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OPTIMIZATION OF PARTICLE IMAGE VELOCIMETRY USING CROSS-CORRELATION ANALYSIS WITH APPLICATION TO HIGH RESOLUTION TURBULENT PIPE FLOW

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ABSTRACT

The performance of PIV in measuring instantaneous velocity fields can be improved by using cross-correlation analysis of image fields in place of auto-correlation methods. By knowing the image shifting in double-exposure recordings and by an *a priori* estimate of the mean flow-field locally, the cross-correlation of different size interrogation spots of double-exposure images with known separation can be optimized in terms of spatial resolution, detection rate, accuracy and reliability. Optimal system parameters are recommended, for a range of velocity fields to eliminate signal bias and to minimize the loss of signal strength.

The cross-correlation algorithm, executed on a 640 MFlop computer-based system, utilizes high resolution image acquisition equipment and eight parallel array processors to achieve high input and data processing rates necessary for analysis of very large sets of PIV images. The architecture, algorithms and performance characteristics of the system are described and applied to fully-developed turbulent pipe flow using adaptive windowing techniques.

INTRODUCTION

Particle Image Velocimetry (PIV) measures instantaneous velocity fields in experimental fluid mechanics by measuring the displacement of images of particles moving with the fluid flow. The analysis of images to measure particle displacements is dependent upon the density of recorded images with many approaches proposed and explored experimentally. In experiments with high image density, correlation methods can be used to obtain regular arrays of velocity vectors averaged over small areas, the interrogation spots. Keane and Adrian (1990,1991) have performed theoretical and numerical simulations to optimize experimental parameters on double-pulse and multiple-pulse PIV systems, where auto-correlation analysis is based on digital Fast Fourier Transforms of digitized image data.

As CCD cameras have become a useful tool in recording single images on successive frames, cross-correlation analysis of pairs of image frames yields image displacements, (Kimura and Takemori, 1986; Willert and Gharib, 1991). Furthermore, in cases of double-pulse single-exposure recordings where estimates of local mean flow are available, cross-correlation analysis of two different regions within the frame can yield advantages over auto-correlation analysis, (Utami *et al.*, 1991). In this present work, a theoretical analysis of the mean value of the cross-correlation function of particle image fields is given for velocity fields with locally linear velocity variation and a Monte Carlo simulation is used to estimate the properties of the cross-correlation function. The spatial resolution, the detection probability and the accuracy of the in-plane velocity measurements determine the performance of the cross-correlation method and are in turn determined by experimental recording and interrogation parameters. Optimal system parameters are recommended for a range of velocity fields in order to eliminate signal bias and to minimize the loss of signal strength (i.e. maximize the detection probability).

The cross-correlation algorithm is executed on a computer system that employs eight parallel array processors with a combined theoretical peak speed of 640 MFlops. Each array processor uses an Intel i860 micro-processor with a theoretical peak speed of 80 MFlops. A VIDEK Megaplus CCD camera and an Imaging Technology VSI 150 frame grabber digitizes 1024 x 1024 pixel images. The high image resolution and array processing speed is necessary for processing large image fields in relatively small amounts of time. For example, a single high-image-density double-pulse photograph of fully-developed turbulent pipe flow, with Reynolds number $Re_D = 50,000$, was interrogated in less than 3 minutes, yielding over 14,000 vectors.

SPATIAL CROSS-CORRELATION ANALYSIS

By extending the theory described by Adrian (1988) and Keane and Adrian (1990,1991) for the analysis of the auto-correlation function, the cross-correlation function $C(\mathbf{s})$ of two particle image fields $I_1(\mathbf{X})$ and $I_2(\mathbf{X})$ is defined by

$$C(\mathbf{s}) = \int I_1(\mathbf{X}) I_2(\mathbf{X}+\mathbf{s}) d\mathbf{X} \qquad (1)$$

where $I_1(\mathbf{X})$ and $I_2(\mathbf{X})$ are the transmitted light intensities from two particle image fields from either separate single-exposure recordings such as a video frame or subsets of a larger double-exposure single-frame image field. The cross-correlation function C of double-exposure frames can be decomposed into five components, similar to the auto-correlation function R.

