TENTH AUSTRALASIAN FLUID MECHANICS
CONFERENCE - UNIVERSITY OF MELBOURNE
11-15 DECEMBER 1989

LAGRANGIAN STATISTICAL SIMULATION OF THE TURBULENT MOTION OF HEAVY PARTICLES

B.L. SAWFORD and F.M. GUEST

CSIRO Division of Atmospheric Research
Private Bag #1, P.O. Mordialloc, Vic. 3195
AUSTRALIA

ABSTRACT

A Lagrangian stochastic model for the motion of heavy particles has been developed by coupling a stochastic model for the motion of fluid elements to the Stokes equations of motion of a particle in a turbulent flow. The model has been used particularly to examine the effects of turbulence nonstationarity through simulations of the dispersion of heavy particles in decaying homogeneous turbulence.

INTRODUCTION

Many previous theoretical studies of heavy particle motion assumed stationary, homogeneous turbulence while experimental measurements and practical applications involve inhomogeneous and nonstationary flows. There is thus a need to develop models which can accommodate more realistic flows. One such approach is Lagrangian statistical modelling, which for passive scalar (i.e. fluid element) dispersion has recently developed beyond its heuristic beginnings to an objective, almost universal theory (Thomson, 1987).

In this paper we describe modifications to the Lagrangian statistical approach to represent the motion of heavy particles. These modifications, which essentially extend Csanady's (1963) approach for stationary homogeneous turbulence, are far from rigorous. Our philosophy is that heavy particle statistics represent in some sense a perturbation to fluid statistics and that the effects of turbulence inhomogeneity and nonstationarity will be reflected mainly through changes to the 'reference state' fluid statistics. Thus we have attempted to combine the understanding of heavy particle effects gained from studies in idealized stationary homogeneous turbulence with a good model of passive scalar dispersion for more realistic flows.

THE EQUATIONS OF MOTION FOR PARTICLES

We assume that the equations of motion of a particle are

$$\frac{dv_i}{dt} = \frac{1}{\tau_a}\left(u_i(x,t) - v_i\right) + g_i \qquad (1)$$

and

$$\frac{dx_i}{dt} = v_i \qquad (2)$$

where v is the particle velocity, u is the fluid velocity (and is evaluated at the location of the particle at time t), τ_a is the particle's aerodynamic response time and g represents external body forces such as gravity. Equation (1), which represents Stokes flow around the particle, is a considerable simplification of the exact equation for particle motion. However, it is a trivial extension of the present approach to use, for example, a non-linear drag law in (1). The major differences between the particle motion and fluid motion are evident from (1). External forces simply give the particle a mean velocity relative to the fluid (which we represent by the terminal velocity, $v^t = g\tau_a$) thus causing the particle trajectory to deviate from that of the fluid element which originally contained it. This is known as the 'crossing trajectories' effect. Inertia, represented by the time scale τ_a, has two effects; it also causes the trajectory of the particle to depart from that of the fluid element which originally contained it, and in addition it slows the response of the particle motion to the fluctuating velocity of its environment. Consequently, the statistics of u along the particle trajectory are in general different from those along the trajectory of a fluid element as a result of both inertia and external forces, and the statistics of fluctuations in v differ from those in u because of the damping effect of inertia.

In order to complete the specification of the particle trajectories we require a model for the fluid velocity u. We provide this through modification of a Lagrangian statistical model for the motion of fluid elements.

LAGRANGIAN STATISTICAL MODEL

Thomson (1987) recently presented a very general Lagrangian statistical theory of turbulent dispersion in which the velocity u and position x of a marked fluid element are jointly represented by a six-dimensional continuous Markov process specified by the Ito equation (Gardiner, 1983)

$$du_i = a_i(x,u,t) + \sqrt{C_o\varepsilon}\, dW_i(t) \qquad (3)$$

$$dx_i = u_i dt \qquad (4)$$

where ε is the dissipation rate and C_o is a universal constant. The random term in (18), $dW_i(t)$, is the incremental Wiener process (Gaussian white noise).

For an arbitrary turbulence field, a can be specified in terms of the assumed known Eulerian velocity statistics. Details are given in Thomson (1987) and Sawford and Guest (1988).

