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RELATIONSHIP BETWEEN LDA VELOCITY BIAS AND FLOW TIME SCALES

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ABSTRACT

Sample and hold processors are used to avoid velocity bias errors in laser Doppler anemometer (LDA) measurements. Such processors provide results free of bias error if the ratio of flow time scale to measurement time scale is sufficiently high, typically greater than 5. It has been widely assumed that the flow time scale refers to the Taylor time microscale. This paper shows the appropriate flow time scale is, in fact, the integral time scale of the flow. Furthermore, it shows the velocity bias associated with a sample and hold processor can in many cases be predicted and hence corrected.

INTRODUCTION

Since LDA flow statistics are approximated from the statistics of the measured seed particles, incorrect interpretation of these may lead to what is known as velocity bias. This is due to the higher probability of detection of a high velocity particle than a low velocity particle. If the individual realizations are ensemble averaged, the resultant mean is often higher than that of the flow, McLaughlin and Tiederman (1973).

Methods of avoiding velocity bias fall into two categories. The first attempts to generate a correction factor which when applied to the individual realizations gives unbiased results (inverse velocity weighting by McLaughlin and Tiederman (1973) and residence time weighting by Buchave and George (1979)).

The second category attempts to reconstruct the original signal or sample the signal in such a way as to obtain unbiased statistics, Edwards (1987). One of the most commonly used is the sample and hold processor which holds the last value obtained and updates the output when a new signal is recorded. Dimotakis (1976) and Edwards and Jensen (1983) put forward that given a sufficiently high validation rate of measurements, this method will produce bias free sampling statistics even at high turbulence intensities.

TERMINOLOGY

The terminology used in this work has been standardized to that of Edwards (1987).

$\dot{N}_2$ - Validation rate. The rate at which the processor provides valid measurements.

$\dot{N}_3$ - Stored data rate. The rate at which the measurements are stored in computer memory for processing.

T_λ - Taylor time microscale. It provides a measure inversely proportional to the root mean square (rms) of the flow acceleration. It is defined as:

$$T_\lambda = \sigma_u \Big/ \left(\left\langle \left(\frac{du}{dt} \right)^2 \right\rangle \right)^{1/2}$$

where σ_u is the rms of the flow velocity.

T_u - Flow integral time scale or macro scale. It is defined by:

$$T_u = \int_0^\infty R_{uu}(\tau)\, d\tau$$

where R_{uu} is the normalized autocorrelation function. It is the average period of correlation of the flow if R_{uu} is approximated by a step function of height one which has the same area under the curve as R_{uu}.

τ_m - Measurement time scale. Equal to the inverse of the validation rate before being subjected to any correction method, that is $1/\dot{N}_2$.

τ_c - A characteristic flow time scale.

τ_c/τ_m Data density.

THEORETICAL CONSIDERATIONS

McLaughlin an Tiederman (1973) found that given the basic assumptions of uniform seeding density, spherical measuring volume, and velocity independent detector sensitivity, the direct relationship between instantaneous flow velocity vector magnitude, $|V_i|$, and probability of particle arrival per unit time, holds for all data densities. This, in the form of the inverse velocity weighting, can be used to correct the particle statistics but requires knowledge of all three components of the velocity vector. As this is rarely possible they proposed a simplified correction using only the major directional component of velocity. This provided accurate results for turbulence intensities up to 30%.

Sample and hold processors approximate the three dimensional correction by using the time between particle arrivals, T_i, as the weighting factor applied to each measurement point instead of $1/|V_i|$. This does not require knowledge of all three components of V_i and works provided that T_i is proportional to the probability of particle arrival per unit time. When all values of T_i are small enough for there to be only a small change in V_i during T_i, the sample and hold processor will accurately reconstruct a velocity signal with identical statistics to that of the original flow, Roesler *et al* (1980), Edwards (1987), Adrian and Yao (1987). At low data densities the relationship between T_i and particle arrivals breaks down and sample and hold processors produce a velocity bias whereas the inverse velocity weighting based on $|V_i|$ does not (given the above basic assumptions).

