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THE EFFECTS OF FEEDBACK AMPLIFIER CHARACTERISTICS ON CONSTANT TEMPERATURE HOT-WIRE ANEMOMETER SYSTEMS

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ABSTRACT

The 3^{rd}-order analysis of Perry and Morrison (1971) was extended to 7^{th}-order by Watmuff (1987) by including both the bridge capacitance and the frequency response characteristics of the feedback amplifier. In this paper the bridge capacitance has been excluded from the analysis. The influence of the gain K, roll-off frequency f_A and offset voltage E_{qi} of the feedback amplifier are examined in more detail together with their interactions with the bridge inductance.

INTRODUCTION

Attempts to increase the overall frequency response of a hot-wire system by increasing the gain and frequency response of the feedback amplifier are invariably frustrated because the system develops instabilities. While some of these instabilities can be explained by the 3^{rd}-order model of Perry and Morrison (1971) others appear to be of higher-order and cannot be accounted for. Some workers have reported difficulty obtaining system stability when using subminiature (e.g. d=0.6μm) hot-wire probes, e.g. Miller, Shah and Antonia (1987). With these small wires the frequency response is usually more than adequate and of secondary importance compared to the frequent probe breakage caused by the instabilities. It appears that the dynamics of the constant temperature hot-wire anemometer are not sufficiently well understood. Yet a more complete understanding could possibly lead to ways of avoiding instabilities and achieving a higher frequency response.

MODEL WITH AMPLIFIER FREQUENCY RESPONSE

Watmuff (1987) derived a 7^{th}-order model based on the configuration shown in figure 1. In this paper the capacitance is excluded from the analysis and the frequency response of the amplifier f_A is assumed to be flat through to a simple 2nd-order roll-off i.e.

$$\frac{e_o}{e_i} = \frac{K}{(Ms+1)(Ms+1)} \qquad (1)$$

This leads to 5^{th}-order transfer functions for velocity fluctuations u' and offset voltage perturbations e_s i.e.

$$\frac{e_o}{u'} = \frac{KxR_a(L_b s + R_b + R_c)}{A(s)} \qquad (2)$$

$$\frac{e_o}{e_s} = \frac{KB(s)}{A(s)} \qquad (3)$$

where x is a constant associated with the wire sensitivity (see Perry and Morrison 1971) and

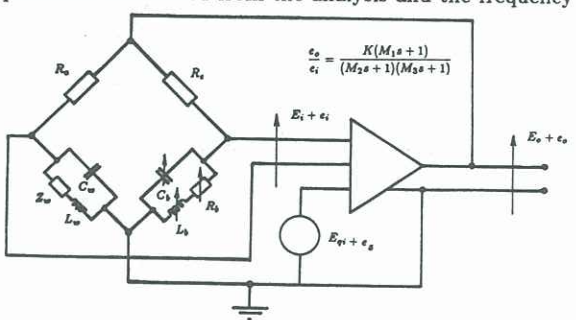

Figure 1. Schematic of constant temperature hot-wire anemometer used by Watmuff (1987) to derive a 7^{th}-order model. In this paper $C_w = C_b = 0$ i.e. system is 5^{th}-order. All results use $R_a = 100\Omega$, $R_c = 1000\Omega$, $R_b = 160\Omega$ and $M_1 = 0$ and $M_2 = M_3 = M$. Platinum filaments, length = 1mm, diameter= 5μm (i.e. $R_g \approx 8\Omega$), U= 20m/s.