$$C(\mathbf{s}) = C_C(\mathbf{s}) + C_F(\mathbf{s}) + C_{D^+}(\mathbf{s}) + C_{D^-}(\mathbf{s}) + C_P(\mathbf{s}) \qquad (2)$$

where C_C is the convolution of the mean intensities, C_F is the fluctuating noise component, C_{D^+} and C_{D^-} are the positive displacement and negative displacement peaks and C_P is the self correlation peak. The components of the spatial auto-correlation function and the cross-correlation function for a constant velocity field are shown in Fig. 1 and illustrate the improved signal peak for cross-correlation.

The statistical properties of the cross-correlation function are determined for a given velocity field $\mathbf{u}(\mathbf{x})$ by conditionally averaging equation (2) over an ensemble of realizations containing different sets of randomly located particles. The important parameters of $< C(\mathbf{s}) \mid \mathbf{u} >$ are determined by choosing practical models for the experimental parameters in analyzing the components of $< C(\mathbf{s}) \mid \mathbf{u} >$ for a range of velocity fields.

The light sheet pulses of amplitude I_0 are modelled as Gaussian or top-hat functions, the interrogation beam intensities, A_1, A_2, are modelled as Gaussian functions or top-hat functions of diameters d_{I1} and d_{I2} respectively while all particle image transmissivities are modelled as Gaussian functions of diameter, d_τ.

For constant or slowly varying velocity fields, the mean displacement component can be written as

$$< C_{D^+}(\mathbf{s}) > = \frac{1}{\pi} A_1 A_2 I_0^2 \frac{\tau_{00}^2}{d_\tau^2} \{N_I^* F_I(\mathbf{s}) F_O(\Delta z)\} \exp\{-4 \,|\, \mathbf{s} - M \Delta \mathbf{x} \,|^2 / d_\tau^2\} \tag{3}$$

where τ_{00} is an average photographic transmissivity, M is the image magnification, N_I^* is a weighted average of the image densities of the two spots N_{I1}, N_{I2}, and F_O and F_I measure the reduction in signal strength due to loss-of-image pairs due to out-of-plane and in-plane motion respectively. As $N_I^* F_I F_O$ represents the mean effective number of image pairs weighted by the image intensity contributing to $< C_{D^+}(\mathbf{s}) >$, the loss of pairs can be minimized by choosing a relative window separation, $\mathbf{X}_{I2} - \mathbf{X}_{I1}$, to maximize F_I and a change of second light sheet thickness and position to maximize $F_O = 1$. By selecting d_{I2} sufficiently larger than d_{I1}, all particles with images in the first image spot have images again in second spot so that $F_I = F_O = 1$. In such a case the signal strength is maximized to be

$$< C_{D^+}(\mathbf{s}) \,|\, \mathbf{u} > \;=\; \frac{1}{\pi} A_1 A_2 I_0^2 \frac{\tau_{00}^2}{d_\tau^2} N_{I1} \tag{4}$$

when $\mathbf{s} = M \Delta \mathbf{x}$ as shown in Fig. 1. Due to the generally sparse nature of particle images in high-image-density PIV in comparison to speckle velocimetry, the estimate $< C_C(\mathbf{s}) \,|\, \mathbf{u} >$ is an unreliable one for determining the average peak noise amplitude, so that Monte Carlo simulation is more reliable.

When the velocity variations within the interrogation volume cannot be ignored, spatial cross-correlation of single-exposure and double-exposure double-pulse PIV can reduce and eliminate velocity gradient bias which in auto-correlation analysis, biased the location of the centroid of $< R_{D^+} >$ towards the displacement of lower speed particles. By choosing the size, d_{I2}, and the relative spot shift, $\mathbf{X}_{I2} - \mathbf{X}_{I1}$, of the second interrogation spot to ensure that there is no loss of image pairs due to in-plane motion, gradient bias due to in-plane motion is eliminated because no images of more rapidly moving particles are lost.

