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THE RESPONSE OF A STRATIFIED LAKE TO A SURFACE SHEAR STRESS

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ABSTRACT

A laboratory experiment is described which examines the two dimensional response of an initially three layered fluid to an applied surface shear stress. The importance of the second baroclinic mode is illustrated and the results are discussed in the context of parameterisation of the response of stratified lakes and reservoirs to wind stress.

§1 INTRODUCTION

The response of a density stratified, three layer, two dimensional, confined fluid to an applied wind stress is studied experimentally. The upper layer represents the surface mixed layer, the lower layer represents the hypolimnion and the middle layer represents the interfacial region (the pycnocline, or thermocline in a temperature stratified lake). An experiment is described and compared with parameterisation, and two first order models are introduced to elucidate the behaviour of the middle layer in the start-up period of the experiment.

The surface stress imparted to the water by the wind must be balanced by the surface deflection, Thorpe (1977) describes maximal values in Loch Ness (length 40 km) of 50mm in 'exceptional conditions'. The internal density distribution subsequently adjusts to balance this pressure gradient. It is this response that is studied here. In a two layered fluid the interface tilts and, if the stress is great enough, the lower layer actually reaches the surface and is transported down-wind by a shear dispersion mechanism (Imberger and Monismith, an appendix to Monismith 1986). The tilting and surfacing of the deeper fluid is refered to as upwelling.

The motion of the fluid can be described as a combination of internal waves, the normal modes, as introduced by Lighthill (1969). An idealised fluid consisting of N discrete layers has $N-1$ baroclinic normal modes (the $N th$ mode is a barotropic or surface wave and is ignored in this study). If any portion of the profile is continous there are an infinite number of modes. However the amplitude of the modes generally decays with the modal index, so approximating a continuous profile with several discrete layers limits the number of modes in the problem. The effect of an N layer discretisation of a continuous profile is to confine the energy to the $N-1$ internal modes. An example of the effect of confining this energy can be illustrated by using the two layer Wedderburn number parameterisation (Spigel and Imberger 1980). With this parameterisation the response of the interface is described by a force balance of the surface stress and the baroclinic restoring force. The Wedderburn number is described thus,

$$W = \frac{\epsilon_{12} g h_1}{u_*^2} . \frac{2h_1}{L}, \qquad (1)$$

where $\epsilon_{ij} = (\rho_j - \rho_i)/\rho_j$ represents the modified gravity of the $i th$ layer relative to the $j th$ layer, u_* is the shear velocity at the surface, g is acceleration due to gravity, ρ_i is the density in the $i th$ layer and h_1 and L are the thickness of the surface layer and the longitudinal length scale for the problem respectively. The Wedderburn number in the form (1) is exact for a two layer fluid, ignoring non-linear terms and representing the stress as a constant body force in the surface layer (see §4.1). The parameterisation implies that the interface reaches the surface (upwells) at the upwind end for $W = 1$; the laboratory experiments of Monismith (1986) show upwelling at values of W greater than 1.

Previous experimental work such as Keulegan and Brame (1960), Kranenburg (1985) and Monismith (1986) have examined the response of the two layer structure. An infinitely thin interface is difficult to achieve in the laboratory and in the field, consequently comparisons of analytic models, experimental models and field observations must be approached carefully. The response of the continuous density interface is examined here by introducing a third discrete layer at the interface to approximate the continuously stratified region (see figure 1). While the motion in the experiment can not be assumed to be confined to two baroclinic modes the effects of ignoring them are greatly reduced.

§2 THE EXPERIMENT

The laboratory experiment was performed in a glass tank ($2.0 \times 0.4 \times 0.4m$) with a horizontal moving belt in contact with the fluid surface to introduce the shear stress and the stratification was achieved using saline fluid. The initial density profile of the experiment described here is shown in figure 1.

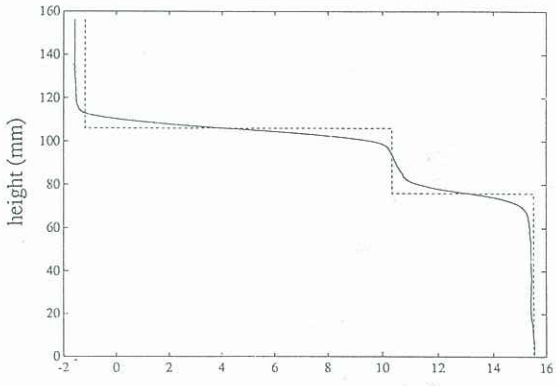


figure 1; the initial density profile and the approximate three layer structure.