CORRECTION FOR CROSSING TRAJECTORIES

The Lagrangian time scale in our stochastic model is $T^{(L)}=2u^2/C_o\varepsilon$ where u is the r.m.s. turbulence velocity. In order to use this trajectory model to represent the velocity of the fluid along a heavy particle trajectory, we follow Csanady (1963) in accounting for the increased rate of decorrelation (compared with the velocity along the trajectory of a fluid element) by decreasing the correlation time scale by a factor $(1 + (\beta v^t/u)^2)^{1/2}$. For stationary homogeneous turbulence, this corresponds to an interpolation between the fluid Lagrangian integral time scale for $v^t/u \to 0$ and the time scale of fluid velocity fluctuations at a point moving along a straight line trajectory with velocity v^t, for $v^t/u \to \infty$. We treat the ratio β as an adjustable constant, but expect it to be of O(1).

DECAYING HOMOGENEOUS TURBULENCE

Similarity Laws

A common approximation in the description of decaying grid turbulence is obtained through the hypothesis of self-preserving development of the energy-containing scales of motion. The turbulence energy decays like

$$u^2 = u_0^2\,(x_1/x_{1,0})^{-m} \tag{5}$$

and the Eulerian length scale grows like

$$L = L_0(x_1/x_{1,0})^{1-m/2} \tag{6}$$

where $x_{1,0}$ is some reference point and m has a value 1.3±0.15. For isotropic turbulence the dissipation rate can be calculated as $\varepsilon = -(3/2)\,U\,du^2/dx_1$, where U is the mean velocity.

It is convenient to replace the inhomogeneous laboratory system by a nonstationary theoretical system, in which a continuous source dispersing with distance down the tunnel is replaced by an instantaneous source which disperses in time. For convenience we choose the reference point $x_{1,0}$ to coincide with the location of the source and the time origin in the transformed system to coincide with the source release. The Taylor transformation relating the laboratory and theoretical systems is therefore

$$x_1/x_{1,0} = 1 + t/t_0 \tag{7}$$

where $t_0 = x_{1,0}/U$. Note that the assumption of cross-stream homogeneity is only reasonable sufficiently far downstream for the wakes and jets behind individual elements of the grid to have fully coalesced. This state is reached about ten grid mesh lengths downstream.

Stochastic Dispersion Model

For decaying homogeneous isotropic turbulence, the Markov model introduced above reduces (Anand and Pope, 1985; Sawford and Guest, 1988) to the rescaled Langevin equation

$$d\left(\frac{u_i}{u}\right) = -\frac{1}{T^{(L)}}\left(\frac{u_i}{u}\right)dt + \sqrt{\frac{2}{T^{(L)}}}\,dW_i(t) \tag{8}$$

where we have identified a Lagrangian time scale

$$T^{(L)}(t;t_0) = \frac{2u^2}{C_0\,\varepsilon} = \frac{4t_0}{3C_0 m}\left(1+\frac{t}{t_0}\right). \tag{9}$$

This model can be solved analytically (Anand and Pope, 1985) for the velocity correlation, the diffusivity and the dispersion. It is important to emphasize that the dispersion process depends explicitly on the location of the source (through u_0 and t_0) even for $t \gg t_0$. It is clear from (5)-(9) that the effects of variation in source location or in turbulence properties due to different grids (i.e. in u_0 and L_0) can be eliminated by nondimensionalization using u_0 and t_0 as velocity and time scales respectively. This is the key to the comparison of dispersion data sets for different source locations or different grids. Warhaft (1984) showed that data for the dispersion of heat downstream of a line source in grid turbulence from different experiments and different source locations are collapsed by the nondimensionalization and that the rescaled Langevin equation with C_0=2.1 and m=1.32 represents the data to within experimental error.

Modification of Time Scales

Following the approach of Csanady (1963) as outlined above we modify the time scale by writing,

$$T^{(L)}(t;t_0) = \frac{4t_0}{3C_0 m}\left[1+(\beta v^t/u)^2\right]^{-\frac{1}{2}}\left(1 + \frac{t}{t_0}\right) \tag{10}$$

Thus, (8) with (10) represents our stochastic model for the fluid velocity along a particle trajectory in decaying homogeneous, isotropic turbulence. When coupled with (1) and (2) it provides a model for the motion of heavy particles in decaying grid turbulence.

