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HEAT TRANSFER IN A TURBULENT BOUNDARY LAYER

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Abstract.

Mean velocity, temperature and turbulence quantities have been measured in a two–dimensional boundary layer with a step in the wall temperature causing a turbulent thermal boundary layer to develop. Eddy diffusivities for heat and momentum, a_t and ν_t, have linear distributions in the wall region, and a_t is larger than ν_t in the outer part of the boundary layer. A new model which is relatively insensitive to effects of thermal intermittency is suggested for the turbulent heat flux.

Introduction.

The present study is concerned about the boundary layer convective heat transfer mechanism in the region after a stepwise change in the boundary conditions. The situation represents a challenge for turbulence modelers when calculating parameters like the thermal layer growth rate and the streamwise distribution of the local wall heat flux. The turbulent heat fluxes need to be modeled or separate transport equations have to be solved for these quantities. In this investigation a 2D turbulent boundary layer with a step in the wall temperature is studied experimentally. The turbulent heat fluxes are measured and some simple closure assumptions are discussed.

Experimental Setup

The experiment was carried out in the 7m long test section of a closed–return wind tunnel having a 0.5 x 1.0m² cross–sectional area. The longitudinal pressure gradient is removed by slightly inclining the roof of the test section to compensate for the growth of the boundary layers. The turbulent boundary layer under investigation develops on the flat floor of the test section such that the parameters characterizing the development of the flow field (skin friction, shape factor and momentum–loss thickness) compare excellent with what is expected using 2D turbulent boundary layer theory (Saetran 1987). After an unheated length the wall temperature has a stepwise change (see Figure 1). The step is produced by a 0.5m wide and 1.0m long hot aluminium plate which is mounted flush with the test section floor. A uniform wall temperature (±0.3°C) is achieved by heating the plate by vapor, produced by boiling Methylene chloride (CH_2Cl_2) at atmospheric pressure in a chamber beneath the plate. The wall temperature is thus 21°C above that of the free–stream.

One–component velocity measurements are made with 2.5μm dia. Wollaston hot–wires operated by constant temperature anemometers at an overheat ratio of 0.8. For the two–component velocity measurements 5μm dia. X–wires are used. Temperature is measured with 0.4mm long, 1μm dia. cold–wires operated in the constant current (0.2mA) mode. The temperature sensitivity of the cold–wires is found by calibrating against thermocouples in the core region of a hot jet. The interface of the hot turbulent and the cold irrotational fluid is assumed to be

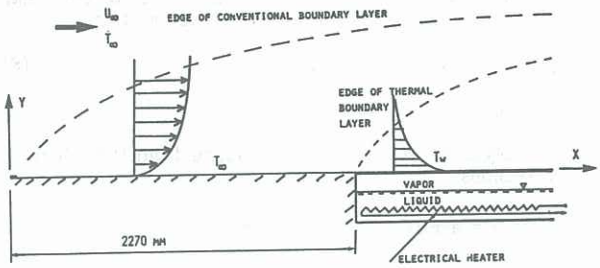

Figure 1. Diagram of the experimental scheme with the constant temperature hot plate.

sharp and the frequency response of the cold wires is estimated to 3.5kHz at a speed U≈10m/s by recording the signal on a storage oscilloscope when the wires are placed in the intermittent part of the jet. The cold–wire length/dia. ratio is 400 and could thus be subject to a systematic error in the temperature intensity measurements due to wire/prong interaction effects. These effects are tested by placing the cold–wire in the intermittent part of the jet, relatively close to the jet exit, where it shares the time between being in the hot fluid from the jet core and in the cold fluid from the surroundings. From this analysis it is found that a 8% correction of the magnitude of signals for the temperature fluctuations was necessary for representing the correct level of the rms. temperature intensity.

A trippel–wire is used to measure the turbulent heat fluxes $\overline{u\theta}$ and $\overline{v\theta}$. The two 5μm dia. hot–wires are arranged in a conventional X–array, i.e. ±45° to the perpendicular of the wind tunnel floor. The axis of the 0.4mm long, 1μm dia. temperature sensing cold–wire is parallel to the plane of the X–array at a position: z = 1mm from the closest hot–wire, and x = at the same streamwise position as the geometrical center of the X–array. The spatial resolution of the trippel–wire probe is thus approximately 2mm. The two hot–wire signals are analog linearized and the two resulting signals and the cold–wire signal are low–pass filtered at 8kHz before digitized and sampled by a computer system. The sampling rate is 10kHz at each channel and typically $0.3 - 1 \cdot 10^6$ samples are taken (depending on the flow conditions) before time–averaged quantities are calculated. The cold–wire signal is low–pass filtered at 8kHz for instrumentation convenience although the frequency response is found to be lower (3.5kHz). The choice for the filter setting was justified by band–pass filtering the signal showing that there is a negligible contribution to the rms. values in the 3.5 – 8kHz frequency range.

