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MACH REFLECTION OF A WEAK PLANE SHOCK WAVE

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ABSTRACT

Oblique reflection of a weak shock wave was investigated. A modified three-shock theory was introduced to explain the well-known von Neumann paradox for weak Mach reflection. The effect of divergence of slipstream behind the triple point was taken into account. The angle of divergence is given parametrically to calculate some characteristics around the triple point, e.g. the angle of reflection. Numerical results were compared with experiment and characteristics of solutions was examined. It is found that for weak Mach reflection, the modified three-shock theory gives physically realistic solutions, even when von Neumann's three-shock theory has no solution. All the experimental data were found to exist in the domain given by the modified three-shock theory proposed here.

NOTATION

I	incident shock
M_j	Mach number of the flow in region (j) in the reference frame attached to the triple point
Ms	incident shock Mach number: $Ms = M_1 \sin\gamma_i$
M	Mach stem
p_j	pressure in region (j)
R	reflected wave
S	slipstream
T	triple point
γ_i, γ_m	angle of incidence
γ_r	angle of reflection
δ	angle of divergence of slipstream
ε	pressure difference between the flows behind the reflected shock and Mach stem
θ_i	deflection angle across the incident shock
θ_m	deflection angle across the Mach stem
θ_r	deflection angle across the reflected shock
θ_w	reflecting wedge angle
κ	ratio of specific heats
χ	angle of triple point trajectory

INTRODUCTION

When a planar shock wave encounters a sharp wedge in a shock tube, a reflected wave develops. The overall wave configuration is roughly classified into two categories, i.e., regular reflection and Mach reflection (see, for example, von Neumann, 1963; Hornung, 1986; Ben-Dor, 1988). In regular reflection, the intersection of incident and reflected waves coincide with the wedge surface. In Mach reflection, an intersection of incident and reflected waves (triple point T) is above the wedge surface, and a third shock wave called Mach stem M and a slipstream S appear (see Fig. 1). If the wedge surface is smooth and plane, the wave configuration is determined by the incident shock Mach number Ms and the reflecting wedge angle θ_w, as well as a thermodynamic property of the medium (say, the ratio of specific heats κ). In unsteady Mach reflection, it is usually assumed that the triple point T moves with constant velocity M_T along a straight line making an angle χ with the wedge surface. Then the flow fields relative to the triple point can be regarded pseudosteady, and not only the overall wave geometry but the flow field is self-similar about a point where the incident shock is reflected for the first time (e.g., a tip of the wedge).

In these flow fields near the triple point, von Neumann's three-shock theory (von Neumann, 1963) can be applied, and in the case of strong shock waves in air (Ms>1.48) the results are in good agreement with experiments. However, for weak shock waves (Ms<1.48), there is large discrepancy between solutions from the three-shock theory and experimental data. For Ms<1.25, there are no physically realistic solutions for Mach reflection. On the other hand, for 1.25<Ms<1.48, the theoretical results agree with experimental ones for lower angle of incidence. However, as the angle of incidence increases, the three-shock theory tends to have no realistic solution(Sakurai et al.,1989; Colella and Henderson, 1990). If we interpret this phenomenon in terms of angles of incidence and reflection, experimental results exist in the domain of (γ_i, γ_r)-plane where no solution is possible theoretically (N.B. The symbols γ_i and γ_r denote angles of incidence and reflection, respectively). These discrepancies are referred to as the von Neumann paradox. Many efforts have been made on the solution of this paradox in the past forty years. In his earlier work, Sakurai (1964) had pointed out the divergence of slipstream as one of the causes of the paradox. Dewey et al.(1985) also suggested the divergence of slipstream from his experimental works. But it has not been solved perfectly (Sakurai et al.,1989).

In the present paper, we introduce a modified three-shock theory by assuming a fraction of slipstream divergence behind the triple point, and examine the characteristics of the solutions obtained by the modified theory. The numerical results based on this theory are compared with experimental data.