MONTE CARLO SIMULATION

Although the analysis of the mean cross-correlation function reveals the important parameters to maximize mean signal strength, it needs to be complemented by a Monte Carlo simulation of the PIV image field and interrogation analysis to address questions of probability of signal measurement and the fluctuation in the statistics of individual velocity measurements, particularly the instantaneous noise amplitude and signal-to-noise ratio. The simulation which has been detailed by Keane and Adrian (1992) has been performed for three-dimensional velocity fields over a range of three-dimensional linear velocity gradients.

In the case of a constant or variable velocity field, the mean effective number of image pairs $N_I F_I F_O$ can be maximized by choosing $F_I = F_O = 1$. By comparing the valid detection probability for a range of image displacements, $|\Delta \mathbf{X}| / d_{I1}$, of constant three dimensional velocity fields and a range of interrogation spot sizes and separations for cross-correlation analysis of single-exposure and double-exposure images, it can be seen in Fig. 2 that the valid detection probability depends upon the mean effective image density $N_I F_I F_O$. To ensure a 95% valid detection rate for cross-correlation of single-exposure and double-exposure images, it is sufficient to choose $N_{I1} > 6$ and $N_{I1} > 8$ respectively provided $(\mathbf{X}_{I2} - \mathbf{X}_{I1})$ and d_{I2} ensure $F_I = 1$, which is an improvement over double-pulse auto-correlation results, (Keane and Adrian, 1990, 1991).

