet.

spread out as a result, forming the observed structure.

The divergence structure is studied in detail in the present paper by isolating the intrusion from the remainder of the flow. This is accomplished by simulating the intrusion as a stratified jet, with the same velocity and temperature structure as the intrusion itself, ejecting into a cavity. In this manner it has been possible to demonstrate that the divergence is a local function of the intrusion and is generated by the conduction mechanism hypothesised by Patterson and Armfield (1990). The remainder of the paper is as follows. The numerical scheme is briefly described in the next section, results are presented for an isothermal jet, a stratified jet with non-zero temperature diffusivity and a stratified jet with zero temperature diffusivity. These results are discussed in the last section.

## Numerical Method

The Navier-Stoke equations, together with the temperature advection-diffusion equation, are expressed in non-dimensional form in cartesian coordinates $(x, y)$ with corresponding velocity components $(U, V)$ and $P$ the pressure as follows,

$$U_t + UU_x + VU_y = -P_x + \frac{1}{Re}(U_{xx} + U_{yy}),$$

$$V_t + UV_x + VV_y = -P_y + \frac{1}{Re}(V_{xx} + V_{yy}) + \frac{Gr}{Re^2},$$

$$U_x + V_y = 0,$$

$$T_t + TV_x + TV_y = \frac{1}{RePr}(T_{xx} + T_{yy}),$$

where subscripts indicate partial differentiation and the Boussinesq assumption for buoyancy has been made. The Grashof number is defined as,

$$Gr = g\beta H^3(\Delta T)/\nu^2,$$

where $g$ is the acceleration due to gravity, $\beta$ the coefficient of thermal expansion, $H$ the domain height, $\nu$ the kinematic viscosity and $\Delta T$ the total temperature variation in the domain, that is the free-stream temperature minus the lowest temperature in the intrusion jet. The Prantdl number is $Pr = \nu/\kappa$, where $\kappa$ is the thermal diffusivity. In these equations velocity is non-dimensionalized by the inlet velcoity, length by $H$ and the temperature relative to the free stream temperature by $\Delta T$.

The equations are discretized using an implicit second order time accurate method defined on a non-staggered mesh in which all the variables are stored at the same grid locations. The method of obtaining the pressure and satisfying continuity is similar to the SIMPLE schemes used with the conventional staggered mesh (Patankar, 1980). To enable such an approach to be used with a non-staggered mesh, without the occurrence of a grid scale oscillation in the pressure field, elliptic correction terms are included in the continuity equation (Armfield, 1990). Finite volumes are used to convert differential terms in the governing equations as follows. All second derivative, pressure gradient and divergence terms are approximate using second order centred differences. The advective terms are approximated using QUICK (Leonard, 1979). The domain is discretized using approximately 100 points in the $x$ direction and 50 points in the $y$ direction, with grid stretching being used to concentrate points in the region of the jet.

Boundary values are specified for the four sides of the domain as follows. The pressure on all the boundaries is obtained via a second order extrapolation from the interior of the domain. At the left hand boundary, the entrance, the velocity and temperature profiles are set as shown in Fig. 2. Thus the jet is approximately parabolic with a stable exponential temperature in its lower half and a constant temperature in its upper half. The velocity and temperature profiles were chosen to match those of the cold intrusion in the cavity flow described above. On the top boundary the normal derivatives of $T$ and $U$ are set to zero, while $V$ is set to zero. At the right hand boundary, the exit, the normal derivatives of $U$, $V$ and $T$ are set to zero. The bottom is a non-slip insulated boundary and therefore $U$ and $V$ are set to zero, with the normal derivative of $T$ set to zero.

Initially the fluid in the cavity is stationary and isothermal. At time $t = 0$ the entrance velocity and temperature profiles are switched on and the resulting jet is observed to travel across the domain. Results shown in the next section were obtained at approximately the same time as the cavity temperature contours shown above. In the remainder of the paper distances and locations will be given in non-dimensional terms with respect to the origin at the bottom left corner of the computational domain.

## Results

Results are presented for an isothermal jet with the inlet velocity profile shown in Fig. 2, a stratified jet with the inlet velocity and temperature profiles shown in Figures 2 and 8, and a jet with the same velocity and temperature inlet profiles, but with zero temperature diffusivity. The Reynolds number for all three cases considered is $Re = 160$, while the Grashof number for the non-isothermal cases is $Gr = 2.9 \times 10^6$. For the stratified jet with non-zero temperature diffusivity the fluid is assumed to be water with Prandtl number $Pr = 7$. For the stratified jet with zero temperature diffusivity the parameters are identical but the diffusivity is set to zero. For the two stratified jets the internal Froude number based on the jet width and stratification at the entrance is $Fr = 0.7$. This is less than 1.0 and the intrusion flow is therefore subcritical and would not be expected to undergo a hydraulic jump.