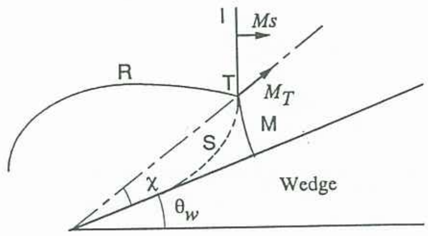

Fig.1. Mach reflection of a plane shock over a wedge

EXPERIMENT

The experiment was performed in a conventional shock tube which had 65x30mm rectangular cross section. Test section had a pair of 62x94mm optical viewing windows on both sides. The working gas was air and the condition ahead of the incident shock was set at room temperature and atmospheric pressure. Wave systems were photographed with a schlieren apparatus. The angles of incidence, reflection and flow deflection were measured directly from the negative filmes, using a profile projector (V-12, Nikkon Inc.). The contact surface and all the waves except incident shock are curved near the triple point so there was some uncertinity about where to draw the tangents at this point for the angle measurement. The trajecory angle χ was measured as indicated in Fig. 1, that is on the assumption of pseudo-steadiness. Measured angles were corrected by comparing the photographic image of the shocks with that of circular protractor which was taken at the test section.

MODIFICATION OF THE VON NEUMANN'S THREE-SHOCK THEORY

Von Neumann's three-shock theory is a simple application of oblique shock wave relations to three plane shocks (i.e., incident and reflected shocks and a Mach stem) intersecting at a triple point, under the assumption of pseudosteadiness of the flow field. The flow field relative to the moving triple point, whose Mach number is denoted by M_1, is illustrated in Fig. 2(a). The incident and reflected shocks, the Mach stem, and the slipstream are denoted by I, R, M and S, respectively. Therefore, the incident shock I propagates perpendicular to its front at the Mach number of Ms, where $Ms = M_1 \sin\gamma_i$. The flow field in the vicinity of the triple point is divided into four regions: (1) the region ahead of the incident shock I and the Mach stem M, (2) the region between the incident shock I and the reflected shock R, (3) the region bounded by the reflected shock R and the slipstream S, (4) the region bounded by the Mach stem M and the slipstream S. The sign of the flow

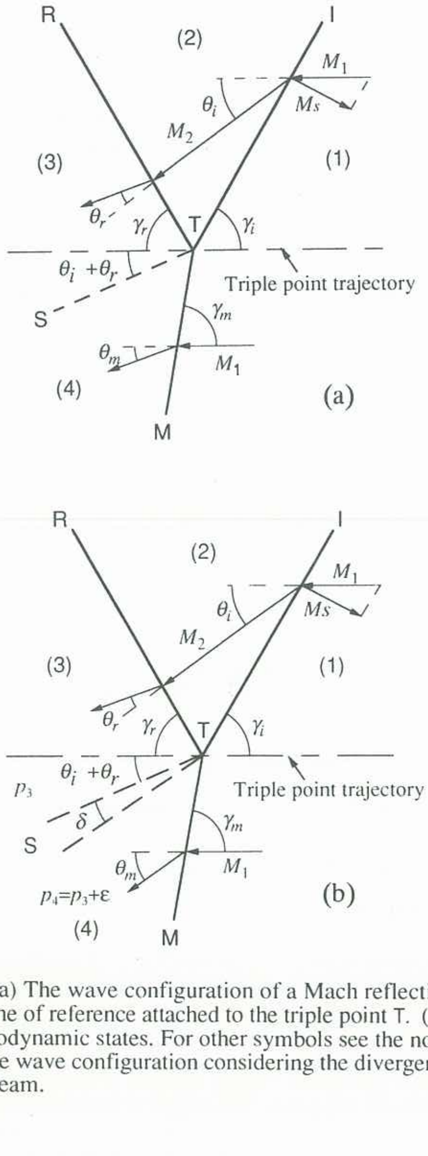

Fig.2. (a) The wave configuration of a Mach reflection from a frame of reference attached to the triple point T. (1)-(4), thermodynamic states. For other symbols see the notation. (b) The wave configuration considering the divergence of slipstream.