The mean and rms of a sample and hold processor are given by :

$$\bar{U}_{SH} = \frac{\sum U_i T_i}{\sum T_i} \qquad (1)$$

$$\sigma_{SH} = \left(\frac{\sum U_i^2 T_i - (\sum U_i T_i)^2 / \sum T_i}{\sum T_i} \right)^{1/2} . \qquad (2)$$

Edwards and Jensen (1983) derived a criterion for unbiased measurement which required the ratio of Taylor microscale of the flow to measurement time scale to be greater than 10. It was redefined more stringently in terms of validation rate by Edwards (1987) as

$$\dot{N}_2 T_\lambda > 5 \qquad (3)$$

An alternate model to that used by Edwards and Jensen (1983) can be obtained by use of the following assumptions.

(1) The particle spacing, D_J, along a streak line can be described by a Poisson process with the mean equal to D_M, the average number of particles per unit distance. Eq. (4) is the corresponding exponential distribution giving the probability density function for D_J. The average particle spacing is given by $\tau_m \cdot |V_M|$, and is the same for all velocities. Effects due to a non-spherical measuring volume or non-uniform seeding are assumed negligible. V_M is the mean velocity of the flow.

$$p(D_J) = \frac{1}{D_M} \exp\,(-\,D_J/D_M\,) \qquad (4)$$

(2) The probability of a particle arrival per unit time is proportional to $|V_i|$ where V_i is the velocity vector of the particle.

(3) The velocity persistence time for the flow is given by a value τ_c. After the arrival of a particle i, with a velocity V_i the flow remains at V_i for a time τ_c (that is, perfectly correlated) then becomes equal to the mean flow velocity, V_M (perfectly uncorrelated). The value of τ_c is assumed constant for a particular flow and thus independent of the value of V_i. By definition, τ_c is the integral time scale of the flow.

(4). The time between two particle arrivals for a given spacing D_J and velocity of the first particle V_i, is given by

$$T_{V_i} = \frac{D_J}{|V_i|} \qquad D_J/|V_i| \le \tau_c \qquad (5)$$

$$T_{V_i} = \tau_c + \frac{D_J - \tau_c\,|V_i|}{|V_M|} \qquad D_J/|V_i| > \tau_c \qquad (6)$$

The expected time between particle arrivals $\langle T_{V_i} \rangle$ for a given velocity, V_i, at the start of an interval can then be calculated using Eqs. (4), (5), and (6) and integrating over all values of D_J.

$$\langle T_{V_i} \rangle = \frac{D_M}{|V_i|} \left[1 - \left(1 - \frac{|V_i|}{|V_M|} \right) \exp\left(-\frac{\tau_c}{\tau_m} \cdot \frac{|V_i|}{|V_M|} \right) \right] \qquad (7)$$

Eq. (8) may be used to give an expression for the expected probability density for the output of a sample and hold processor if it is assumed that time between particles from integration along a streak line (Lagrangian view) is equivalent to time between particles at a point (Eulerian view):

$$\langle g_{SH}(V_i) \rangle = p(V_i) \left[1 - \left(1 - \frac{|V_i|}{|V_M|} \right) \exp\left(-\frac{\tau_c}{\tau_m} \cdot \frac{|V_i|}{|V_M|} \right) \right] \qquad (8)$$

At lower turbulence intensities it will be sufficient to model the velocity vector as equal to the streamwise component of velocity (1-D case). As the turbulence intensity increases the magnitude of the full velocity vector will be necessary to accurately determine the probability of particle arrivals and the mean interarrival time (3-D case).

Fig. 1 shows the simulated bias of streamwise velocity for the sample and hold processor as predicted by Eq. (8) for both one and three dimensional estimates of velocity, over a range of data densities. For the 3-D case, Gaussian distributions of equal rms were used for all three components of velocity. Zero means were assigned to the lateral components of velocity and the streamwise component was given a value of one. The results show the ratio of time scales required for unbiased measurement is such that $\tau_c/\tau_m > 5$. At low data densities the bias reaches the maximum value when $\tau_c/\tau_m \le 0.05$. The same Gaussian distribution used for the streamwise component in the 3-D case was used to approximate the total velocity vector magnitude in the 1-D case. Results using this approximation are useful below 30% turbulence intensity but at higher levels of turbulence, they overestimate both the bias and the data density required for unbiased measurement.

