TENTH AUSTRALASIAN FLUID MECHANICS
CONFERENCE - UNIVERSITY OF MELBOURNE
11-15 DECEMBER 1989

NUMERICAL PREDICTION OF TRIPPED FLOW PAST A RESONATOR TUBE

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ABSTRACT

The flow and acoustic field around a trip rod located upstream of a resonator tube are investigated numerically. The separated flow from the rod is modelled using a vortex method with the surface vorticity technique employed to satisfy the no-slip condition at the rod surface. The acoustic field, largely defined by the organ pipe mode of the tube, is approximated in the region around the rod and the tube opening by an oscillating potential flow solution. Howe's theory of aerodynamic sound is used to determine the acoustic power output resulting from the interaction of shed vortices with the resonant sound. It is shown that the excitation of the tube resonance can result in the locking of the vortex shedding from the rod, in line with recent experimental flow visualisation using the free surface hydraulic analogy.

INTRODUCTION

Numerous studies have investigated the acoustic resonances induced by vortex shedding from bluff bodies located inside flow ducts. The acoustic resonance frequency depends on the dimensions of the body as well as the dimensions of the duct. For single plates or cascades, it is common for a transverse mode of the modified duct to be excited (e.g. Parker (1966, 1967, 1969); Parker and Griffiths (1968); Welsh and Gibson (1979); Cumpsty and Whitehead (1971); Archibald (1975)).

Welsh *et al.* (1984) examined the fluid mechanics of the resonant process in a duct containing a plate with semicircular leading edges using Howe's (1975, 1984) theory of aerodynamic sound. The resonant process is described in terms of an interchange of energy between the flow and acoustic fields and has three basic components: (a) a sound source (the vortex street), (b) a feedback effect of the sound on the vortex shedding, and (c) a damping process whereby acoustic energy is transferred out of the duct system.

The resonant acoustic field of a cavity can also be excited when a vortex shedding body is located externally, as demonstrated by Vrebalovich (1962) for a tube with a single open end facing a high-velocity wind tunnel and either a ring trip or a wedge trip placed upstream. A detailed study of the strong acoustic pulsations generated in such a resonator tube using wedge trips was undertaken by Brocher and Duport (1988). Flow visualisation of the vortex shedding from a wedge trip and the coupling of the shedding with the tube resonance has been presented by Kawahashi *et al.* (1988) using the hydraulic analogy. The resonator tube has a number of valuable practical applications, including the acoustic stimulation of boundary layers on plates located upstream of the tube (Brocher 1985). At present, little understanding exists of the mechanism that sustains the acoustic resonance of the tube when the flow is tripped upstream.

The aim of this paper is to describe the acoustic sources in the flow around a trip rod in front of a resonator tube in terms of the flow and the acoustic field near the rod using Howe's theory of aerodynamic sound and the solution of the flow field. A vortex model simulates the shedding of vortex clouds from the trip rod and their passage past the resonator tube, showing how the large-scale vortex shedding rate is locked to the sound frequency and predicting the generation of acoustic power.

MATHEMATICAL MODELLING

In the following model, the tube and rod geometries are based on the experiments of Kawahashi *et al.* (1988). Figure 1 shows a schematic of the geometry of the resonator tube with the trip rod placed upstream symmetrically about the centreline of the tube.

Acoustic Field

The acoustic mode to be modelled is a standing wave corresponding to an organ pipe mode of the tube. In the flows of interest here, the Mach number is small and the acoustic pressure p satisfies the wave equation:

$$\frac{\partial^2 p}{\partial \tau^2} = c^2 \nabla^2 p, \qquad (1)$$

where c denotes the velocity of sound and τ is time. The time-independent amplitude function ϕ can be extracted from a standing wave solution $p = \phi e^{i2\pi f\tau}$, where f is the frequency. Then ϕ satisfies the Helmholtz equation:

$$\nabla^2 \phi + (2\pi f / c)^2 \phi = 0. \qquad (2)$$

For the low frequency modes, which are symmetrical about the horizontal mid-plane, the following boundary condition applies on the rigid surfaces and also on the mid-plane:

$$\underline{k} \cdot \nabla \phi = 0, \qquad (3)$$

where $\underline{k}$ is the unit vector normal to the surface.