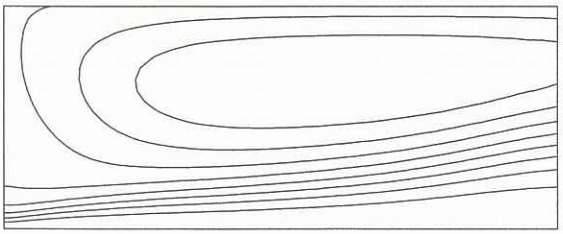

Fig. 3. Streamlines for the isothermal flow.

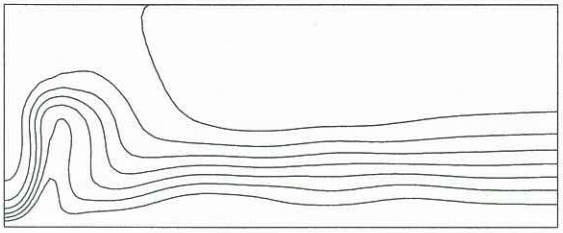

Fig. 4. Streamfunction contours for the stratified jet with non-zero temperature diffusivity.

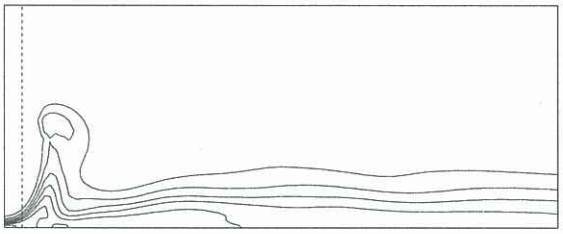

Fig. 5. Temperature contours for the stratified jet with non-zero temperature diffusivity, with the location of the $x = 0.08$ profile presented in Fig. 8 shown as the dotted line.

The streamfunction contours for the isothermal jet, shown in Fig. 3, exhibit the expected characteristics of a laminar jet, with viscous spreading and entrainment. In this case there is no indication of the divergence region observed in the cavity intrusion flow. This behaviour may be contrasted with that of the stratified jet with non-zero temperature diffusivity, for which streamfunction contours are shown in Fig. 4 and temperature contours in Fig. 5. For this case the jet has developed a significant crest immediately downstream of the entrance, several times the height of the jet itself. This structure is also seen in the temperature contours, and its location and general form is seen to correspond well to that observed in the cavity intrusion flow. It is observed that the formation of the divergence structure has lead to small regions of potential instability in the flow. Analysis of the cavity convection flow at higher Grashof numbers indicates that the divergence structure may act to trigger turbulence, possibly as a result of the initial development of unstable regions as observed here.

When the same stratified jet is run, but with zero temperature diffusivity, no divergence structure is observed in

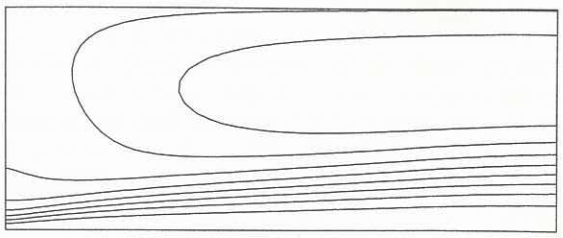

Fig. 6. Streamfunction contours for the stratified jet with zero temperature diffusivity.

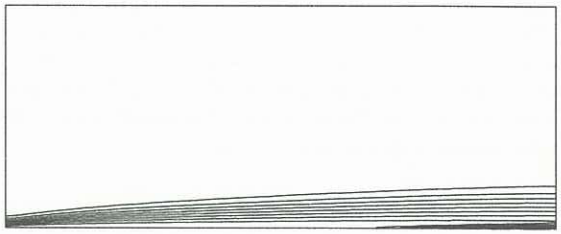

Fig. 7. Temperature contours for the stratified jet with zero temperature diffusivity.

the streamfunction contours, or in the temperature contours. The streamfunction contours for this case are presented in Fig. 6 and the temperature contours in Fig. 7. The streamfunction contours indicate that the jet is now behaving in approximately the same manner as the isothermal flow, but with the stable stratification acting to slightly damp the viscous spreading observed previously. It should be noted that this flow has exactly the same Froude number and buoyancy as the previous flow, the only change being to the diffusivity, however, this change has completely eliminated the divergence structure.