NUMERICAL RESULTS

Figure 1 shows the effect of inertia (represented by $\hat{\tau}_a = \tau_a/t_0$) on the particle velocity variance for the case $v^t = 0$. The particle initial velocity has been chosen randomly from a normal distribution with variance u_0^2 but is uncorrelated with the initial fluid velocity. Contributions due to the decay of the initial conditions and to the turbulence interaction term have been plotted separately; the former is unaffected by the nonstationarity of the turbulence. For comparison, the turbulence velocity variance is also shown as the bold solid line.

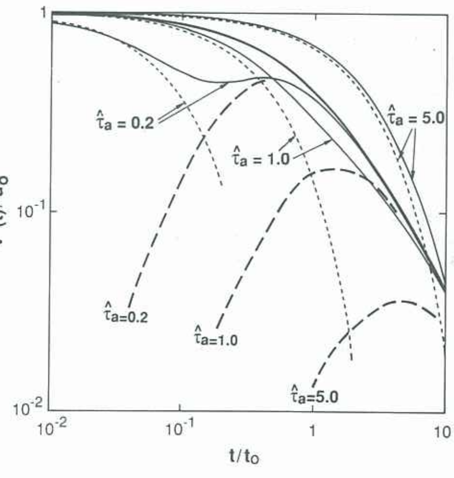

Figure 1. Effect of inertia on particle velocity variance in grid turbulence.
(- - -), source term
(— —), turbulence interaction term
(——), total
(——), fluid/passive scalar result

For small times such that $t < 0.1t_0$, the turbulence is approximately stationary and for small τ_a the particle variance approaches its asymptotic state before the effects of turbulence nonstationarity are felt. However, for large times the particle velocity variance approaches that of the fluid. This is quite different from the stationary case where the particle velocity variance asymptotes to the constant value $u^2/(1+\tau_a/T^{(L)})$. This difference occurs because in the nonstationary case the turbulence time scale increases with time and therefore the ratio of the particle's inertial time scale to the (local) time scale of the turbulence, $\tau_a/T^{(L)}$, decreases with time. The effect of inertia therefore vanishes asymptotically. The dispersion of particles is similarly affected.

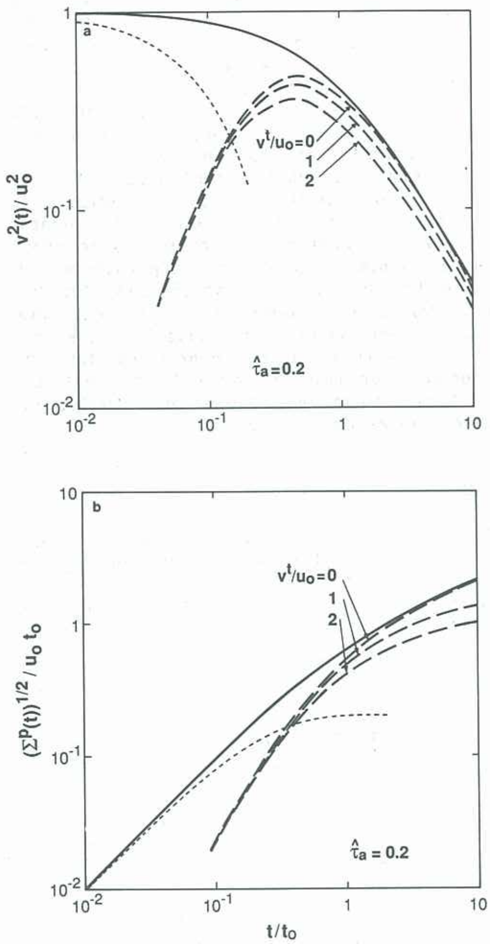

Figure 2. Effect of external force on (a), particle velocity variance and (b), dispersion, in grid turbulence.
(- - -), source term
(— —), turbulence interaction term
(——), fluid/passive scalar result

Figures 2(a) and (b) show the effect of an external force on the particle velocity variance and dispersion. For clarity of presentation we have not shown the combined contributions of the initial conditions and the turbulence interaction terms. The bold solid lines represent fluid statistics. For small time such that $t/t_o < 0.5$ the external force has little effect since then the motion is insensitive to the Lagrangian time scale. At large times the particle velocity variance approaches that of the turbulence, but the particle dispersion asymptote clearly diverges from that for passive scalar dispersion.