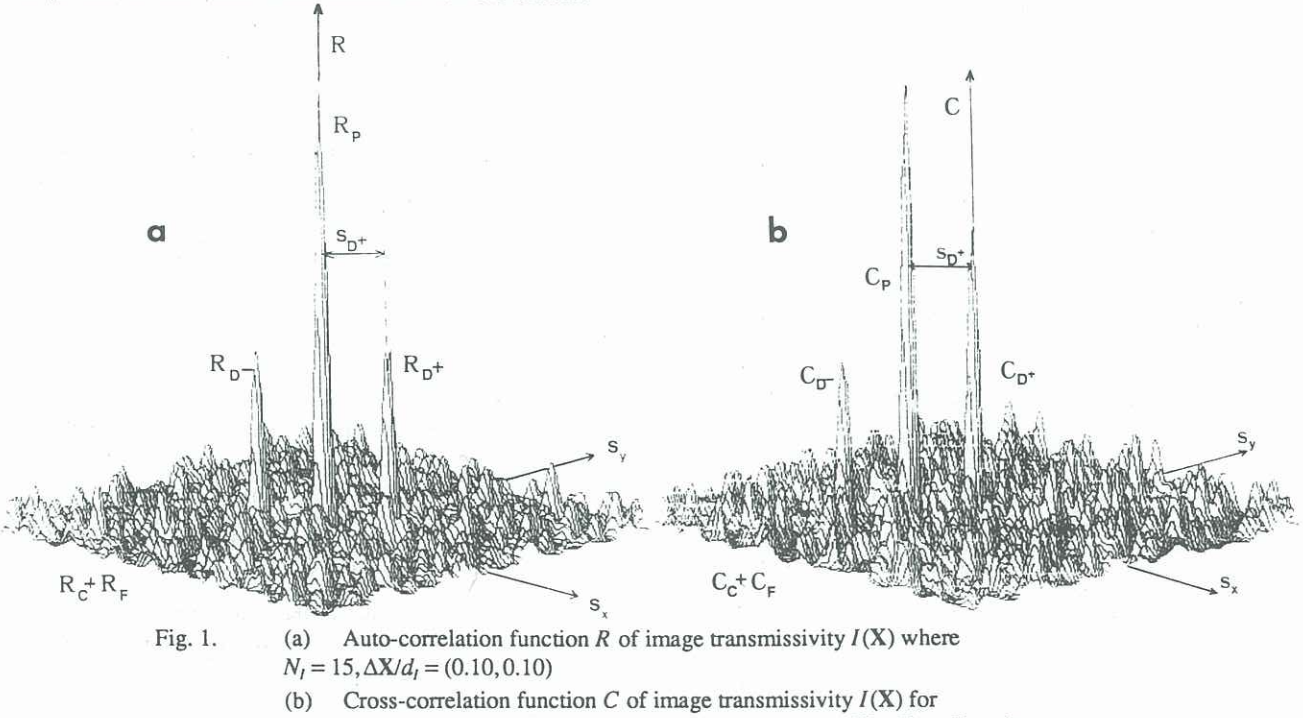

Fig. 1. (a) Auto-correlation function R of image transmissivity $I(\mathbf{X})$ where $N_I = 15, \Delta\mathbf{X}/d_I = (0.10, 0.10)$

(b) Cross-correlation function C of image transmissivity $I(\mathbf{X})$ for double-exposure frames with optimal window separation $\Delta\mathbf{X} = \mathbf{X}_{I2} - \mathbf{X}_{I1}$ where $N_{I1} = 15, \Delta\mathbf{X}/d_{I1} = (0.10, 0.10)$

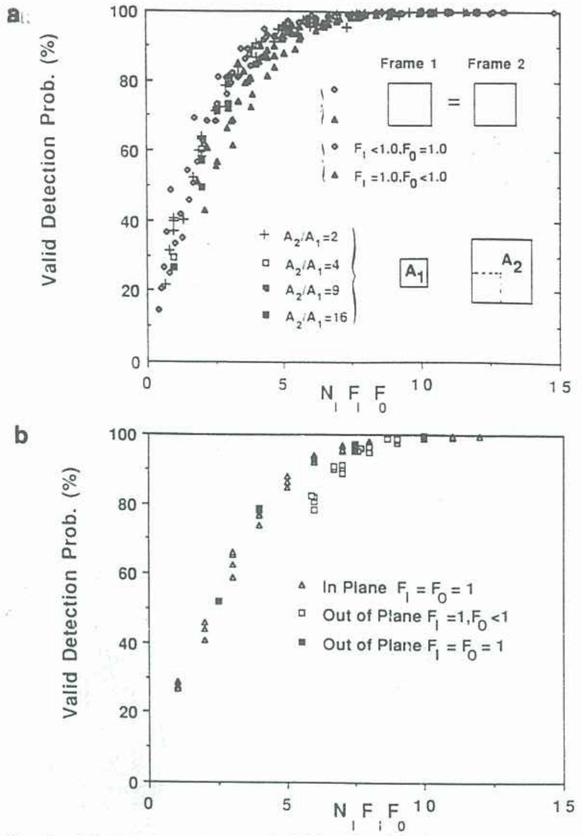

Fig. 2. Valid detection probability for (a) single-exposure and (b) double-exposure cross-correlation analysis as a function of effective image density $N_I F_I F_O$

Velocity gradients diminish and broaden the mean cross-correlation peak $< C_{D^+} >$ necessitating higher particle image densities to overcome peak splintering in individual realizations. In order to compare different correlation methods the simplest case of a velocity gradient, a simple shear in which $\partial u/\partial y \neq 0$, has been examined for variable interrogation spot sizes. Figure 3 illustrates the effect of peak splintering upon detection probability for cross correlation of single- and double-exposures. The mean effective image density N_{II}, must be larger than 10 for double exposure images and larger than 7.5 for single exposure images to achieve an acceptable detection probability of 95%, (assuming $F_I = F_O = 1$ by suitable interrogation spot choices). Thus there is less spatial resolution possible with a double-exposure image for a given fluid flow as the first interrogation spot size cannot be reduced to the same extent as for single-exposure images. In-plane velocity gradient bias can be removed in cross-correlation analysis by choosing d_{I2} sufficiently large so that $F_I = 1$, as shown in Fig. 4 for a simple shear where F_I increases as d_{I2} is increased and the second spot location is optimized. The measured velocity component u_m relative to the known velocity u_I is shown also for auto-correlation analysis in which case $F_I = 0.75$, for $|\Delta \mathbf{X}|/d_{II} = 0.2$.

