

**ACTIVE CONTROL OF NOISE TRANSMISSION INTO AN ENCLOSURE -
 EFFECT OF ACTUATOR LOCATION**

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Abstract

A point force applied to a panel can control the sound transmission through the panel into an internal cavity. This paper is an analytical study of the effect of actuator location on the sound transmission. Although the simple quadratic relationship between the actuator phase and amplitude and the resulting interior noise level is well known, there is no such quadratic relationship between actuator location and sound energy in the cavity. Results obtained demonstrate that for a particular type of incident sound field, the optimal actuator locations can be obtained by a global evaluation of the total sound energy in the cavity as a function of those locations. A controller which drives an actuator placed at these optimal locations will always produce a better attenuation performance than one which drives a randomly located actuator. There might be more than one location at which an actuator may achieve a similar minimum sound energy level in the cavity. However, the best actuator location can be determined by considering a second constraint — the input mechanical impedance. Thus the final selection is made for those locations at which the actuator can maximally reduce the interior noise level while requiring a small input force amplitude. The system response (sound field and panel vibration) is analysed to show the physical mechanism involved in the control.

1. Introduction

For the case of active control of sound transmission through a boundary structure, such as an aircraft fuselage, into the interior, the sound energy in the interior is a quadratic function of the strengths of the secondary sources (which may be acoustic or vibration actuators). This simple relationship ensures that a local search over the amplitude-phase space of the actuators will find the amplitudes and phases corresponding to the minimum value of sound energy in the cavity. However, the optimal control of sound transmission will not only depend upon the strength of the control forces but also upon the location of the actuators and the nature of the external sound field. Evaluation of the effect of actuator location upon sound transmission provides a method for discovering the best actuator locations on a structure exposed to a particular type of sound field.

In this paper, a rectangular box with 5 rigid walls and a single modally reactive wall is used as a model for the investigation. The modal coupling analysis technique has been used for calculating the system response for both controlled and uncontrolled conditions. A quadratic relationship between the total sound energy in the cavity and the control force strength is derived. This relationship is then used to

obtain an expression for the minimum energy corresponding to the fixed actuator location. A detailed discussion about the minimized energy as a function of the actuator location is then presented. The control force output, the input mechanical impedance and the system response are all estimated as a measure of the effectiveness of the actuator location.

2. System response & optimization

Figure 1 is the panel-cavity system used for this investigation. The dimensions of the cavity are $L_x = 0.868\text{m}$, $L_y = 1.150\text{m}$ and $L_z = 1.000\text{m}$. The top aluminium panel is simply supported and has the dimensions of $0.868\text{m} \times 1.150\text{m} \times 0.006\text{m}$. A plane incident sound wave is assumed and the control point forces on the panel surface are represented by the delta functions of the actuator location. In Figure 1, α and θ are respectively the elevation and the azimuth of the incident plane wave, with respect to the panel centre point.

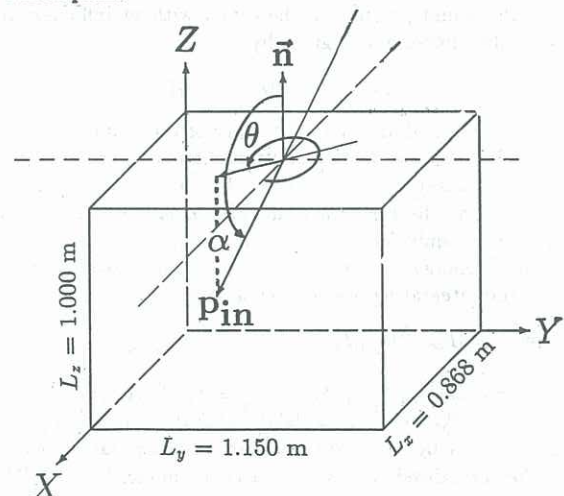


Figure 1. Coordinate system of the panel-cavity model.