deflection angle across a discontinuity (or shock front) is consistently defined; that is, counterclockwise deflection is positive, clockwise negative.The flow across the incident shock is deflected counterclockwise by θ_i and this flow is deflected in turn by θ_r when crossing the reflected shock. Therefore, the net flow deflection angle relative to the incident flow is $\theta_i + \theta_r$. On the other hand, the flow across the Mach stem is deflected by θ_m. When the oblique shock relations are applied to these adjacent regions under the assumption that the fluid is a perfect gas, we have the following relations (Ikui and Matsuo, 1983);

$$\frac{p_2}{p_1} = \frac{2\kappa M_1^2 \sin^2\gamma_i - (\kappa-1)}{\kappa+1}, \tag{1}$$

$$\frac{p_3}{p_2} = \frac{2\kappa M_2^2 \sin^2(\gamma_r + \theta_i) - (\kappa-1)}{\kappa+1}, \tag{2}$$

$$\frac{p_4}{p_1} = \frac{2\kappa M_1^2 \sin^2\gamma_m - (\kappa-1)}{\kappa+1}, \tag{3}$$

$$M_2 = \frac{(\kappa-1)M_1^2 \sin^2\gamma_i + 2}{\sin^2(\gamma_i - \theta_i)\{2\kappa M_1^2 \sin^2\gamma_i - (\kappa-1)\}}, \tag{4}$$

$$\tan\theta_i = \frac{2\cot\gamma_i\,(M_1^2 \sin^2\gamma_i - 1)}{M_1^2(\kappa + \cos 2\gamma_i) + 2}, \tag{5}$$

$$\tan\theta_r = \frac{2\cot(\gamma_r + \theta_i)\{M_2^2 \sin^2(\gamma_r + \theta_i) - 1\}}{M_2^2\{\kappa + \cos 2(\gamma_r + \theta_i)\} + 2}, \tag{6}$$

$$\tan\theta_m = \frac{2\cot\gamma_m (M_1^2 \sin^2\gamma_m - 1)}{M_1^2(\kappa + \cos 2\gamma_m) + 2}, \tag{7}$$

where p_j and M_j are the pressure and the Mach number in the region (j), respectively, γ_m the incident angle of Mach stem and κ the specific heat ratio.

Since the pressures on both sides of contact surface S must be equal, we have

$$p_4 = p_3 \tag{8}$$

Furthermore, if the gas flows above and below the contact surface are assumed to be parallel, then we obtain

$$\theta_i + \theta_r = \theta_m. \tag{9}$$

These two complementary equations are called the Neumann condition. The above set of 9 equations contains 11 variables, namely, p_2/p_1, p_3/p_2, p_4/p_1, M_1, M_2, θ_i, θ_r, θ_m, γ_i, γ_r and γ_m. If the shock Mach number Ms and the angle of incidence γ_i are given, the entire set of 9 equations becomes solvable through the geometrical relation, $Ms = M_1 \sin\gamma_i$ (See Fig.2(a)).

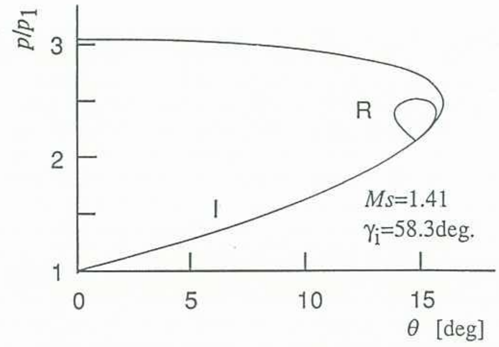

Fig.3. Shock polar diagrams for the weak shock.

Applying the three-shock theory to the reflection of a weak shock with Ms = 1.41 and γ_i =58.3 deg., one obtains shock polars illustrated in Fig. 3, in which there is no intersection of the I-polar and R-polar, whereas the experiment shows that the Mach reflection is possible. Thus, the classical three-shock theory is inadequate to describe the Mach reflection of weak shock. Although many causes for this contradiction had been considered, it may be the most prominent one among them that von Neumann's theory theassumes the slipstream as a mathematical surface. Indeed, when two parallel flows of different velocities interact, boundary layers develop in both flows according to the viscous effect. Physically, divergence of flow behind the triple point can be occured due to the boundary layer between two parallel flows which move with different velocities, in which the flow has the velocity component perpendicular to contact surface (Schlichting, 1979; Ben-Dor,1987). Moreover, it is expected that the slipstream is strongly affected by the reflecting wedge surface at small Mach stem length. Therefore, we introduce a modified Neumann condition which allows pressure difference and flow divergence between the flows behind the reflected shock and the Mach stem as illustrated in Fig. 2(b). Then the conditions (8) and (9) are modified as

$$p_4 = p_3 + \varepsilon, \qquad (10)$$

$$\theta_i + \theta_r = \theta_m + \delta, \qquad (11)$$

where ε and δ are pressure difference and angle of divergence, respectively.