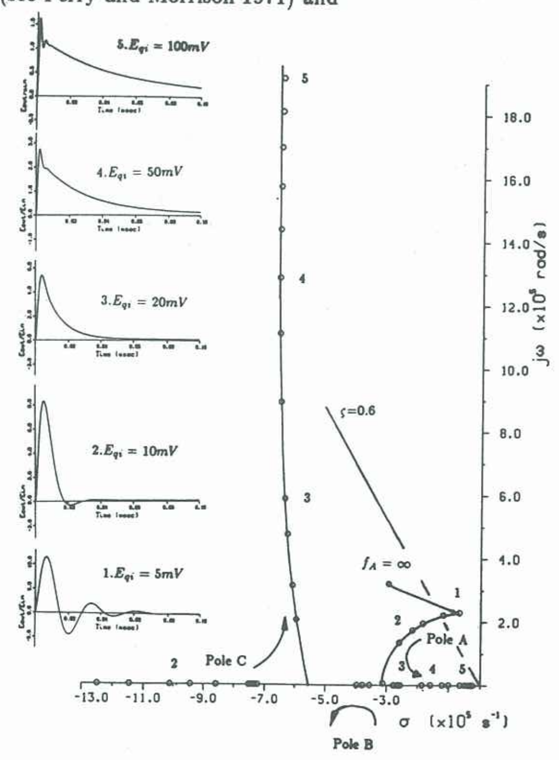

Figure 2. Calculated s-plane trajectories (2^{nd}-quadrant only) of dominant poles with increasing offset voltage E_{qi}. Inductor values $L_b = 8\mu$H, $L_w = 1\mu$H, amplifier frequency response $f_A \approx$ 79.6kHz and gain K=1000 are constant. Point $f_A = \infty$ corresponds to model of Perry and Morrison (i.e. $M = 0$) for conditions at point 1. Square-wave response for operating points 1 to 5 also shown. Line ς=0.6 shows optimum damping for complex poles.

$$A(s) = A_5 s^5 + A_4 s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0 \quad (4)$$

$$B(s) = C_3 s^3 + C_2 s^2 + C_1 s + C_0. \quad (5)$$

The coefficients of the polynomial $A(s)$ are given by,

$$\begin{pmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2M & 1 & 0 & 0 \\ M^2 & 2M & 1 & 0 \\ 0 & M^2 & 2M & 1 \\ 0 & 0 & M^2 & 2M \\ 0 & 0 & 0 & M^2 \end{pmatrix} \begin{pmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \end{pmatrix} + K \begin{pmatrix} C_{K0} \\ C_{K1} \\ C_{K2} \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (6)$$

$C_0 \cdots C_3$ and $C_{K0} \cdots C_{K2}$ are constants that depend on the operating point and system parameters:

$$\begin{aligned} C_0 &= (R_b + R_c)(R_a + R_w + \alpha) \\ C_1 &= (R_a + R_w + \alpha)L_b + (R_b + R_c)\left[(R_a + R_w)T_w + L_w\right] \\ C_2 &= \left[(R_a + R_w)T_w + L_w\right]L_b + (R_b + R_c)L_w T_w \\ C_3 &= L_b L_w T_w \\ C_{K0} &= (\mathring{R} + R_c\alpha) \\ C_{K1} &= (\mathring{R}T_w + R_c L_w - R_a L_b) \\ C_{K2} &= T_w(R_c L_w - R_a L_b) \end{aligned} \quad (7)$$

$\mathring{R} = R_w R_c - R_a R_b$ is the bridge imbalance and $\alpha = R_w(R_w - R_g)/R_g$ where R_g is the wire resistance at gas temperature. T_w is the lumped time constant of the wire filament arising from its thermal inertia. The transfer functions (2) and (3) are identical to those derived by Perry and Morrison for $M = 0$.

AMPLIFER OFFSET VOLTAGE

The results of a systematic parametric study suggest that two types of dominant pole s-plane trajectories are observed as the amplifier offset voltage E_{qi} is varied. The type of trajectory depends on the nature of these poles when $f_A \to \infty$. The effect of varying E_{qi} on the higher-order poles is usually very small.

An example of the first type of trajectory is shown in figure 2. As E_{qi} is increased poles A and B move towards the real axis where they meet and split to form two simple poles. With further increases of E_{qi}, simple pole C continues moving towards the origin and eventually merges with simple pole B to form a new complex conjugate pair. However pole A remains closest to the origin so that it dominates the frequency response. This behaviour is typical of systems in which the dominant poles remain complex as $f_A \to \infty$. Note that the ringing frequency of the square-wave response is around 320kHz at operating point 5 shown in the figure but the simple pole A limits the system frequency response to about 3.6kHz. The Bode diagrams and the step-response shown in figure 3 clearly demonstrate why this type of behaviour is undesirable.