The response of the sound field and the panel vibration to the incident sound wave and the control forces can be calculated using modal coupling analyses¹. In this analysis, the sound pressure and the panel velocity can be expressed by their normal mode expansions as

$$p(\vec{r}, \omega) = [\Phi_N]^T [P_N], \quad (1)$$

and

$$v(\vec{\sigma}, \omega) = [S_M]^T [V_M]. \quad (2)$$

where $[\Phi_N]$ and $[S_M]$ are respectively, mode shape matrices of the cavity and the panel. $[P_N]$ and $[V_M]$ are matrices representing, respectively the modal amplitudes of the sound pressure in the cavity and the panel velocity. They relate to the external sound pressure matrix $[P_M^{ext}]$ by $[Z_A]$, the internal modal radiation impedance matrix of the panel and the panel modal input impedance matrix $[Z_P]$:

$$[P_N] = [Z_A][V_M]. \quad (3)$$

$$[P_M^{ext}] = [Z_P][V_M], \quad (4)$$

The external sound pressure matrix $[P_M^{ext}]$ includes contributions from both the primary sound pressure $[P_M^P]$ due to the incident wave and the control point forces $[P_M^S]$; that is,

$$[P_M^{ext}] = [P_M^P] + [P_M^S]. \quad (5)$$

If M point force actuators are used for control, and they are located at $\vec{\sigma}_1, \dots, \vec{\sigma}_{M1}$, the matrix $[P_M^S]$ may be expressed as

$$[P_M^S] = [S_{con}][F_{con}] \quad (6)$$

where $[S_{con}]$ and $[F_{con}]$ are the control force location matrix and control force matrix respectively.

Substituting Equations (4) and (5), into Equation (2), the velocity v of the simply supported panel due to the incident sound field and the control forces can be written as

$$v = v_0 + [S_M]^T [Z_P]^{-1} [S_{con}][F_{con}] \quad (7)$$

where v_0 is the displacement of the panel without influence of the control forces,

$$v_0 = [S_M]^T [Z_P]^{-1} [P_M^S]. \quad (8)$$

Substituting Equations (3), (4) and (5), into Equation (1), the sound pressure in the cavity due to the distributed panel velocity v can be represented by

$$p = p_0 + [\Phi]^T [Z_A][Z_P]^{-1} [S_{con}][F_{con}] \quad (9)$$

p_0 is the sound pressure in the cavity without influence of the control forces and is given by

$$p_0 = [\Phi]^T [Z_A][Z_P]^{-1} [P_M^S]. \quad (10)$$

The temporal and spatial average of the sound pressure squared $\langle pp^* \rangle$ in the cavity is related to the acoustic potential energy in the cavity. In this calculation $\langle pp^* \rangle$ is defined as the cost function. Minimization of this cost function is equivalent to the minimization of the acoustic potential energy in the cavity. Using Equations (9) and (10), and integrating over the whole cavity volume, we have

$$\langle pp^* \rangle = [F_{con}]^H [a_A][F_{con}] + [F_{con}]^H [b_A] + [b_A]^H [F_{con}] + [c_A] \quad (11)$$

where $[a_A] = [S_{con}]^T [A_A][S_{con}]$, $[b_A] = [S_{con}]^T [A_A][P_M^S]$, $[c_A] = [P_M^S]^H [A_A][P_M^S]$ and $[A_A] = ([Z_P]^{-1})^H [Z_A]^H [\Delta_A][Z_A]([Z_P]^{-1})$. $[\Delta_A]$ is an N by N diagonal matrix, the diagonal elements are the normalized factors for each cavity mode. $[]^H = []^{*T}$ (the transpose of the complex conjugate of a matrix).

From the quadratic property of the Equation (11), the optimal control force value for the minimum acoustical potential energy in the cavity is determined by

$$[F_{con}]_{opt} = -[a_A]^{-1}[b_A] \quad (12)$$

The minimum value for $\langle pp^* \rangle$ is then obtained as:

$$\langle pp^* \rangle_{min} = [c_A] - [b_A]^H [a_A]^{-1}[b_A] \quad (13)$$

Because $[S_{con}]$ in $[a_A]$ and $[b_A]$ is determined by the location of the actuators, the minimized average sound pressure $\langle pp^* \rangle_{min}$ is a function of the actuator locations. To

achieve the best control result, further minimization of the sound pressure in terms of the actuator location needs to be undertaken. In this further minimization, the cost function (referred to as second cost function) becomes $\langle pp^* \rangle_{min}$. The actuator location matrix $[S_{con}]$ is related to the eigenfunctions of the panel (sine functions); therefore a quadratic relationship does not hold between the second cost function and the actuator locations.