The stress was introduced linearly over a time comparable with the first baroclinic period ($T_1 = 55.6$ seconds and the second internal period is $T_2 = 132$ seconds, these periods are calculated using the normal mode analysis Csanady 1982) to remove the seiching from the response. The combination of belt speed and roughness is sufficient for the transition to a turbulent boundary layer beneath the belt to occur at the very upwind end of the belt. As the flow develops the surface layer becomes fully turbulent and the surface stress is transferred to the upper interface. Consequently the interfacial layer upwells within $T_2/4$ (as suggested by Spigel and Imberger 1980). This is shown at the upwind end in figure 2 (a) where the isopycnals intersect the maximum height of data line at $t \approx 40$ seconds ($T_2/4 = 33$ seconds). However the deeper fluid never upwells, instead it reaches its maximum height at $t \approx 80$ seconds. At the downwind end the picture is more straight forward, the interfacial layer is simply removed by $T_2/4$. The ramp-up in the stress input was intended to remove any inertial seiching, it appears to have not been completely successful, although the seiching may also be attributed to the fact that the the stress input over-shot the desired value. After the initial adjustment the interface slowly deepens by a combination of surface stirring, interfacial shear entrainment and most importantly via shear dispersion of the upwelled fluid into the surface layer.

The quasi-steady structure (ignoring the gradual deepening) is of a mean tilt over the entire basin, with the interfacial layer forced to the upwind end. Consequently the middle layer spreads at the upwind end causing a diffuse interface and its removal from the downwind end generates a very sharp interface. The most striking observations from the experiment are how turbulent the surface layer is under such forcing and also the very obvious transport in the middle layer from the downwind end to the upwind end.

The stress was switched off at $t = 440$ seconds. The mean tilt relaxed and importantly the spreading/sharpening of the interfacial layer also relaxed, finally having approximately the same thickness throughout the fluid. The initial and final profiles are shown in figure 3.

To compare the experiment with one dimensional surface layer entrainment models, two layer parameters must be estimated. Using the inverse Richardson number relationship and the coefficient from Kranenburg (1985) the entrainment velocity (non-dimensionalised by u_*) of the base of the surface layer is given as

$$\frac{u_e}{u_*} = 0.07\frac{u_*^2}{\epsilon_{13} g h_1}. \qquad (2)$$

The entrainment law (2) is comparable to the observations of figure 3, however the observed Δh_1 is approximately three times as large as that suggested by (2).

§3 PARAMETERISATION

The Wedderburn parameterisation discussed earlier uses the parameter W based on the steady state response of the fluid to infer the dynamics of the response. If $W << 1$ the fluid is very unstable and will 'turn over' and mix rapidly. If $W \simeq 1$, as in the experiment described above, there is a balance between the surface stress and the stratification, even so the mixing/entrainment still occurs via by a number of mechanisms. For $W >> 1$ the fluid is very stable and little change will occur to its structure. Note that all parameterisation discussed here uses initial conditions.

As a first attempt to compare the experimental response with this existing two layer parameterisation two Wedderburn numbers (W_{12} and W_{23}) are introduced, one for each interface, each assuming the other interface doesn't exist. These parameters cannot be directly employed in the way that W is. The stress cannot do the same work twice, instead W_{12} must be evaluated and if it indicates upwelling then there is a path for the stress to be transferred to the lower interface. Consequently W_{23} can be employed to categorise the response of this lower interface. If $W_{23} >> 1$ the lower interface is very strong and the motion is confined to the upper two layers, if on the other hand, $W_{23} \sim 1$ then large baroclinic motions are setup in the lower two layers