The validity of the modified condition should be proved by the experimental evidences. Schlieren photograph in Fig. 4 may be a typical example among the evidences that certifies the divergence of slipstream. The following procedure was adopted to prove quantitatively whether the modified condition is valid or not. Let ε (or δ) be unknown variable and another deviation δ (or ε) be zero. If Ms, γ_i and γ_r ,which are measured data obtained from experiment, are given, equations (1)-(7), (10) and (11) can be solved. Then measurable angles γ_m and $\theta_i + \theta_r$ are compared with the solution obtained above. (It is very difficult to measure the other properties such as p_3, p_4 or θ_m, accurately.) Representative solutions with modified conditions are shown in Table I. It is found from this table that ,while calculated values of γ_m and $\theta_i + \theta_r$ with $\delta \neq 0$ and $\varepsilon = 0$ agree well with corresponding experimental data, there is large discrepancy with respect to γ_m between the experimental value and calculated value, if we set $\varepsilon \neq 0$ and $\delta = 0$. This fact suggests that pressure difference is negligible small and only the divergence of slipstream is dominant in the phenomena. Henceforth, we anlyze the modified three-shock theory setting $\delta \neq 0$ and $\varepsilon = 0$. In the numerical calculation, the incident shock Mach number Ms ranges from 1.10 to 1.41 and the ratio of specific heats κ is 1.4.

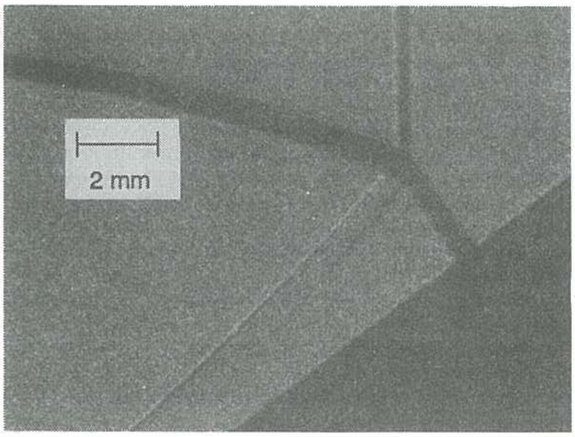

Fig.4. Schlieren photograph of a weak shock reflection. Ms=1.41 and θ_w=37.5deg.

Table I. Comparison of flow properties between measurement and calculation (Ms=1.41, θ_w=22.5deg.)

properties	measurement	calculation (δ≠0, ε=0)	calculation (δ=0, ε≠0)
γ_i	58.2±0.2	58.2±0.2	58.2±0.2
γ_r	77.3±0.2	77.3±0.2	77.3±0.2
γ_m	66.8±0.5	66.0±0.3	71.6±0.1
$\theta_i + \theta_r$	15.2±0.3	15.1	15.1
δ	-	-0.97	0
ε/p_3	-	0	0.083±0.05

RESULTS AND DISCUSSION

Figures 5 shows the comparison of the experimental results with numerical results, where the value of divergence δ is not obtained by solving the shock relations but given parametrically *a priori* to solve the set of equations (1)-(7) with modified conditions (10) and (11). A dashed curve stands for the two-shock theory (RR). The open circle represents Mach reflection, whereas the solid circle regular reflection. As the angle of incidence increases, the difference between the classical three-shock theory (MR), which