To optimize the PIV system performance using cross correlation analysis for double-exposure images, the following broad criteria are suggested. Choose (a) $N_{II} > 10$ and $F_O = F_I = 1$, (b) $M\,|\Delta \mathbf{u}|\,\Delta t/d_{II} < 0.03$, (c) $M\,|\Delta \mathbf{u}|\,\Delta t/d_\tau < 1$ and (d) $1.0 < D_0 < 1.2$.

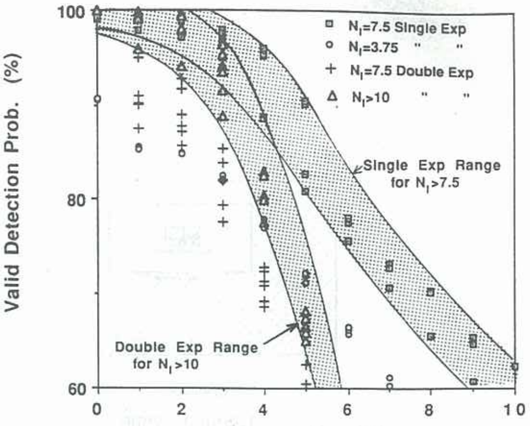

Fig. 3. Valid detection probability in terms of relative variation of image displacements $M\Delta u_y \Delta t/d_{II}$ for a plane shear flow with single-exposure and double-exposure imaging.

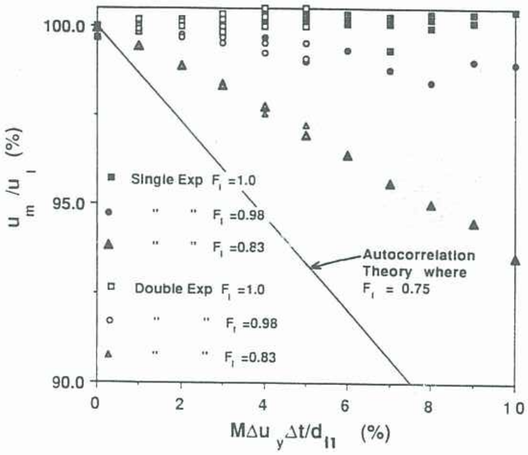

Fig. 4. Relative measured mean velocity *vs.* relative variation of image displacements for a range of cross-correlation window choices.

APPLICATION TO TURBULENT PIPE FLOW

Interrogations using cross-correlation techniques with varying interrogation spot sizes have been carried out for high image density double-pulse photographic recordings of fully developed turbulent pipe flow with $Re_D = 50{,}000$. The light sheet spanned the pipes radial streamwise plane illuminating $1\mu m$ olive oil particles which were imaged onto a 100mm × 125mm photograph with a magnification M of six.

During interrogation, a small region of the photograph is imaged onto the CCD camera and digitized by the frame grabber. This 1024 x 1024 pixel image, referred to as a "camera frame", is divided into eight smaller sub-images known as "processor images" (nominally 512 x 256, see Fig. 5). Each processor image is sent to a single array processor to be analyzed in parallel. Each array processor further divides its image into smaller cross-correlation windows.