The temperature contamination on the velocity signal is removed by a correction method. For a single hot–wire the linearized mean and fluctuating voltage signals may be written as

$$\overline{E} = K(1+b_1\Delta T)\overline{U} \quad (1)$$

$$e = \alpha u - \beta\theta \quad (2)$$

where ΔT is the temperature difference between the wire and the air, K is an amplification factor and the coefficients b_1, α and β are

$$b_1 = \frac{\alpha_c}{a_w n}\left[1 + \frac{A}{B\overline{U}^n}\right] \quad (3)$$

$$\alpha = K\left[1 + \left[\frac{\alpha_c}{a_w} + (1-n)b_1\right]\Delta T\right] \quad (4)$$

$$\beta = K b_1 \overline{U} \quad (5)$$

The constants A, B and n are the hot–wire calibration constants, α_c the temperature coefficient of wire resistance and a_w is the overheat ratio. For a X–wire with matched resistances and the wires orientated at $\pm 45^0$ to the mean velocity direction the X–wire voltages are written

$$e_1 = \alpha u + \alpha v - \beta\theta \quad (6)$$

$$e_2 = \alpha u - \alpha v - \beta\theta \quad (7)$$

The Reynolds' stresses are then found from the following expressions

$$\overline{(e_1+e_2)^2} = 4\alpha^2\overline{u^2} - 8\alpha\beta\overline{u\theta} + 4\beta^2\overline{\theta^2} \quad (8)$$

$$\overline{(e_1-e_2)^2} = 4\alpha^2\overline{v^2} \quad (9)$$

$$\overline{(e_1+e_2)(e_1-e_2)} = 4\alpha^2\overline{uv} - 4\alpha\beta\overline{v\theta} \quad (10)$$

The error by using this method is found, by direct calibration, to be less than 2% when the mean temperature deviates up to 20^0C from the calibration temperature (Saetran 1983).

Experimental Results

The step in the wall temperature causes a thermal boundary layer to develop. The thickness δ_t of the thermal layer is for all measurement stations well inside the thickness δ of the velocity boundary layer (Saetran 1989). The growth rate is well predicted by

$$\frac{\delta_t}{\delta_0} = 0.096\left[\frac{x-x_t}{\delta_0}\right]^{0.69} \quad (11)$$

where δ_0 (=44.1mm) is the velocity boundary layer thickness at the temperature step. The stream wise variation of the wall heat flux and the mean temperature profiles are well described by analytical methods as shown by Saetran (1989) – methods that will be useful to start numerical calculations in the immediate region after singularities in the boundary conditions. The mean velocity and mean temperature profiles in the logarithmic wall region of the boundary layer correspond well to the following expressions

$$u^+ = \frac{1}{0.41}\ln(y^+) + 5.4 \quad (12)$$

$$T^+ = \frac{1}{0.55}\ln(y^+) + 5.4 \quad (13)$$

where $u^+=\overline{U}/v_*$, $v_*^2=\tau_w/\rho$, $y^+=yv_*/\nu$, $T^+=(\overline{T}-T_w)/T_*$, $T_*=q_w/(\rho c_p v_*)$ and q_w is the local wall heat flux. The boundary layer equations for steady, 2D, incompressible turbulent flow may be integrated, such that for the case with constant fluid properties

$$\tau(y) = \tau_w + \rho\overline{U}\frac{d}{dx}\left[\int_w^y \overline{U}dy\right] - \rho\frac{d}{dx}\left[\int_w^y \overline{U}^2 dy\right] \quad (14)$$

$$q(y) = q_w + \rho c_p \overline{T}\frac{d}{dx}\left[\int_w^y \overline{U}dy\right] - \rho c_p\frac{d}{dx}\left[\int_w^y \overline{U}\overline{T}dy\right] \quad (15)$$