These effects are readily interpreted in terms of the Lagrangian time scale (10). Since the turbulence energy, decays with time, the ratio $\beta v^t/u$ eventually dominates in the 'square-root' term in (10) with the result that asymptotic behaviour of the time scale for the fluid velocity along a particle trajectory varies like $(t/t_o)^{1-m/2}$, which is a slower rate of growth than that of the corresponding time scale for fluid elements, (9). This smaller time scale has the direct result of reducing the dispersion of particles compared with passive scalar dispersion.

Comparison of Theory and Observation

We have considered three sets of laboratory data reported from wind tunnel experiments by Snyder and Lumley (1971), Wells and Stock (1983) and Ferguson (1986). According to the similarity scaling introduced above, dispersion and velocity statistics from these different data sets can be compared quantitatively only after scaling by conditions at the source as represented by u_o and t_o. Unfortunately, such a comparison is not possible because the Wells and Stock and Ferguson data were obtained with a source located at the grid, where the similarity description is inapplicable. The ambiguity in the source conditions in these data sets also prevents quantitative comparison with theory. We therefore focus on the Snyder and Lumley data for which the source was located 20 mesh lengths downstream of the grid.

Figure 3 shows a comparison of our model calculations for the particle velocity variance with the laboratory data of Snyder and Lumley. We have used a non-dimensional inertia time scale $\hat{\tau}_a = \tau_a/t_o = 0.63$ corresponding to the solid-glass and copper particles used by Snyder and Lumley. Since our model has an undetermined constant β in the parameterization of the time scale we have made calculations for a range of values of $\beta v^t/u_o$. In the absence of any information on the initial conditions for the particle trajectories, we have assumed that the initial particle velocity has the same variance as the fluid at the source location but is uncorrelated with it.

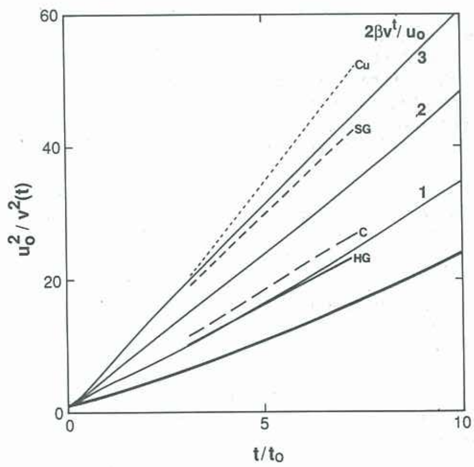

Figure 3. Comparison of present theory with Snyder and Lumley data for the particle velocity variance. Snyder and Lumley data: HG = hollow glass, C = corn, SG = solid glass, Cu = copper. Theory; $\hat{\tau}_a = 0.63$, (——); passive scalar, (——).

It can be seen that we obtain reasonable agreement with the solid glass and copper data for $2\beta v^t/u_o = 3$, which corresponds to $\beta \cong 1$. The solid line is the fluid velocity variance. Snyder and Lumley ascribe the difference between the variance for the hollow glass particles (which with such a small inertia time scale would be expected to behave like fluid elements) and the fluid to inadequate sampling of the particle velocity.

We compare our dispersion calculations with the Snyder and Lumley data in Figure 4. In general our results lie closer to the fluid dispersion curve (the solid line) than do the data so that our theory tends to overpredict the particle dispersion, at least under these conditions. Notice that the hollow-glass particle dispersion also falls well below the fluid dispersion curve (which we recall is a good representation of passive scalar dispersion data). The reason for this discrepancy is not clear, but it may point to a sampling problem which results in an underestimate of the particle dispersion.

