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VISCOUS PLUMES AND SHEETS FALLING ON A DENSITY OR VISCOSITY INTERFACE

J.S. TURNER and R.W. GRIFFITHS

Research School of Earth Science
Australian National University
G.P.O. Box 4, Canberra, ACT 2601
AUSTRALIA

ABSTRACT

Axisymmetric plumes and two-dimensional sheets of very viscous fluid, impinging on a free surface or fluid interface, have been studied in the laboratory. The aim is to understand the behaviour of subducted oceanic plates as they sink through the earth's mantle, where they may encounter a distinct change in density or viscosity at a depth of 670 km. Concentration on the interfacial case has enabled us to eliminate the irrelevant effects of surface tension and introduce a more realistic viscous environment into the upper layer. We have shown that stability to buckling depends mainly on the length to thickness ratio of the plumes, but that the density difference across the interface and the viscosity contrast between the plume and the upper layer also have an influence. The amount of entrainment of the upper fluid which is carried down into the lower layer also depends on whether buckling occurs, and on the interfacial density difference.

INTRODUCTION

Convection in the Earth's mantle, and the associated motion of rigid surface plates which plunge downwards at subduction zones, are now widely accepted features of the large scale properties of the Earth. There is still disagreement about the form of this convection, whether it takes place through the whole depth of the mantle or whether the upper 670 km of the mantle convects as a separate layer, partially isolated from the lower mantle (see Davies 1987). There is certainly a seismic velocity discontinuity at 670 km, but this could be associated with a phase change or an increase in viscosity, rather than being a compositional and density step which acts as a barrier to convection across it.

There are several lines of geochemical evidence which suggest that there are at least two distinct 'reservoirs' in which materials of different composition have remained separated for billions of years (Sun 1984), and a common simple approach has been to identify these with the upper and lower mantle. In a two-layer model of the mantle, the transport between these chemical and isotopic reservoirs is an important process which will depend upon the nature of the dividing interface and the interaction with convective motions above and below. In particular, the dynamical behaviour of sinking plates or slabs needs to be understood. Under what conditions can they penetrate the interface, and how much of the upper layer will be carried down with them? Do slabs remain stable as they approach the interface, or if not, under what conditions can they buckle and fold and come to rest at that level?

In the present study, different aspects of which have been described in more detail by Griffiths and Turner (1988a,b), we have set aside the unresolved questions about the geometry of mantle convection and have taken an experimental approach to the problem. We have used simple laboratory experiments to explore the behaviour of very viscous plumes and sheets of golden syrup falling through layers composed of glycerine and mixtures of glycerine and syrup. Preliminary experiments with plumes falling through air extend the range of slab-to-surroundings viscosity ratios which can be studied, once the unwanted effects of surface tension have been allowed for.

EXPERIMENTS WITH A FREE SURFACE

Axisymmetric Plumes

We have chosen to study plumes of syrup falling onto a deep pool of the same fluid, rather than onto a solid surface as in previous experiments. The range of phenomena observed as the length of the very viscous plume is increased is shown in Fig. 1. For a given nozzle and a small height to diameter ratio the plume remains stable and merges smoothly with the free surface. As the height of the nozzle is increased a critical height H_c is reached at which a spiral instability sets in; this can be likened to the buckling of an elastic column under an axial loading (Taylor 1969). The whole length of the plume then begins to oscillate with a well-defined period. Measurements of H_c and the frequency just above this height, as functions of the external parameters, are the main data recorded in these experiments.

The frequency of the oscillation increases as the height is increased (linearly with height, but with a finite value at the onset). At larger heights a second period can be detected, and as the height is increased still further the coiling frequency continues to increase

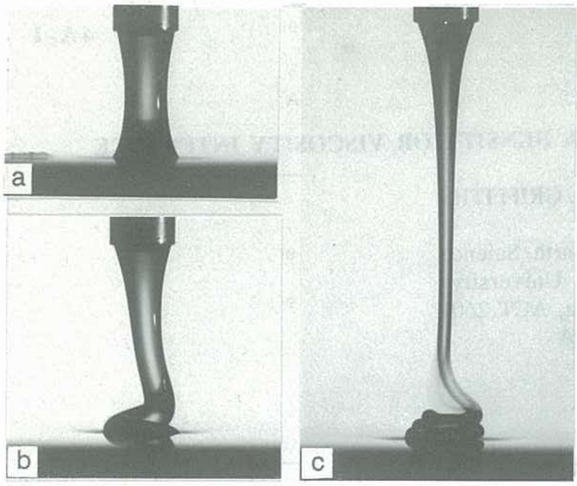

Fig.1. Axisymmetric plumes of syrup falling onto a free surface of syrup, for constant flowrate and various heights.
a) H = 2.1 cm : the plume is stable b) H = 3.3 cm: the plume is coiling c) H = 7.8 cm: a second period of oscillation is observed.

while the motion is again described by a single period. At this stage the upper part of the plume is completely steady, and the oscillations are confined to a short length of plume just above the surface where the plume is in compression, and a tight columnar coil builds up and periodically collapses. This last stage is not of direct interest in the present context.