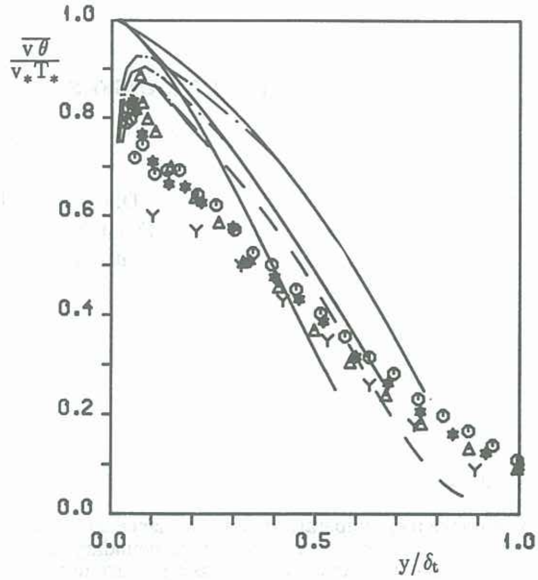

Figure 2. Distributions of turbulent heat fluxes. Δ: $(x-x_t)/\delta_0=6.8$, ∗: $(x-x_t)/\delta_0=12.5$, o: $(x-x_t)/\delta_0=21.1$. — · —: q_t from integral method. ————: total heat flux. — —: Fulachier (1972). Y: Subramanian and Antonia (1981).

where the total shear stress and heat flux consist of a molecular part and a turbulent part

$$\tau(y) = \tau_m + \tau_t = \rho\nu\frac{\partial\overline{U}}{\partial y} - \rho\overline{uv} \quad (16)$$

$$q(y) = q_m + q_t = \rho c_p a\frac{\partial\overline{T}}{\partial y} - \rho c_p\overline{v\theta} \quad (17)$$

The Reynolds' shear stress $\rho\overline{uv}$ and the turbulent heat flux $\rho c_p\overline{v\theta}$ may thus be determined from the experimental data in two ways: a) by directly measuring them, or b) by measuring $\overline{U}(y)$, $\overline{T}(y)$, τ_w and q_w at different x–values along the plate and using eqs. (14) and (15) to find $\tau(y)$ and q(y). The desired quantities are then found by applying eqs. (16) and (17). Profiles of q_t measured at 3 stations downstream the temperature step are shown in Figure 2. The symbols shown are those found using method a), while the dash–dot lines show the results when method b) is used. The solid lines show q(y) (=q_m+q_t) at the 3 stations. The results are obviously dependent upon the method used although much effort was put into performing the measurements with accuracy and repeatability. It is not obvious that one method is superior to the other: In using method b) it is rather difficult to obtain high accuracy in the determination of stream wise gradients of the integral quantities, while the spatial resolution of the trippel–wire probe (≈2mm) may not be good enough to correctly resolve the large time–dependent spatial gradients of velocity and temperature close to the wall. Also shown in Figure 2 are results by Subramanian and Antonia (1981) (q_w=const. along the entire plate), and Fulachier (1972) (T_w=const. along the entire plate). The latter reference is a report on the only (to the author's knowledge) experiment, with similarities to the present experiment, where one has obtained good agreement for the turbulent heat flux using the two different methods.

The Reynolds' shear stress and the turbulent heat flux are often modeled using the gradient transport hypothesis

$$\tau_t = -\rho\overline{uv} = \rho\nu_t\frac{\partial\overline{U}}{\partial y} \quad (18)$$

$$q_t = -\rho c_p\overline{v\theta} = \rho c_p a_t\frac{\partial\overline{T}}{\partial y} \quad (19)$$

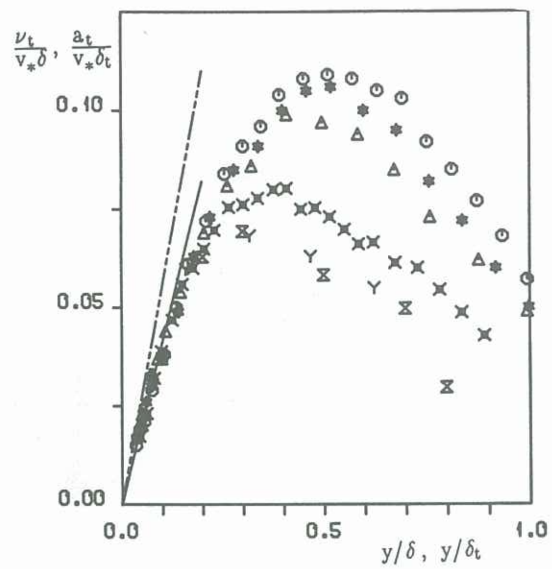

Figure 3. Eddy diffusivities a_t and ν_t. a_t with symbols as in Fig.2. *: ν_t at Re_{δ_2} =4390. ———: eqn.(20). —·—: eqn.(21). ⋈: Klebanoff (1955). Y: Townsend (1951).