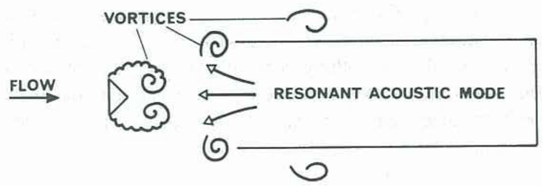

Figure 1 Schematic of resonator tube and trip rod.

Attention is focussed on the region near the rod and the tube opening where the shed vortices can do work on the sound field. The resonant mode of the tube of interest has a wavelength greater than the length of the tube, which in turn is much greater than the vortex shedding region. Therefore, the sound field is approximated well in this region by the Laplace equation. The tube is assumed to be semi-infinite with a potential flow source at infinity. The magnitude of the potential flow source is set at a value that opposes the upstream flow and results in zero net flow into the tube.

Flow Modelling

The flow is modelled by a two-dimensional inviscid incompressible flow, irrotational everywhere except at the centres of elemental vortices. Because of symmetry, only the flow and the resonant acoustic field above the longitudinal centreline is explicitly modelled. The shedding of vorticity is modelled by the creation of the elemental vortices, which are convected under the influence of other elemental vortices and the irrotational flow.

The surface of the trip rod is modelled using the surface vorticity method (see, for example, Lewis (1981), Stoneman *et al.* (1988)). The surface vorticity method, in brief, requires that the contour along the body surface is a streamline and that the tangential velocity on the inside of the vortex sheet is zero. Denoting the distance along the body surface by s, discretisation of a vortex sheet into M segments, with the nth segment having length Δs_n and linear vorticity density $\gamma(s_n)$, produces a set of linear equations:

$$\sum_{n=1}^{M} \gamma(s_n)K(s_n,s_m)\,\Delta s_n - \frac{1}{2}\gamma(s_m) = -(v_x + v_{x,a})\left(\frac{dx}{ds}\right)_m - (v_y + v_{y,a})\left(\frac{dy}{ds}\right)_m - \sum_{n=1}^{N_v}\Gamma_n T(n,m), \quad (m = 1,M) \qquad (4)$$

where the last term in (4) gives the contribution to the velocity field at the surface due to N_v free vortices of circulation Γ_n in the flow. The coupling coefficient $K(s_n,s_m)$ has the value of the surface tangential velocity at s_m induced by a vortex of unit circulation at s_n. The coupling coefficient $T(n,s_m)$ has the value of the surface tangential velocity at s_m due to a vortex of unit circulation at the position of the nth free vortex. The velocity of the irrotational flow has been separated into a steady component (v_x,v_y) and an unsteady component $(v_{x,a},v_{y,a})$ contributed by the local potential approximation to the acoustic field. The solution of (4) gives the surface vorticity density at the pivotal points on the surface of the body, which are taken to be the centre of each discrete element.

At each time step, the elemental vortices are created having the circulation of each vortex segment on the surface of the trip rod; vortices on the surface of the trip rod are released into the flow to simulate the formation of the boundary layers.

The elemental vortices are potential vortices with smoothed cores (Rankine profile) of radius $0.03b$, where b is the dimension of the leading edge of the triangular trip rod. Test cases with larger and smaller smoothing cores showed that the formation of the large-scale vortex structures was insensitive to the exact smoothing value used. The vortices are advected using a second-order Adams-Bashford scheme; a time-step of $0.09(b/v_\infty)$ is used, where v_∞ is the upstream flow velocity. As mentioned above, prediction of the level of the resonant acoustic particle velocity amplitudes was not attempted; these amplitudes were determined from the empirical data. For input to the numerical model, the value for the amplitude of the acoustic particle velocity was set at the level such that the maximum mean outflow velocity from the resonator tube was equal to the upstream velocity at infinity.

Interaction of Flow and Sound

Howe (1975) showed that when an acoustic oscillation occurs in an inviscid, isentropic fluid, but with regions of rotational flow (vortices), an acoustic power P is generated in a volume V, which is given by:

$$P = -\rho_0 \int \underline{\omega} \cdot (\underline{v} \times \underline{u})\, dV, \qquad (5)$$

where $\underline{v}$ is the fluid velocity, $\underline{\omega} = \nabla \times \underline{v}$ is the vorticity, $\underline{u}$ is the acoustic particle velocity and ρ_0 is the mean density of the fluid.