It can be shown that surface tension has the dominant influence on the height at which coiling occurs for small nozzle diameters (d_0), whereas experiments with the largest nozzles approach the state where surface tension can be neglected. Plotting H_c/d_0 against $1/d_0$ allows an extrapolation to give the critical height in the absence of surface tension: $H_c/d_0 = 1.35$. It is also important to note that coiling or folding is a low Reynolds number phenomenon: when inertia effects dominate over the longitudinal compressive stress the stream can no longer become unstable.

Two-dimensional Sheets

Sheet-like plumes are of more direct interest because of their possible application to subducting plates. A two-dimensional plume is stable for small outlet-to-surface heights, with the sheet contracting in both horizontal directions at first and then expanding again as it approaches the surface and comes under compression. When the height is increased instability sets in abruptly at a critical height. The motion now consists of a two-dimensional folding, as shown in Fig. 2.

The dimensionless critical height can again be extrapolated to the condition of zero surface tension to give $H_c/d_0 = 1.1$. Oscillations begin at a finite frequency at the critical height, and the frequency increases linearly with height above that. The most important result, which continues to hold for the interfacial experiments, is that the critical height depends only on the plume width, while the frequency depends on d_0, the exit velocity, and the ratio of gravitational and viscous forces.

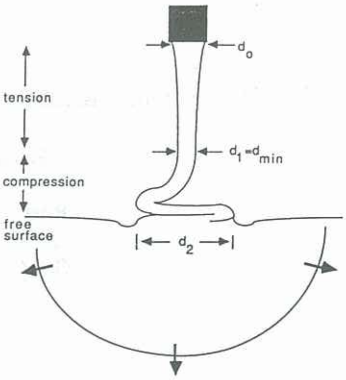

Fig.2. Sketch of the folding instability of a sheet plume falling onto a free surface.

PLUMES AT A DENSITY INTERFACE

Axisymmetric plumes of pure syrup have been released into an upper layer of glycerol, above a lower layer composed of mixtures of syrup and glycerol. In addition to the height H of the source above the interface the behaviour now depends on the density ratio β defined by $\beta = (\rho - \rho_B)/(\rho - \rho_T)$ where ρ is the density of the plume fluid, ρ_B is the bottom layer density and ρ_T the top layer density. With $\beta = 0$ and H small the plume is stable and spreads as a hemispherical intrusion at the top of the lower layer of its own density. With small but non-zero values of β and small H, a stable plume forms a thickened bulbous front which sinks through the lower layer, carrying some upper fluid with it. Plumes falling from greater heights again spiral as they approach the interface. Only for very small β can the mixture become neutrally buoyant at the interface and spread out along it, since relatively little upper layer fluid is trapped between the coils. For still larger values of β the coiled mixture is denser than the lower layer fluid and descends steadily, carrying the spiral structure with it (Fig.3). At the highest values of β used the turns of the spiral remain separated, and a larger volume of upper layer fluid is pulled down by the spiral.

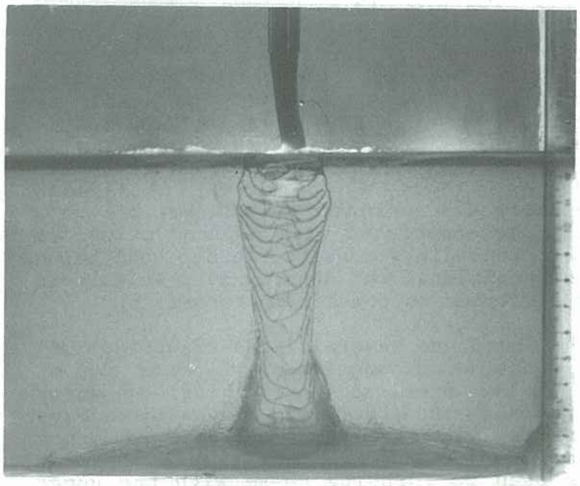

Fig.3. Axisymmetric plume of syrup falling through a layer of glycerol onto a lower layer of glycerol-syrup mixture. The plume is unstable at the interface and carries trapped fluid down between the coils.