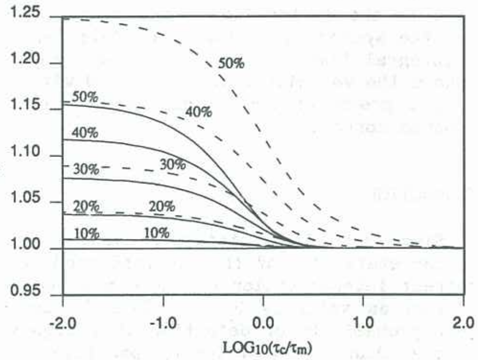

Fig. 1 Velocity bias based on Eq. (8)
– – – – 1-D model ——— 3-D model

Sensitivity of the model to the velocity distribution was examined by a comparison of the results of Gaussian and uniform distributions with the same mean and rms. The onset and point of maximum bias occurred at the same data densities. A 7% increase in the difference between the biased and unbiased velocity for the uniform distribution was the only variation.

Bias may be corrected when values of τ_c/τ_m, $\bar{V}_{SH}$, and σ_{vSH} are known. By using $\bar{V}_{SH}$ and σ_{vSH} as first approximations to the flow statistics the true mean and rms velocities can be calculated iteratively using numerical integration of Eq. (8) and the relevant approximate velocity probability density distribution (the distribution of particle statistics should in most cases be sufficiently accurate). Using the calculated change in the mean and rms due to bias, a closer estimate of the true flow statistics can be made. Convergence is reached within two to three iterations.

APPARATUS

Measurements were made in an axisymmetric steady air jet having nozzle diameter of 15.79 mm. The measurements were taken at a non-dimensionalized axial distance of x/d = 30 on the centreline and at a distance 25 mm (y/x = 0.0528) from the centreline. By adjustment of upstream pressures approximately equal mean velocities at two different turbulence intensities were obtained.

One channel of a TSI two component polarization LDA operating at 5 MHz shift frequency was used for all measurements. An argon-ion laser operating with a wavelength of 514 nm was used with a beam intersection angle of 11.03^0 and a fringe spacing of 3.785 μm. The processor-computer interface had a time resolution of 6 μs and a cycle time of 96 μs (thus $\dot{N}_3$ is limited to 10.416 kHz), taking only the first point to arrive in the cycle.

The flow was seeded using aerosol generators with a 20:1 water-glycerine mix. Measurements with a Malvern type 2600 spray and droplet sizer had shown 97% of particles to be below 3.8 μm at the seeder. Significant evaporation takes place in the flow so that all droplets are of a size small enough for successful LDA measurement.

RESULTS AND DISCUSSION

The integral and micro time scales of the flow vector magnitude were approximated using discrete forms of the definitions given previously, applied to the streamwise component of velocity. As predicted by Kolodzy and Edwards (1986), the LDA autocorrelation and hence the integral time scales show some bias when compared to the hot-wire results, Fig. 2 (refer Hinze (1959) for methods). However, differences for all data densities were of the order of 10% which produces negligible effects when used in Eq. (8). The LDA Taylor microscale results are very data rate sensitive but seem to reach an asymptote at very high data densities at the value given by the hot-wire signal.