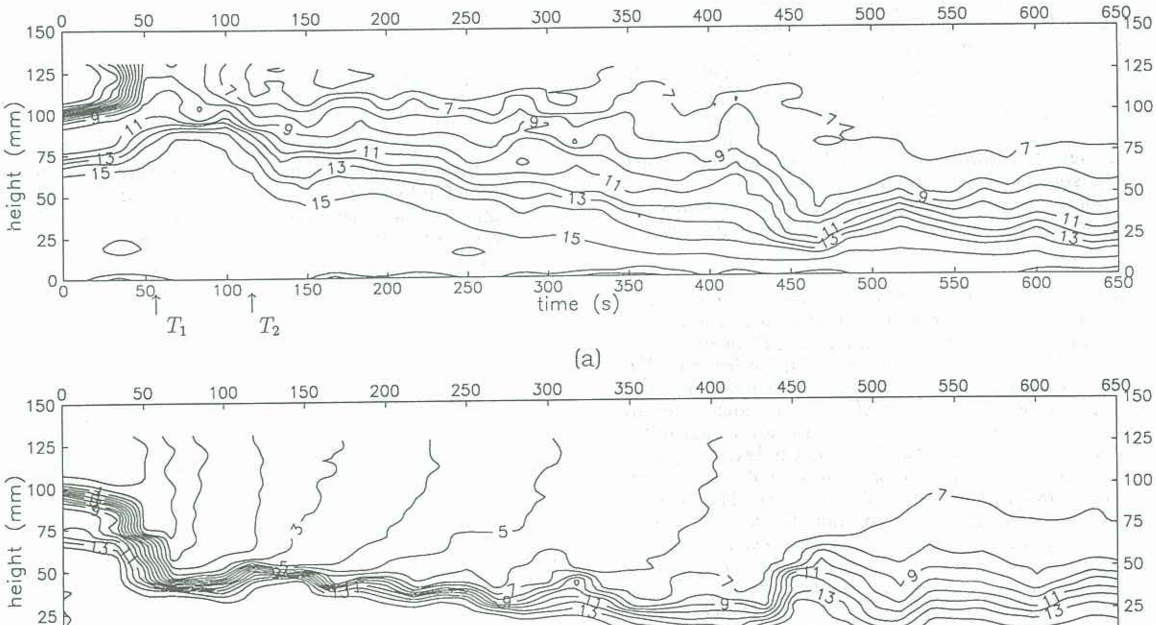


figure 2; contours of the density structure in depth and time, at (a), $x/L = 0.26$ and (b), $x/L = 0.89$ (n.b. the depth contours start at 6 mm above the tank floor).

of the basin. The two parameters classify the response of the entire water body and are given, along with their mean value in the experiment, as

$$W_{12} = \frac{\epsilon_{12} g h_1}{u_*^2} . \frac{2h_1}{L} = 0.9,$$
$$\text{and } W_{23} = \frac{\epsilon_{23} g (h_1 + h_2)}{u_*^2} . \frac{2(h_1 + h_2)}{L} = 1.1. \quad (3)$$

The extension from three layers to a continuous profile is embodied in the Lake number (Imberger and Patterson 1989).

The parameters (3) for this experiment suggest firstly, the intermediate layer will upwell and secondly, the bottom layer will probably also upwell. The transition to these baroclinic states must drive significant motions in the lower layers as well as the expected surface layer response. The observations agree with these results, although the bottom layer never reaches the surface (figure 2a). The lower interface is not preserved and cannot be identified in the final profile (figure 3), suggesting that significant mixing has occured deep in the water column.

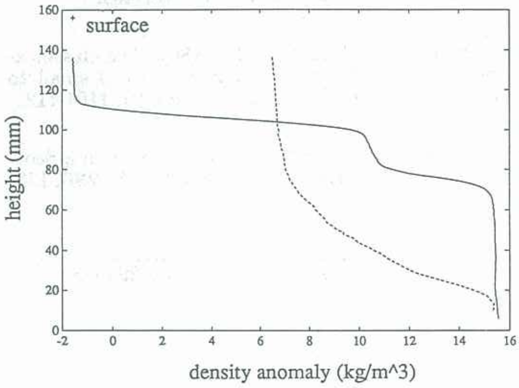


figure 3; the initial (solid) and final (dashed) density profiles.

§4 SIMPLE MODELLING OF THE DYNAMICS OF THE INTERFACIAL LAYER

§4.1 The stress distribution

Simple two layer models often assume the stress decays linearly to zero from the surface to the base of the surface layer and that the stress is represented by a body force in the horizontal momentum equation. Consequently in any first order model of the interface tilt such as the one following it is inappropriate to use a model higher than first order for the stress input. However extending the stress input to the base of the middle layer provides an insight to the reponse to a stress input that penetrates past the base of the surface layer. Figure 4 illustrates the two stress distributions.

For the case where the stress decays to zero at the base of the upper layer, the steady linearised momentum equation for this layer is

$$0 = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + F, \quad (4)$$

where ρ_0 is a reference density and F is a body force exerted by the stress. Equation (4) is inviscid apart from the stress input and by definition $u_*^2 = K u_z$, where K is a kinematic eddy viscosity. Integrating over the surface layer gives the steady state interfacial response as,

$$\frac{d\zeta_1}{dx} = -\frac{u_*^2}{\epsilon_{12} g h_1} = -Ri^{-1}, \quad (5)$$

where ζ_1 is defined as positive upwards and the x origin is at the upwind end of the fluid.

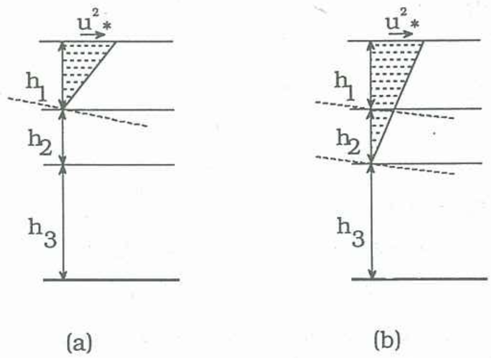


figure 4; model stress distributions, decaying to zero, (a) at the base of the surface layer and (b) at the base of the intermediate layer.