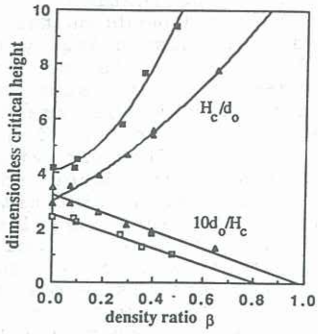

Fig.4. Dimensionless critical heights H_c/d_0 (solid) and the inverse (open points) plotted against density ratio β. Results are shown for sheets from a 6 mm wide slit (squares) and axisymmetric plumes from a 1.1 mm diameter nozzle.

The critical height H_c, for a given nozzle size d_0, increases with increasing β, as shown in Fig.4. This is a result of the decreasing resistance of the interface to penetration by the plume. The critical height at $\beta=0$ is $H_c/d_0 = 2.9$, about twice that for axisymmetric plumes in air after extrapolation to zero surface tension. This difference is due to the smaller viscosity contrast between the plume and its surroundings, as discussed by Griffiths and Turner (1988a). Onset of the coiling instability again occurs with a well-defined frequency, and above H_c the frequency is independent of β and proportional to height. The ratio of plume diameters just above and below the region of coiling is 3.3 ± 0.3, independent of all parameters and close to the value obtained by Cruikshank and Munson (1982) for plumes in air. It can also be shown that this result implies a constant fractional energy loss on buckling which is remarkably insensitive to all parameters.

SHEETS AT A DENSITY INTERFACE

A comparable series of runs has been carried out using 10cm long slit sources of various widths to release a line plume or sheet of syrup into an upper layer of glycerine. The density and viscosity of the lower layer are varied simultaneously by using a lower layer consisting of syrup or mixtures of syrup and glycerol; the density ratio and viscosity ratio between the plume and the upper layer are not changed. It is much more clearly seen in this case how viscous drag results in circulation cells being formed in the upper layer, one each side of the sheet, and causes a layer of glycerol to be pulled down through the interface by the descending plume. A boundary layer of this kind can also be seen in numerical simulations of mantle plumes penetrating the transition zone, such as that reported by Christensen and Yuen (1984).

When the distance between the slit and the interface is increased above a critical level H_c the sheet plume buckles and folds, as it does on a free surface. The value of H_c depends again on the density ratio β defined above, and the dimensionless height H_c/d_0 is plotted against β in Fig.4 for sheets as well as for axisymmetric plumes. The value at $\beta=0$ is about four times the free surface value at zero surface tension.

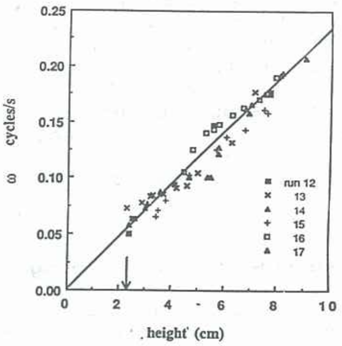

Fig.5. Plots of frequency versus height of fall for six experiments on sheet plumes.

The dimensional plot of frequency versus height for folding sheets shown in Fig.5 indicates that ω depends linearly on height, with no systematic dependence on β. The wavelength L of the folds defined by $L=V_{max}/\omega$, where V_{max} is the maximum velocity just above the interface, is also independent of H or β.

The presence of the surrounding glycerol layer and the lower layer of denser viscous mixtures of glycerol and syrup make a profound difference to the behaviour below the interface. The two-dimensional folding traps much more fluid than does the coiling of axi-symmetric plumes, and so the decrease in density of the 'mixture' is larger. Thus spreading of the folded sheet and the incorporated material occurs over a wider range of β values (> zero) than is possible in the axi-symmetric case. (See Fig.6.) Because the Reynolds number is so low the upper layer fluid is only trapped by the folds rather than being thoroughly mixed into them, and given long enough, the two fluids can separate again. Glycerol bubbles up into the upper layer, and the remaining plume material, being denser, sinks to the bottom of the tank.