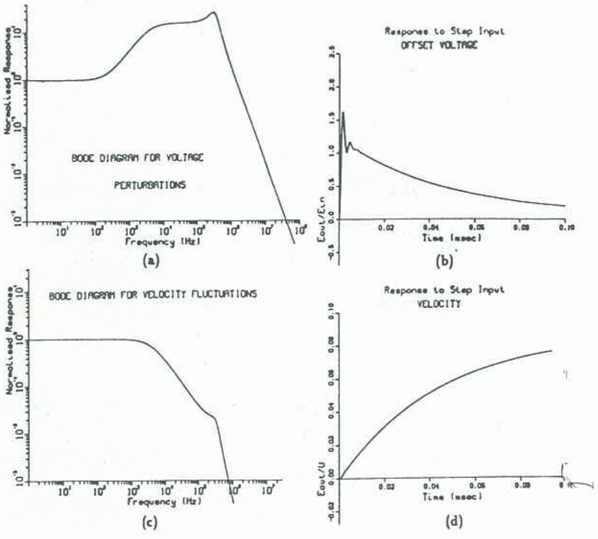

Figure 3. Bode diagrams and step-response for point 5 in figure 2 where E_{qi} =100mV, (a) and (b) offset voltage perturbations e_s, (c) and (d) for velocity fluctuations u'.

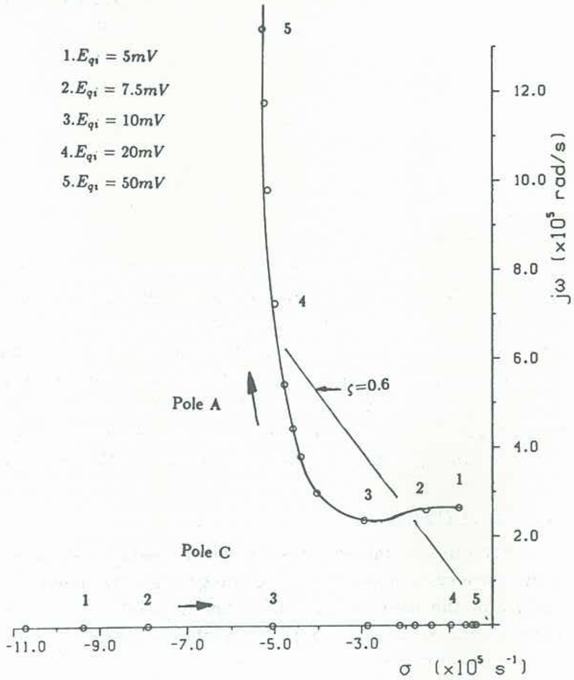

Figure 4. Same as figure 2 but with $L_b = 0.5\mu$H and $L_w = 0.1\mu$H. Despite the different s-plane trajectories the end result is much the same as in figure 2 for large E_{qi}.

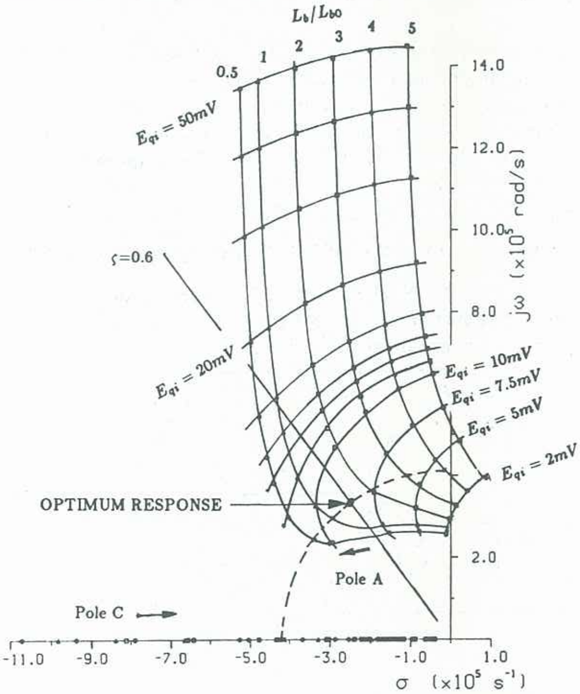

Figure 5. Same as figure 4 where $L_w = 0.1\mu$H and $L_{b0} = (R_c/R_a)L_w = 1\mu$H. Trajectories for L_b/L_{b0}= 0.5, 1, 2, 3, 4 and 5 also shown. Dashed line shows locus of points where poles A, (B) and C have equal characteristic frequencies. Having E_{qi} and L_b as the only tunable parameters leads to an optimum response when $L_b > L_{b0}$.