The correlation windows were chosen to be rectangular, 128 x 96 pixels to provide higher spatial resolution in the wall normal direction than the streamwise direction, with zero padding used to make the window 128 x 128 pixels. The relative window offset is continuously varied during interrogation to maintain signal strength by eliminating loss of in-plane image pairs. Figure 6 illustrates the instantaneous in-plane velocity vectors and the fluctuating velocity vectors determined by removing line averaged mean velocities, where spatial resolution of 160 μm resolves the smallest scale motion in the flow.

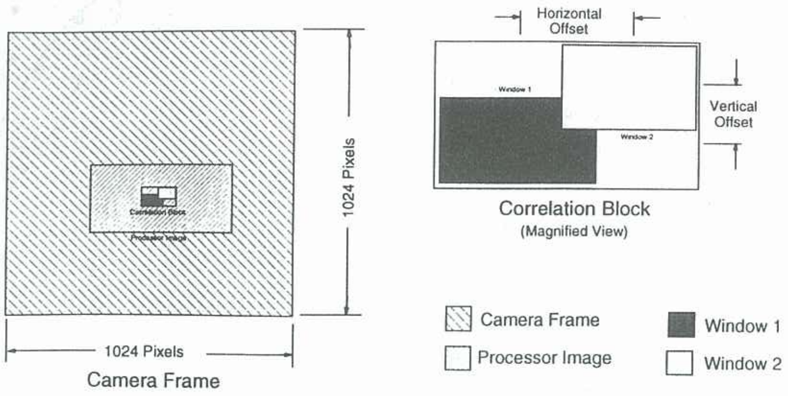

Fig. 5. Cross-correlation interrogation spots within the camera frame for a particular array processor.

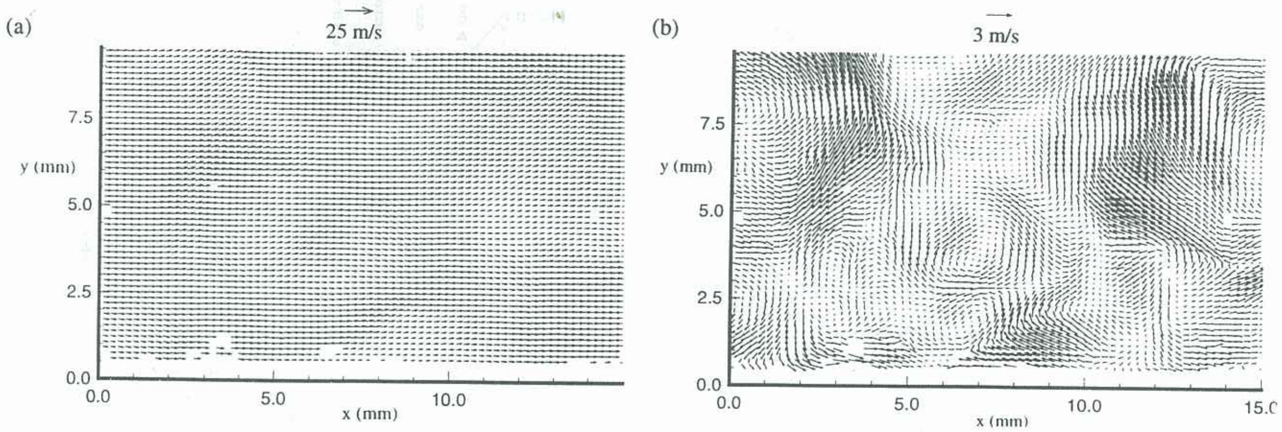

Fig. 6. Instantaneous velocity vector field of pipe flow with $Re_D = 50,000$ (a) Total velocity, (b) Fluctuating velocity.

CONCLUSIONS

Theoretical modeling and Monte Carlo simulation of cross-correlation analysis of PIV systems show that the increase in signal-to-noise ratio, the elimination of in-plane velocity gradient bias and the improvement of dynamic range of measurements provide advantages over auto-correlation analysis. A PIV interrogation of fully developed turbulent pipe flow has illustrated the feasibility of a high speed high resolution parallel processing system using cross-correlation of photographic images.

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