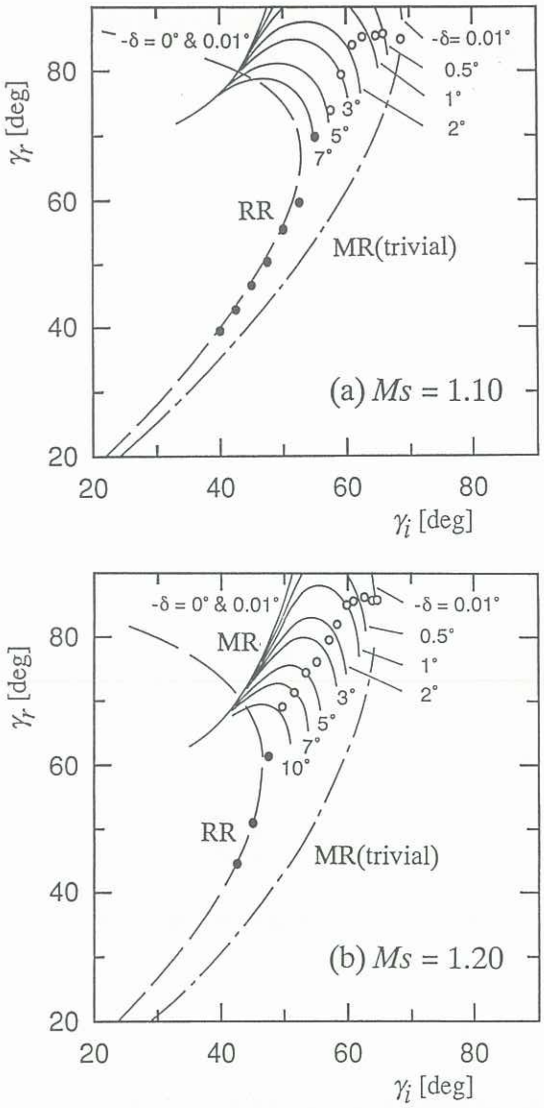

Fig.5. Relation between angles of incidence and reflection. (a) Ms=1.10, (b) Ms=1.20. ○ , Mach reflection (expt.); ● ,regular reflection(expt.); ——— , modified three shock theory; — - —,trivial solution; — — —, two shock theory (continued).

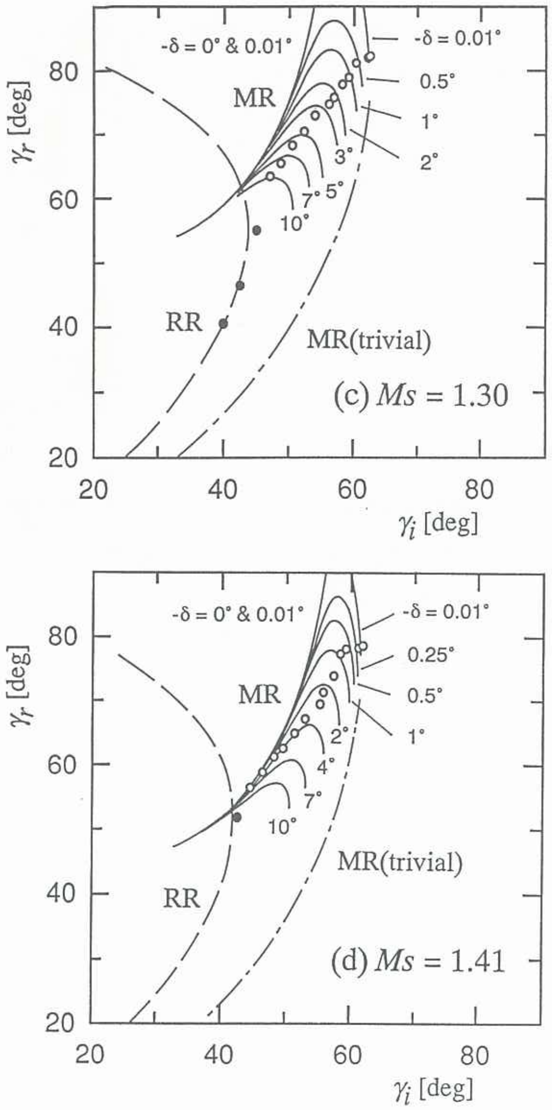

Fig.5. Relation between angles of incidence and reflection. (a) Ms=1.30, (b) Ms=1.41. For the symbols see previous page.