An example of the second type of trajectory is shown in figure 4. With increasing E_{qi} poles A and B remain complex conjugate while simple pole C moves towards to the origin and eventually dominates the system frequency response. This behaviour is typical of systems which possess only simple poles when $f_A = \infty$. Despite these different trajectories the end result is much the same as in the first example and the system possesses similar Bode diagrams and step-reponse characteristics.

Many of the rules frequently given for estimating the frequency response from square-wave tests assume a 2^{nd}-order approximate response. While the square-wave test is an invaluable aid for tuning hot-wire systems it is open to misinterpretation as shown above. This type of behaviour can be observed in real systems over a wide range of balance inductor and offset voltage settings. The only way to reproduce these observations in the model is to include the effects of amplifier frequency response.

Increasing f_A		Increasing K	
f_A (kHz)	f_0 (kHz)	K	f_0 (kHz)
15.9	17.8	200	16.3
106.4	27.2 *	1875	28.6 *
526.2	30.2 †	29770	31.5 †
∞	30,6	10^{12}	31.8

* Higher-order poles transformed into complex conjugates.
† Higher-order poles become unstable.

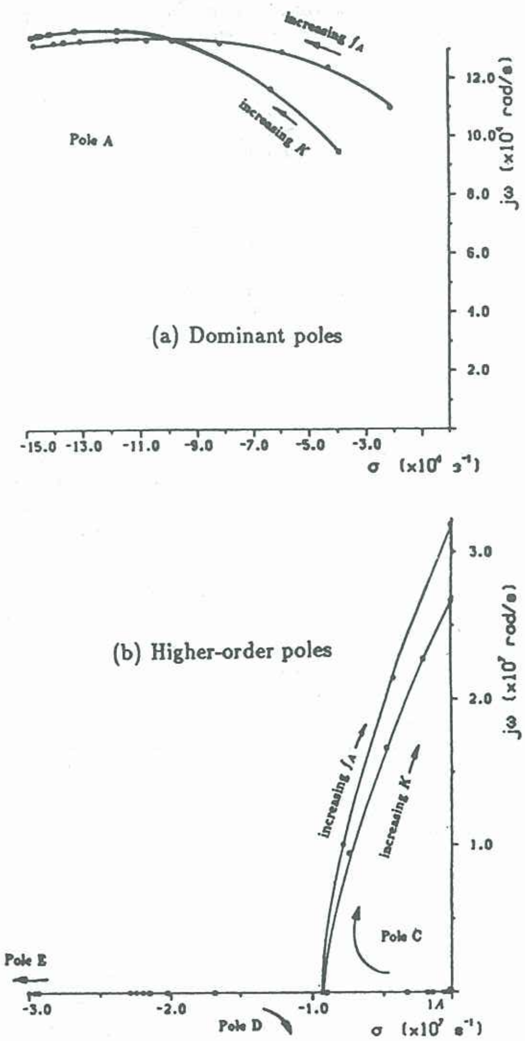

Figure 6. Calculated s-plane trajectories of system poles for increasing f_A (with constant K=1000) and for increasing K (with constant $f_A \approx 79.6$ kHz). $E_{qi} = 12.5$mV, $L_w = 5\mu$H and $L_b = 40\mu$H. (a) Dominant poles (b) Higher order poles.

BALANCE INDUCTOR

The analysis of Perry and Morrison predicts instability when the balance inductor L_b is in excess of the value required for *a.c.* bridge balance L_{b0}. However systems with finite frequency response amplifiers can be stable in this situation. More significantly, having control of only E_{qi} and L_b leads to an optimum system response when $L_b > L_{b0}$. For example, the system in figure 5 has an optimum reponse when $L_b \approx 1.5 L_{b0}$. However if L_b is too large it may be impossible to obtain a satisfactory response.