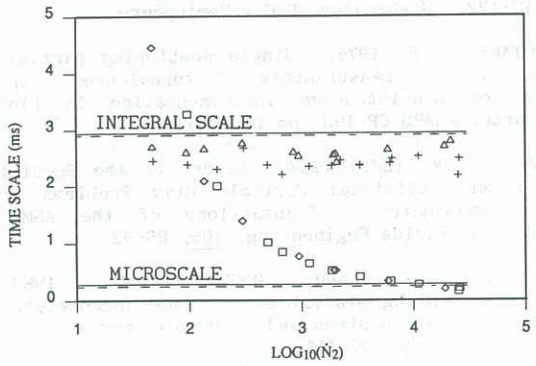

Fig. 2 Flow time scales
Δ T_u, ◇ T_λ LDA at centreline
\+ T_u, □ T_λ LDA at y/x=0.0528
——— Hotwire at centreline
\- - - - Hotwire at y/x=0.0528

Figs. 3 and 4 show velocity and turbulence intensity for two different turbulence intensity flows and different weighting methods together with the rates corresponding to the LDA Taylor microscales and integral time scales. The high data density results for mean velocity and turbulence intensity results can be considered free of velocity bias, Adrian and Yao (1987), Edwards (1987). The lower turbulence intensity results (26%) were independently confirmed with the LDA data using the one dimensional McLaughlin and Tiederman correction. In this particular flow, accurate results can be expected from this correction method, McLaughlin and Tiederman (1973), Buchave and George (1979).

Having the true flow statistics and time scales available, the performance of sample and hold processor simulation and correction methods may be assessed. In Figs. 3a and 3b the expected bias determined from Eq. (8) was plotted using both time scales and assuming a Gaussian velocity distribution with the 1-D version of the model. It

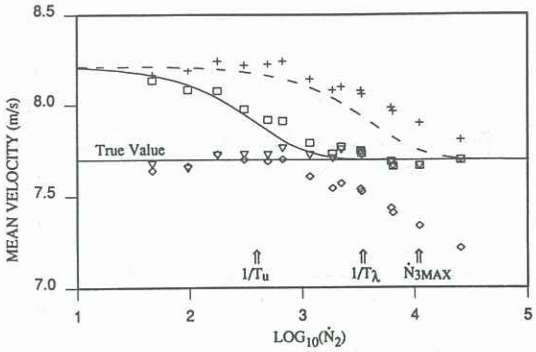

Fig. 3a Mean velocity at jet centreline

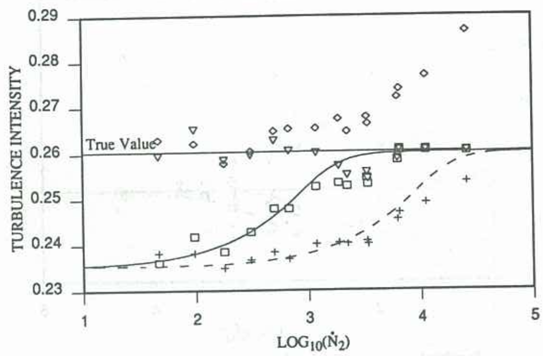

Fig. 3b Turbulence intensity at jet centreline
□ Sample and hold processor ∇ Correction using 1-D model based on eqn. (8) ◇ 1-D M&T correction
\+ Arithmetic average processor
——— 1-D model using integral time scale
\- - - 1-D model using Taylor microscale

is quite clear that the integral time scale is the appropriate one with the Taylor microscale producing an order of magnitude error in the onset of bias when compared with the LDA data. Sample and hold data corrected using the one dimensional approximation model produced excellent agreement with the true values at all data densities.

In the higher turbulence intensity flow, Figs. 4a and 4b, the one dimensional approximation produced an unsatisfactory result at low $\dot{N}_2$. This is because the streamwise component of velocity no longer suitably approximates the vector magnitude. Here, the 3-D model produced very good results, at all data densities. In both flows the sample and hold processor produced unbiased results when the data density reaches a value of about 5 and $\tau_c = T_u$. When the data density approaches 0.05 the statistics of the sample and hold and unweighted processors become the same.

Arithmetic averaging of the data produces a biased velocity result, Figs 3 and 4. Bias is seen to be independent of data density at low values of $\dot{N}_2$ but reduces as $\dot{N}_2$ approaches and passes the maximum value of $\dot{N}_3$. This reduction in bias error can be attributed to the effect of the controlled processor, Edwards (1987), which is created by the limited sampling rate of 10.416 kHz.