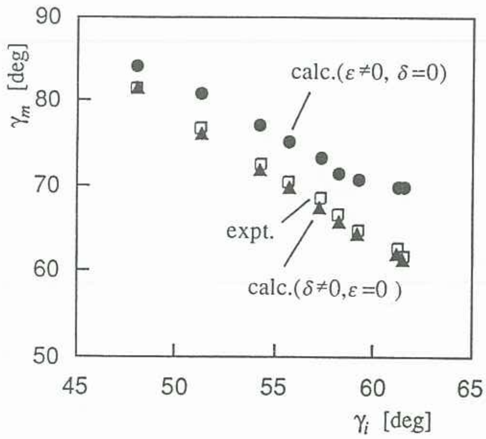

Fig.6. Relation between angles of incidence of incident shock I and Mach stem M. Ms=1.41.

corresponds to $-\delta = 0$, and the experiment becomes distinguished. This difference also increases as the incident shock Mach number decreases. By assuming the flow divergence, Mach reflection persists even for higher angle of incidence where no Mach reflection is possible according to the classical theory, and the domain where realistic solutions of modified theory exist is restricted by two curves. They are the solution of the von Neumann's theory ($-\delta = 0$) and the trivial solution which is obtained by the classical theory with the assumption that $\gamma_i = \gamma_m$, $\theta_i = \theta_m$ and $\theta_r = 0$. The upper limit of angle of incidence γ_i for Mach reflection seems to be bounded by the solution of modified theory with $-\delta \rightarrow 0$ which may be connected with trivial solution of Mach reflection. In fact, for the experimental values, Ms=1.20, γ_i = 64.3 deg.and γ_r =86.2 deg., we obtain $-\delta$=0.013 deg., γ_m = 64.7 deg. and p_3/p_2 = 1.0019. Thus the Mach number M_2 in region (2) is almost unity which is the lower limit of M_2 under which no reflected shock exists. Figure 6 also shows that, as the shock Mach number increases, solution curve for each δ gathers to that of von Neumann's three-shock theory. This fact suggests that even though experiment agrees well to the solution of Neumann's theory at relatively high Mach number Ms=1.41 and low incident angle about 43-50 degrees, there is no denying the existence of diverging slipstream. Comparing the numerical results with experiment, the angle of divergence appears to depends on the angle of incidence for constant shock Mach number. Consequently, it seems that the modified theory can offer a clue to solution of the von Neumann paradox.

Figure 6 illustrates the relation between the angles of incidence for the incident shock and the Mach stem. Calculations were performed with measured data of Ms, γ_i and γ_r. It is found that by assuming only a fractional quantity of flow divergence, the modified theory and the experiment agree quite well with respect to shock configuration. Therefore, there is no pressure difference across the slipstream and only the assumption of ε=0 and $\delta \neq 0$ is valid.

CONCLUSION

The modified theory which takes the downstream flow divergence into account was proposed and comparison with the experiment was made. Conclusions are as follows;

1) By assuming the flow divergence, Mach reflection persists even for higher angle of incidence where no Mach reflection is possible according to the classical theory.

2) The modified theory could offer a reasonable solution of the von Neumann paradox. Therefore, the flow divergence is a promising candidate as one of the causes of the von Neumann paradox.

REFERENCES

HORNUNG, H (1986) Regular and Mach reflection of shock waves. Ann. Rev. Fluid Mech., 18, 33-58.

BEN-DOR, G (1988) Steady, pseudo-steady and unsteady shock wave reflections. Prog Aerospace Sci 25 , 329-412.

BEN-DOR, G, (1987) A reconsideration of the three-shock theory for a pseudo-steady Mach reflection. J Fluid Mech, 181, 467-484.

COLELLA, P and HENDERSON, L F (1990) The von Neumann paradox for the diffraction of weak shock waves. J Fluid Mech, 213, 71-94.

DEWEY, J M. and McMILLIN, D J (1985) Observation and analysis of the Mach reflection of weak uniform plane shock waves. Part 2: Analysis, J. Fluid Mech, 152, 67-81.

IKUI, T and MATSUO, K (1983) Mechanics of Shock Waves. Corona-sha, Tokyo (in Japanese).

SAKURAI, A, HENDERSON, L F, TAKAYAMA, K, WALENTA, Z and COLELLA, P (1989) On the von Neumann paradox of weak Mach reflection. Fluid Dynamics Res, 4 , 333-345.

SAKURAI, A (1964) On the problem of weak Mach reflection. J Phys Soc Japan, 19, 1440-1450.

SCHLICHTING, H (1979) Boundary Layer Theory. 7th ed. McGraw-Hill, 183-185.

VON NEUMANN, J (1943) Oblique reflection of shocks. in John von Neumann Collected Works, 6 (ed by A H. Taub), 238-299, Pergamon (1963).