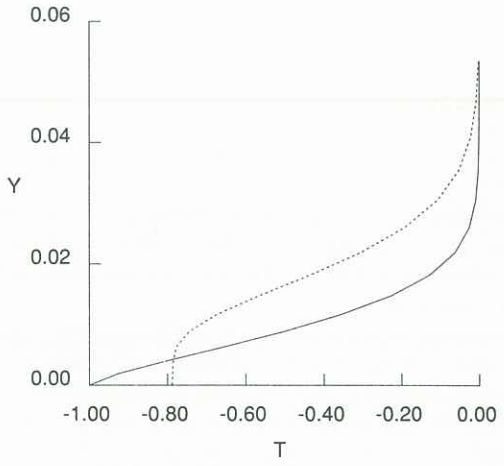


Fig. 8. Temperature profiles in the stratified jet with non-zero temperature diffusivity at the entrance (solid line) and at $x = 0.08$ downstream of the entrance (dashed line).

Figure 8 shows the temperature profile at the entrance and at $x = 0.08$ downstream of the entrance, for the strat-

ified jet with non-zero temperature diffusivity. For clarity this location is shown as the dotted line in Fig. 5. The adjustment made by the temperature profile to suit the changed boundary condition is quite clear at the $x = 0.08$ location. The normal profile resulting from the insulated boundary condition has diffused some distance out into the jet at the downstream location, resulting in a significant increase in the temperature at the boundary, with an associated decrease in the temperature above the point where the two profiles cross over. Heat has also been transported from the middle to the top of the jet as a result of diffusion along the profile in the region from $y = 0.007$ to $y = 0.04$. The result of these processes is to reduce the temperature in the middle region of the jet, that is the interval from $y = 0.004$ to $y = 0.04$, where the temperature at the $x = 0.08$ location is significantly less than at the entrance. A region from $y = 0$ to $y = 0.004$ is also produced in which the temperature at the $x = 0.08$ location is greater than it is at the entrance. The significance of this will be discussed in the next section.

## Discussion

From the results presented in the previous section it is clear that the divergence structure observed in the cavity convection flow is a locally generated feature of the intrusion and is therefore not dependent on other features of the cavity flow associated with the walls or the ceiling. Thus the divergence can be generated when the intrusion only is simulated as a stratified jet with the same velocity and temperature structure as the cavity intrusion.

The generation of the divergence structure however is entirely dependent on the diffusion of heat within the jet and the resultant interaction of the velocity and temperature via the buoyancy, as it was only observed in the case of a stratified jet with non-zero temperature diffusivity. As has been noted for the non-zero temperature diffusion flow, once the jet enters the computational domain the temperature profile immediately begins to adjust, via diffusion, as a result of the changed boundary conditions, at both the top and bottom of the jet. This results in heat being transported away from the the centre of the jet, producing the blocking mecahnism described in the introduction. As was seen in the previous section a region occurs close to $y = 0$ in which the fluid at the downstream location is warmer than it is at the entrance, which could be expected to lead to an acceleration of the flow in the immediate vicinity of the boundary, effectively splitting the jet around the blocking region. This is not observed, and it is hypothesised that such a splitting is prevented by viscous effects. It has however been noted that when the Prandtl number is set to 1, so that temperature diffusion and viscosity are equal, such splitting does occur.

The crest associated with the divergence in the jet flow is considerably larger than the equivalent structure in the cavity instrusion flow. The most likely reason for this is that the other features of the cavity flow, removed when the intrusion is isolated as an intrusion jet, act to damp this mechanism. Thus it is likely that the seiche generated by the intrusion striking the far wall (Patterson and Armfield, 1990), as well as the resulting process of the intrusion filling the cavity core with stratified fluid and the entrainment of fluid by the wall jet, will act to reduce the effective magnitude of the divergence crest.

The blocking mechanism considered here in the context of the intrusions associated with the cavity convection flow, and in a jet with the same temperature and velocity structure as the cavity intrusions, could also exist in other flows. Thus whenever a stratified flow enters a region where, due to a change in boundary conditions, the temperature profile is likely to undergo a rapid and significant adjustment, it is likely that a mechanism of this type will have an effect on the flow field. It is also possible to hypothesise situations in which the change in temperature profile will have the opposite effect to that observed here, that is to accelerate the flow in the interior of the jet and produce a convergence. For instance such would be the case in the the present flow if the jet had an unstable rather than a stable temperature stratification.

Further work on this mechanism will consist of developing analytic tools to predict the location and magnitude of the blocking effect and resultant divergence, or convergence as noted above, for a given velocity and temperature structure, and to find other situations in which this phenomenon may occur.

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