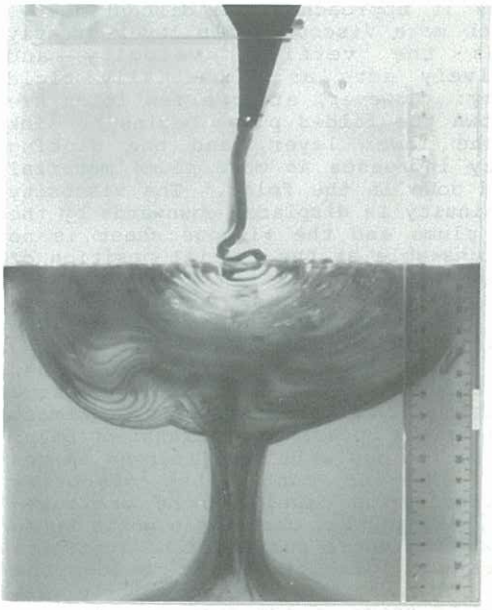

Fig.6. Sheet plume falling on an interface at which there is a large change in buoyancy. Though the plume is denser than both layers the trapping of upper layer fluid between the folds forms a neutrally buoyant mixture at the interface.

As β is increased further, unstable sheets can no longer remain at the interface following buckling, but instead sink through the lower layer. There is a constant ratio between the width of the sheet just above the interface and the folded plume just below. The fractional entrainment (the amount of entrained fluid compared with the flux in the plume) increases with increasing β, with a large increase for density ratios greater than about 0.3.

Viscosity Dependence of Folding

Additional experiments were carried out using upper layers either of water or glycerol plus syrup mixtures, to assess the importance of the viscosity ratio between the plume and upper layer fluid. Folding of plumes in longitudinal compression can only occur if bending of the plume and the induced movement of the outer less viscous fluid causes less dissipation of energy than would smooth spreading of the plume. The critical height for folding should thus be indefinitely large for comparable viscosities and should decrease as the plume becomes much more viscous than its surroundings. This behaviour is predicted by the theory of Biot (1961) and is borne out by our experiments, which also show that the dimensionless critical height approaches an asymptotic value above a viscosity ratio of about 10^4.

EFFECTS OF A VISCOSITY DISCONTINUITY

The experiments described above have already included a viscosity step across the interface, due to the changes in composition used to vary β. To investigate situations dominated by a viscosity step alone we used a lower layer of syrup and glycerol with above it an aqueous solution of much lower viscosity, and a very small density step i.e. β in the range $0.9<\beta<1$.

When the height of the source from the viscosity discontinuity is large enough a sheet of viscous syrup initially begins to fold as it approaches the discontinuity. The much more viscous lower layer greatly reduces the vertical velocity and effectively acts as a partially rigid boundary. However, after a few folds are laid down the folded plume begins to sink into the lower layer, and the sinking velocity increases as more plume material is laid down in the folds. The viscosity discontinuity is displaced downwards by the folded plume and the viscous sheet is no longer unstable at the initial position of the discontinuity. Instead it falls stably through a 'well' of low viscosity fluid until it folds onto the top of the descending composite blob.

Although these effects of a large compositionally induced viscosity contrast between layers appear to be transient only, the situation may well be different in the Earth. In the mantle the viscosity discontinuity is likely to be pressure-induced, and in that case there would be no low-viscosity entrained fluid to lubricate the descent of the plume below the discontinuity, so that the instability will continue to occur at the same level as the flow evolves.

IMPLICATIONS FOR THE EARTH

The detailed application of these results to the mantle depends on the nature of the major seismic discontinuity near a depth of 670 km which is at present imperfectly understood, but some general conclusions can already be drawn. A folding instability will occur if a near-vertical slab has a sufficient viscosity contrast with its surroundings and its length is greater than about five times its thickness. Thus cool slabs are likely to be unstable either if mantle circulation involves two layers separated by a density interface or if there is whole-mantle convection associated with a large viscosity increase with depth near 670 km.

If a slab remains planar as it passes through a density interface, a boundary layer of lighter fluid is pulled into the lower layer, and the consequent mass flux can be predicted. If the slab becomes unstable there is another more efficient entrainment mechanism, as upper layer fluid is trapped between the folds of the slab, and is carried into the lower layer. The effective entrainment increases as the density difference between the upper and lower layers decreases. On the other hand, when there is a substantial density difference between the layers a dense slab can cease to sink through the interface and instead spreads out along the interface because it is unstable and traps enough upper layer fluid between its folds to become neutrally buoyant. Each of these processes has implications for the geochemistry of the mantle which remain to be explored.

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