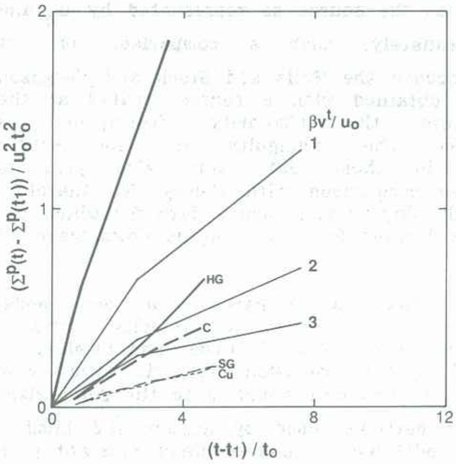

Figure 4. Comparison of present theory with Snyder and Lumley data for particle dispersion relative to that at $t_1 = 2.42t_0$. Symbols as in Figure 3.

CONCLUSIONS

Turbulence non-stationarity has a considerable effect on statistics of the motion of heavy particles. For large times the particle velocity variance relaxes back to the fluid velocity variance because $\tau_a/T^{(L)}$ decreases with time as the fluid time scale increases.

The effect of a mean drift velocity due to an external force is to reduce both the velocity variance and dispersion. However, although the velocity variance asymptotes to that of the fluid for large times, the behaviour of the particle dispersion is qualitatively different with the asymptotic particle dispersion growing at a slower rate than fluid element dispersion.

Our model shows the importance of the source location in influencing both passive scalar and particle dispersion in grid turbulence through velocity and time scales defined at the source. Although it ignores the role of Reynolds number and of the micro-scales of turbulence, similarity scaling works well for passive scalar dispersion. However, in the experiments of Wells and Stock (1983) and Ferguson (1986) the source was located too close to the grid for the similarity assumptions to be appropriate.

We have shown that our model is capable of reasonably matching the velocity and dispersion statistics of the heaviest particles used by Snyder and Lumley (1973) with $2\beta v^t/u_o = 3$, which corresponds to $\beta \cong 1$. For very light particles our model correctly reverts to passive scalar motion although the experimental data do not.

It is apparent from our study that there are limited experimental data with which to test in detail theories such as ours and that there are significant inconsistencies between the data and with the passive scalar limit. We are able to make some recommendations to guide future experimental work. Firstly, in order to facilitate comparison between different data sets and with theory, it is desirable that the source be located well downstream from the grid (at least 10M) in order that the inhomogeneous region of the flow be avoided. Otherwise similarity scaling of the results is not possible. A corollary of this requirement is the proper characterization of the turbulence. Secondly, it is important in our opinion to attempt to match statistics for very light particles to passive scalar statistics. An excellent way to test this (and the general performance of the experimental set up) is to carry out passive scalar dispersion experiments in conjunction with the particle experiments. Finally, particularly for heavy particles, it is desirable to make some measurements very close to the source in order to determine initial conditions for the particle motion. Our calculations indicate that the effects of these initial conditions may persist for significant distances downstream.

REFERENCES

Anand, M.S. and Pope, S.B. (1985) Diffusion behind a line source in grid turbulence. Turbulent shear flows 4, 46-52 (eds. L.J.S. Bradbury, F. Durst, B.E. Launder, F.W. Schmidt and J.H. Whitelaw). Springer-Verlag.

Csanady, G.T. (1963) Turbulent diffusion of heavy particles in the atmosphere. J. Atmos. Sci., 20, 201-208.

Ferguson, J.R. (1986) The effects of fluid continuity on the turbulent dispersion of particles. Ph.D. thesis, Washington State University, Pullman.

Gardiner, C.W. (1983) Handbook of stochastic processes for physics, chemistry and the natural sciences. Springer-Verlag.

Sawford, B.L. and Guest, F.M. (1988) Uniqueness and universality of Lagrangian stochastic models of turbulent dispersion. Extended abstract, 8th Symposium on Turbulence and Diffusion, April 25-29, San Diego, California. AMS.

Snyder, W.H. and Lumley, J.L. (1971) Some measurements of particle velocity autocorrelation functions in a turbulent flow. J. Fluid Mech., 48, 41-71.

Thomson, D.J. (1987) Criteria for the selection of stochastic models of particle trajectories in turbulent flows. J. Fluid Mech., 180, 529-586.

Warhaft, Z. (1984) The interference of thermal fields from line sources in grid turbulence. J. Fluid Mech., 144, 363-387.

Wells, M.R. and Stock, D.E. (1983) The effects of crossing trajectories on the dispersion of particles in a turbulent flow. J. Fluid Mech., 136, 31-62.