When the vorticity is 'compact', that is when the vorticity extends over a region which is small relative to the acoustic wavelength, then the acoustic power/unit length of vortex tube generated by a vortex reduces to:

$$P = -\rho_0 \Gamma \underline{k} \cdot (\underline{v} \times \underline{u}), \qquad (6)$$

where Γ is the circulation of the vortex and $\underline{k}$ is the unit vector normal to the plane of the flow.

RESULTS

Predicted Acoustic Field

The local acoustic particle velocity magnitudes associated with the longitudinal mode, having wavelength approximately four times the length of the tube, with the trip rod in place are shown in Fig. 2.

Vortex Cloud Model

'Snapshots' of the predicted flow are shown at different instants of the acoustic cycle for the acoustic Strouhal number $St_a = 0.42$ in Fig. 3. Here, $St_a = f\,W/V_\infty$, where f is the resonant acoustic frequency, W is the resonator tube width, and v_∞ is the flow velocity at upstream infinity. This case corresponds to the flow in Region 3 of Fig. 6 in Kawahashi *et al.* (1988). The sound field at the opening of the tube is assumed to be 100% efficient. That is, when the acoustic particle velocities are maximum and directed into the tube, the blockage effect of the tube is totally

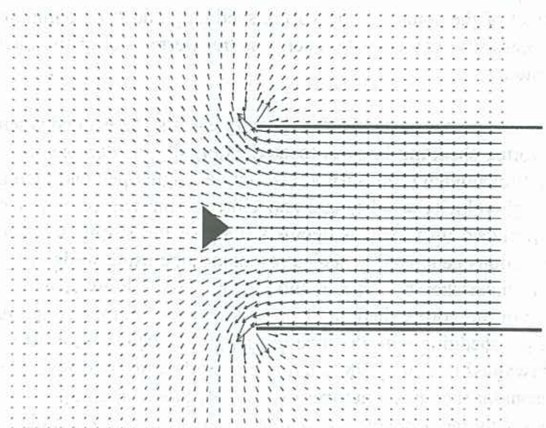

Figure 2 Local acoustic particle magnitudes of longitudinal mode.

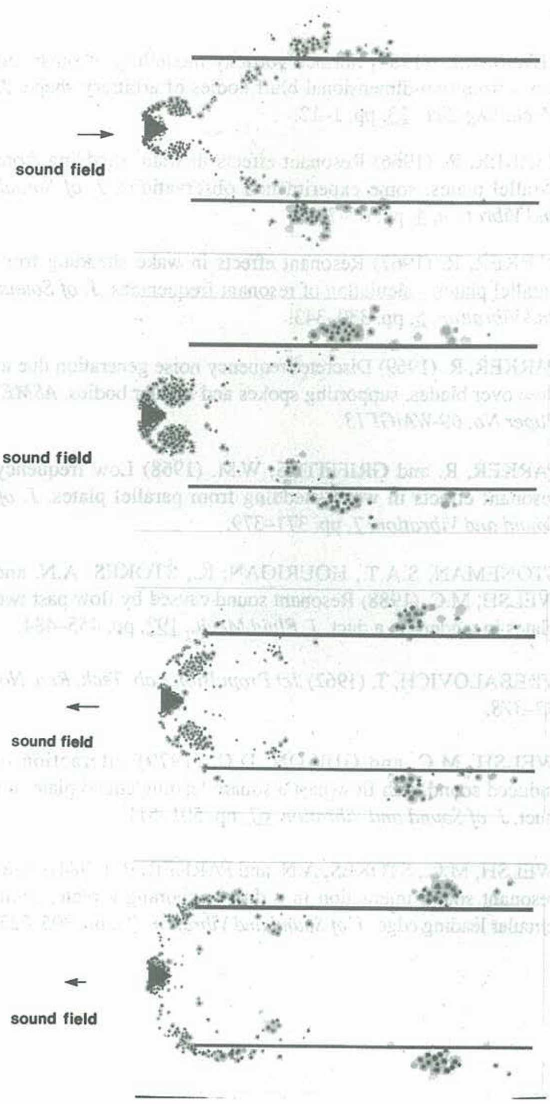

Figure 3 Instantaneous plots of elemental vortex positions at different phases of shedding cycle.

negated. This efficiency is not always obtained in practice, although it is sometimes approached. It is found during resonance that the shedding rate of the vortex clouds from the trip rod is locked to the acoustic frequency.