The one-dimensional McLaughlin and Tiederman corrected results are bias free in the lower turbulence intensity flow (26%) for low data rates. However, the controlled processor effect overcorrects the results as the data rate approaches the maximum sampling rate, showing the importance of ensuring that no data points are lost due to signal processor-computer interface, or computer speed limitations. This means $\dot{N}_2$ must equal $\dot{N}_3$ if overcorrection of the results is not to occur. Furthermore, overcorrection in the higher turbulence intensity flow is observed as predicted

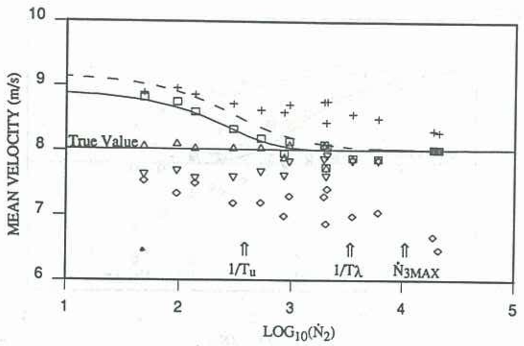

Fig. 4a Mean velocity at $y/x = 0.0528$

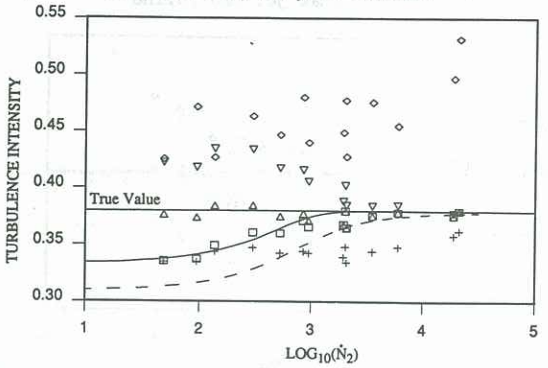

Fig. 4b Turbulence intensity at $y/x = 0.0528$
□ Sample and hold processor ∇ Correction using 1-D model Δ Correction using 3-D model ◇ 1-D M&T correction + Arithmetic average processor
– – – – 1-D model with integral time scale
———— 3-D model with integral time scale

by McLaughlin and Tiederman (1973) due to the same effects that create the error in the 1-D case of the sample and hold model.

The sample and hold results were not affected by the controlled processor because the 10.416 kHz sampling rate is much higher than any significant flow frequencies. Data points eliminated during the 96 μs cycle time of a sample interval have essentially the same value as the first point and therefore do not affect the sample and hold result.

CONCLUSIONS

This work has shown that the velocity bias associated with a sample and hold processor can be modelled successfully up to high levels of turbulence. The proposed model characterizes the flow using a flow correlation time equal to the integral time scale of the flow.

For approximately equal variance in each direction a sample and hold processor can produce mean and rms flow quantities free of bias if:

$$\dot{N}_2 T_u > 5$$

where T_u is the integral time scale of the flow. T_u may be determined using LDA measurement and was independent of data density over the range of $\dot{N}_2$ considered. Bias errors in LDA measurements of T_u are not significant for determination of the $\dot{N}_2 T_u$ threshold for accurate measurements.

The terms giving the density of LDA data should be redefined as follows :

High data density	$\dot{N}_2 T_u > 5$
Intermediate data density	$5 \geq \dot{N}_2 T_u \geq 0.05$
Low data density	$0.05 \geq \dot{N}_2 T_u$

where T_u is the integral time scale of the flow.

If the McLaughlin and Tiederman (1973) correction is used in any of its forms (1D, 2D, or 3D), it must be ensured that $\dot{N}_2$ is equal to $\dot{N}_3$ and that no data are lost in the processor-computer interface. Lost data can cause a constant interval sampling or saturable detector effect which would in turn cause overcorrection of the results. Such effects are most likely to occur at high data densities and will affect all methods which attempt to generate correction factors using the probability distribution of particle arrival per unit time based on velocity.

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