The RHS of (5) is identified as the inverted bulk Richardson number Ri, so as the stabilty of a surface layer increases ($\uparrow Ri$), the interfacial tilt decreases. The stress boundary condition at the base of the surface layer is set to zero for the integation over the depth of the layer. Consequently repeating the integration for the second layer, yields $d\zeta_2/dx = 0$. Hence the second interface must remain horizontal until the stress can act on the layer directly. This is borne out by the observations of figure 2(a) where it can be seen that the lower layer does not move appreciably until $t = 40$ seconds. The upper interfacial tilt (5) is evaluated with initial conditions, as $d\zeta_1/dx = -0.057$ for this experiment (hence an upwind deflection of 55 mm).

Now the analysis is repeated for the case where the stress decays to zero at the base of the second layer. In this case the gradient is weaker so the body force is reduced, but now the stress can directly drive the lower layer. Consequently

$$\frac{d\zeta_1}{dx} = -\frac{u_*^2}{\epsilon_{12} g h_1}\frac{h_1}{h_1 + h_2} = -Ri_{12}^{-1}\left(\frac{h_1}{h_1 + h_2}\right), \quad (6)$$

this reduces to (5) as $h_2 \to 0$. Now additionally

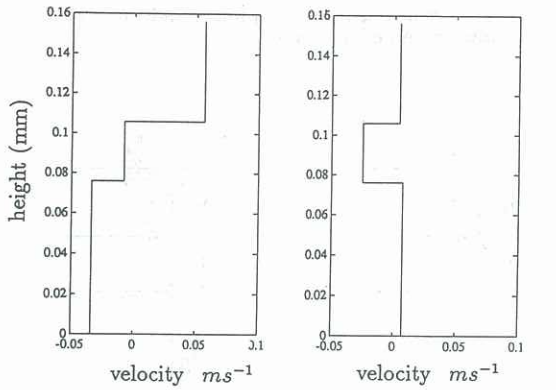
$$\frac{d\zeta_2}{dx} = -\frac{u_*^2}{\epsilon_{23} g h_1}\frac{h_1}{h_1 + h_2} - \frac{u_*^2}{\epsilon_{23} g (h_1 + h_2)},$$
$$= -Ri_{12}^{-1}\frac{\epsilon_{12}}{\epsilon_{23}}\frac{h_1}{h_1 + h_2} - Ri_{23}^{-1}. \quad (7)$$

This in turn reduces to the equivalent of (5) if $\epsilon_{12} = 0$ or if $h_1 = 0$.

For the initial conditions of this experiment the interfacial slopes are

$$\frac{d\zeta_1}{dx} = -0.038,$$
$$\text{and } \frac{d\zeta_2}{dx} = -0.125,$$

while the figure for the upper interface is realistic the second value suggests a very large slope on the lower interface. To evolve to this state implies motion in the middle layer directly opposed to the observations made in the experiment (§2). This is hardly suprising as that is the direction of the additional body force. However if the lower interface had been very stable the result given by (7) would have appeared appropriate, but not neccessarily correct. An alternative model must now be sought.



(a) (b)

figure 5; the peak horizontal baroclinic velocities (a) first mode and (b) second mode

§4.2 The baroclinic response

Another simple model to illustrate the response of the interfacial layer utilises the horizontal velocity structure of the two modes. The velocities of the modes are calculated by the normal mode technique (Csanady 1982 and Monismith 1985) where the surface stress is divided amongst the internal modes. The peak velocity structure of the modes is shown in figure 5. The velocities compare well with initial results from automated particle tracking techniques used in the experiments. The first mode has the two lower layers moving upwind, and the surface layer moving downwind, causing the mean tilt of the density structure. The second mode generates the upwind transport in the middle layer, while the upper and lower layers move downwind.

While the problem can be described with wave models, field observations suggest that the response is often overdamped, especially in weakly stratified environments (Monismith 1985). Consequently each mode is considered to exist for the first 1/4 of its respective wave period. Taking the centre of the tank ($x/L = 0.5$) as an observation point and integrating over time for the quarter-period, approximately 25% of the middle layer is transported past the observation point to the upwind section of the tank. This illustrates simply the upwind transport in the middle layer and the subsequent spreading of the upwind density interface.

§5 DISCUSSION

The importance of the modal structure in determining the response of the fluid has been established. While convenient, it can be misleading to take bulk observations, such as the total top to bottom density difference, and apply them to two layer models when using a model only slightly more complex reveals basin scale dynamics having a significant effect on the fluid motion. The link between the initial density distribution and the subsequent fluid dynamics will be used to take the parameterisation of the fluid response a step further.

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