AMPLIFIER FREQUENCY RESPONSE AND GAIN

Analysis predicts that the frequency response of hot-wire systems $f_0 \to \infty$ in the limits of $f_A \to \infty$, $K \to \infty$, $E_{qi} \to 0$ and $L_b \to L_{b0}$. Smits and Perry (1980) observed that hot-wire systems are prone to instabilities as $L_b \hookrightarrow L_{b0}$ since there is an extreme sensitivity to very small variations in L_b. However this observation was made for 3^{rd}-order systems where $f_A = \infty$. Systems with finite frequency response amplifiers have an optimum frequency response when $L_b > L_{b0}$ and stability can be maintained even when L_b is excessively large. Nevertheless one might suspect that the form of dominant pole instability described by Smits and Perry would eventually occur as f_A is increased. However the results of a systematic parametric study suggest that other higher-order instabilities are more likely to arise

Increasing f_A		Increasing K	
f_A (kHz)	f_0 (kHz)	K	f_0 (kHz)
15.9	21.0	200	16.9
98.4	27.8 ‡	1210	26.9 ‡
107.5	24.3 *	1918	21.3 *
520.4	18.7 †	25400	18.5 †
∞	17.9	10^{12}	18.3

‡ Dominant complex poles transformed into simple poles.
* Higher-order poles transformed into complex conjugates.
† Higher-order poles become unstable.

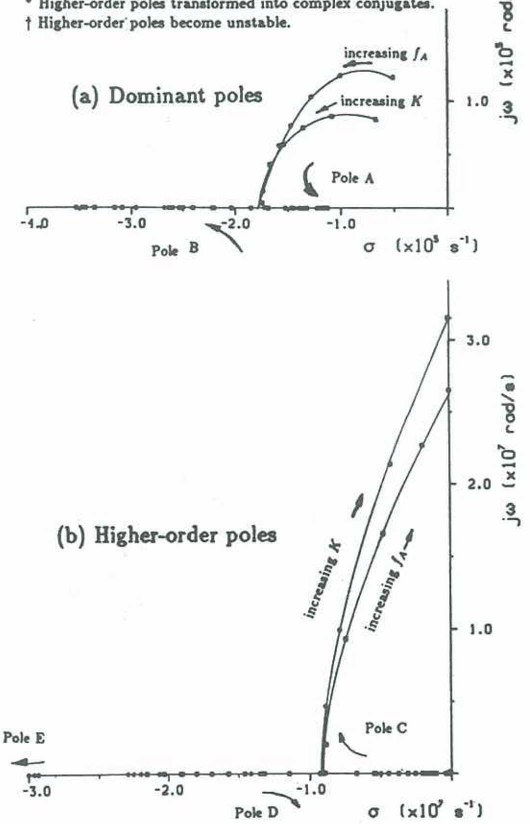

Figure 7. Same system as figure 6 but E_{qi} increased to 20mV. Calculated s-plane trajectories of system poles for increasing f_A (with constant K=1000) and for increasing K (with constant $f_A \approx 79.6$ kHz). (a) Dominant poles (b) Higher order poles.

beforehand as shown in figures 6'and 7. This behaviour is typical of constant temperature hot-wire systems with finite frequency response amplifiers. The effects of increasing K and f_A are quite similar and unexpectedly lead to increased damping of the dominant poles. It is the higher-order poles that are responsible for the system instability.

The higher-order poles exert very little influence on the overall system frequency response and stability provided that they remain stable. However the damping of the higher-order poles is often difficult to determine during a square-wave test, even when they are grossly underdamped, since the oscillations can still decay rapidly when compared to the response of the dominant poles. However a small change in either K or f_A from this point could result in instability. This type of higher-order instability can occur suddenly and without warning to the anemometer operator who can only observe the square-wave response.