Figure 4 shows the predicted acoustic power output P generated by the flow over a number of cycles for the acoustic Strouhal number $St_a = 0.42$. The model predicts that a major contribution to the acoustic power comes from near the trip rod.

DISCUSSION OF RESULTS

The shedding rate of vortex clouds from the trip rod is predicted to be locked to the sound frequency in the presence of the strong sound field. The predicted flow structures show close resemblance to those observed by Kawahashi *et al.* (1988).

According to the theory of Howe (1975) and embodied in Eq. (6), a necessary condition for the generation of acoustic power by the flow field is that vortices must cut across acoustic particle velocity field lines. In the present case, this can occur to the greatest extent in the flow between the trip rod and the opening tips of the resonator tube. This is clear from Fig. 2, which shows

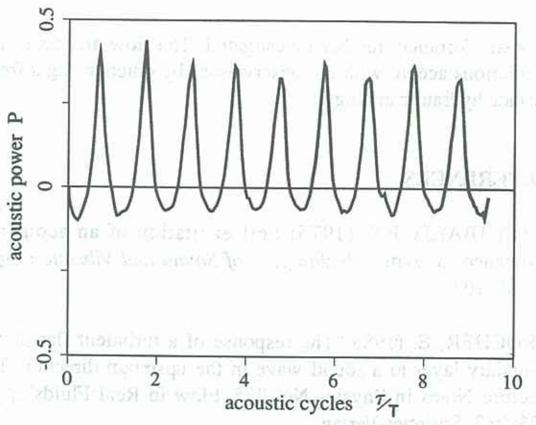

Figure 4 Predicted acoustic power output P versus time τ, normalised to acoustic period T.

the direction of the local acoustic particle velocities, and Fig. 3 which indicates the movements of the large-scale vortex structures shed from the trip rod. The acoustic particle velocities between the trip rod and the tube mouth possess significant components orthogonal to the paths of the vortices shed from the trip rod, thus allowing relatively substantial transfer of energy between the flow field and the resonant acoustic field.

During the first half of the acoustic cycle when a vortex structure is formed on each side of the trip rod, the sense and direction of the local acoustic particle velocities are such that acoustic energy is absorbed. However, the vortices are not fully formed during this half cycle and they are moving relatively slowly whereas in the second half of the initial sound cycle, the vortices are at full strength and have accelerated. According to (6), the positive acoustic power generated during this second half cycle will exceed the acoustic power absorbed during the first half cycle. Therefore, the acoustic resonance can be sustained by the net transfer of positive acoustic energy each sound cycle by the vortices between the trip rod and the tube opening.

Downstream of the tube opening, the acoustic particle velocities are small and are directed almost parallel to the mean flow and the paths of the vortex structures, resulting in little acoustic power generation in this region.

The acoustic source region therefore lies in the region immediately behind the trip rod. The acoustic resonance is sustained by the shedding of vortices in a symmetrical manner, which in turn is due to the 'locking' of the vortex shedding rate by the acoustic field. This is effectively acoustic feedback. The feedback is via a resonance, which means that most of the sound energy which influences vortex shedding at any one time was generated in previous cycles, and has been reflected, usually many times, from the resonator tube termination. In the present study, no attempt has been made to investigate the initial transient excitation of the resonance. This will be the subject of future investigations.

CONCLUSIONS

The resonant sound field of a resonator tube has been shown to couple with the vortex shedding from a trip rod located upstream of the tube. The feedback of the sound field, via a velocity perturbation on the separating shear layers from the trip rod, synchronises the vortex shedding to the sound. Net acoustic power is predicted to be generated at the value of the acoustic Strouhal number investigated. The flow and acoustic predictions accord with the observations by others using a free surface hydraulic analogy.

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