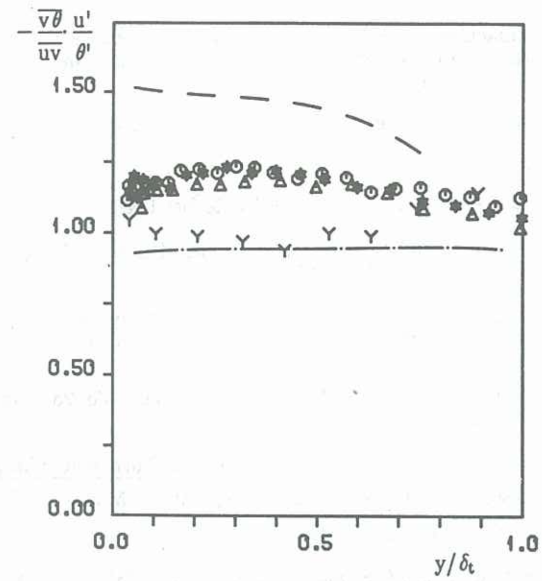

Figure 4. Turbulent heat flux model. b_0 with symbols as in Fig.2. Y: Subramanian and Antonia (1981). — —: Fulachier (1972). —·—: Chen and Blackwelder (1978).

where ν_t and a_t are the so called "eddy diffusivities". Three profiles for a_t and one for ν_t are shown in Figure 3 and the eddy diffusivities are calculated from the measured profiles of the means and the measured fluxes. The distributions of a_t are shown as a function of y/δ_t, while the ν_t distribution is plotted as a function of y/δ. With these normalizations of the distance from the wall the distributions for ν_t and a_t are linear and coincide up to y/δ and $y/\delta_t \approx 0.2$, while an estimate for ν_t and a_t in the wall region of the boundary layer, using eqs. (12) and (13), will give

$$\nu_t = 0.41 v_* y \tag{20}$$

$$a_t = 0.55 v_* y \tag{21}$$

Eqn. (20) is shown in Figure 3 as the solid line and show good agreement with the experimental results, while eqn. (21) predicts a distribution for a_t having a steeper gradient than what is measured. Also shown in the same figure are data for ν_t from the classical experiments of Klebanoff (1955) and Townsend (1951) as presented by Hinze (1975). Their results are in good agreement with the present distribution when $y/\delta \leq 0.2$ but somewhat lower in the outer part of the boundary layer. More recent experiments, i.e. Pimenta et. al. (1975) and Subramanian and Antonia (1981), show larger values for $\overline{uv}$ than the Klebanoff and Townsend results.

The temperature signal is intermittent in the outer part of the thermal layer, just as the velocity signal is intermittent in the outer part of the velocity layer (which in this case is thicker than the temperature layer). This means that the conventional time–averaged temperature signals, and especially turbulence quantities like the heat flux $\overline{v\theta}$ are averaged over time periods where the instantaneous values are negligible. When the heat flux signal is averaged over those time periods the probe is submerged in hot fluid end not over the time periods the probe is surrounded by cold fluid, a higher value for the conditional averaged heat flux will be found than when using the conventional time–averaging. This is most probably the reason for the decreasing values for both ν_t and a_t in the outer part of the boundary layer (see Figure 3). Conditional averaging techniques were not available for the present experiments. The correlation coefficient $R_{v\theta}$ $(=\overline{v\theta}/v'\theta')$, where θ' is the root–mean–square of the temperature fluctuation, is only slightly affected by thermal intermittency in the outer part of the thermal layer since the effects influence both the nominator and the denominator. The turbulent transport mechanism for heat and momentum in the near wall region are supposed to be similar and will thus be reflected in the distributions for the correlation coefficients $R_{v\theta}$ and R_{uv} $(=\overline{uv}/u'v')$. The ratio of the two correlation coefficients may therefore serve as a model for the turbulent heat flux $\overline{v\theta}$

$$\overline{v\theta} = -b_0 \frac{\overline{uv}\cdot \theta'}{u'} \tag{22}$$

where the proportionality coefficient is evaluated from the experimental results: $b_0 = 1.15 \pm 0.08$. The results are plotted in Figure 4, where also results from Fulachier (1972), Chen and Blackwelder (1978) and Subramanian and Antonia (1981) are shown for comparison.

Conclusions.

Turbulent heat fluxes are measured by two different methods in a developing turbulent thermal boundary layer. The direct measurement method and the integral method do not produce results that agree to a satisfactory degree. The reason for this is not clear.

Eddy diffusivities for heat and momentum, a_t and ν_t, have linear distributions in the wall region of the boundary layer. a_t is larger than ν_t in the outer part of the boundary layer and both quantities decrease as the distance from the wall increase. It is thought that this is due to the effects of intermittency.

A new model for the turbulent heat flux is suggested. The proportionality coefficient is from the present results found to be constant. Results from other similar experiments give a fair support to the suggested model.

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