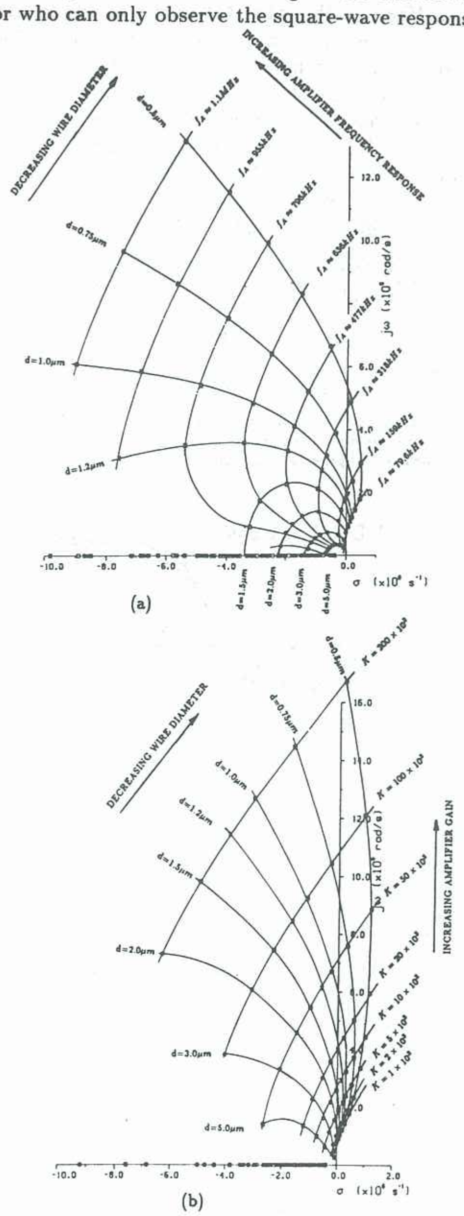

Figure 8. Calculated s-plane trajectories of the dominant poles for a range of wire diameters with the same length to diameter ratio. R_b has been adjusted to closely give the same resistance ratio i.e. $R \approx 2$. $E_{qi} = 5$mV, L_b=0.5 μH and L_w=0.1 μH. (a) Increasing f_A (with constant gain K=1000). (b) Increasing K (with constant $f_A \approx 79.6kHz$). Higher values of K and f_A are required for stability as the wire diameter is reduced.

INSTABILITIES AND SUBMINIATURE WIRES

For fixed amplifier characteristics the ratio of the amplifier time constant to the wire time constant is considerably larger with subminiature wires. This is especially significant since it has been demonstrated that the dominant poles become less damped as f_A is reduced. If f_A is too small then the dominant poles may be unstable. Figure 8 shows the effect of reducing the wire diameter while maintaining the same length-to-diameter ratio (1:200) and the same resistance ratio $R \approx 2$. Higher values of f_A and K are required for stability as the wire diameter is reduced. The effect on the higher-order poles is similar to figures 6(b) and 7(b) and is not shown.

The wire current required for a given resistance ratio is smaller for subminiature wires so that the static output voltage at the top of the bridge is considerably less than that obtained with more usual sized wires. Increasing the bridge resistor values would help to restore the size of the output signal. However this is not recommended since increasing R_a and R_c reduces the damping of the dominant poles as shown in figure 9. An even higher frequency response amplifier would be required to maintain stability under these conditions.

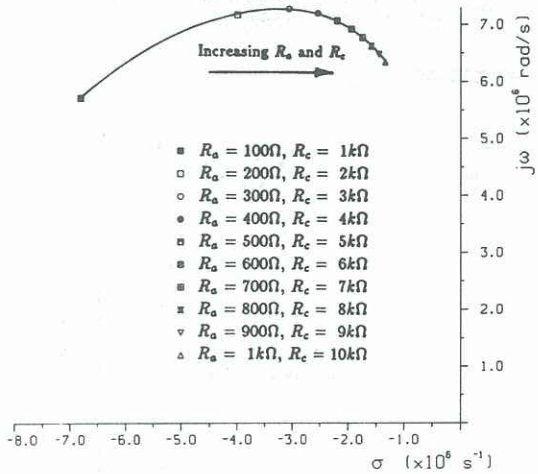

Figure 9. Increasing upper bridge resistors R_a and R_c causes dominant poles to become less damped. Subminiature platinum filament of length 0.1mm and diameter d=0.5μm ($R_g \approx 80\Omega$). Largest values of R_a/R_w and R_c/R_w are approximately the same as for the d=5μm wires in figures 1 to 8. $L_b = 8\mu$H, $L_w = 1\mu$H, $R_b = 1.6k\Omega$ and the amplifier frequency response $f_A \approx 1.6$MHz, gain K=1000 and offset voltage E_{qi} = 5mV are constant. Air velocity is 20 m/s.

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