3. Results and discussion

The noise reduction, defined as

$$N.R. = -10 \log \frac{\langle pp^* \rangle}{4p_{in0}p_{in0}^*}, \quad (14)$$

is used to characterize the transmission of the incident sound field through the panel into the cavity. $2p_{in0}$ is the complex amplitude of the blocked sound pressure.

Figures 2(a) and 2(b) show the noise reduction levels for both controlled and uncontrolled cases. Figure 2(a) is for the normal plane wave incident ($\alpha = 180^\circ$, $\theta = 0^\circ$) and Figure 2(b) is for an asymmetrical plane wave incident with an elevation angle of 135° and an azimuth angle of 45° . Unit amplitude of the incident sound pressure is assumed. The solid curves represent results without control, while the dotted curves represent the optimally controlled result with one actuator at the centre of the panel. The dashed curves and the dashed-dotted curves in the figures are controlled results for an actuator located at $(L_x/4, L_y/4)$ and at $(L_x/8, L_y/8)$ respectively.

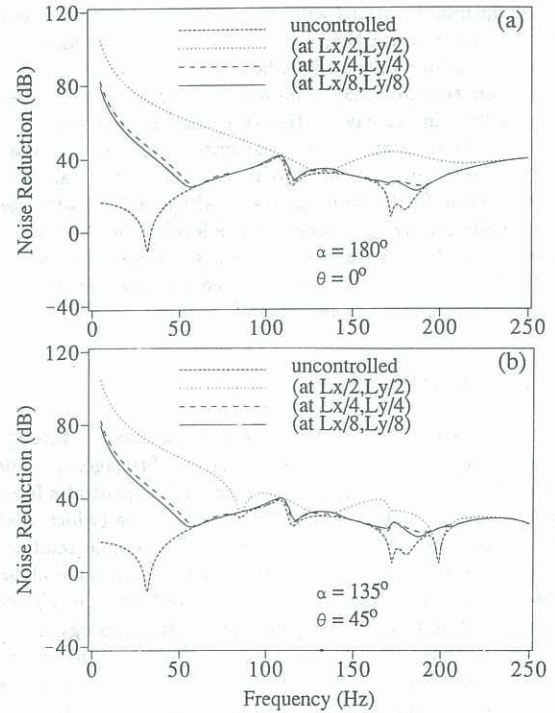


Figure 2. Noise reduction level. (a) for ($\alpha = 180^\circ$, $\theta = 0^\circ$); (b) for ($\alpha = 135^\circ$, $\theta = 45^\circ$).

These results indicate that the maximum achievable noise reduction level of the panel-cavity system by a vibration actuator depends upon the type of the incident sound wave (particularly in the high frequency range), the driving frequency of the incident wave, and the location of the actuator. Close to and above the first resonance frequency of the cavity, the dimensions of the panel are comparable with half of the wavelength of the incident wave. Thus in this fre-

frequency range, the responses of some high order panel modes are sensitive to different angles of the incident sound field, and this is reflected in the variation of the noise reduction level. Because of the complexity of the higher order panel mode shapes, the panel velocity distribution becomes more complicated, which results in a strong dependence of the control results on the actuator location. For example, in the case of Figure 2(b), one actuator at the centre is not able to reduce the sound transmission at all frequencies and particularly at 198 Hz. Close to this frequency, the cavity sound field is dominated by the (1,0,0) cavity mode and the panel modes which couple with this cavity mode have their nodal lines passing through the panel central point. The noise reduction level can be increased if the actuator is located at $(L_x/4, L_y/4)$ or at $(L_x/8, L_y/8)$, or at a position where the nodal lines of the corresponding panel mode do not pass.

The noise reduction level between the controlled and uncontrolled situations is only one criterion upon which to judge the results of applying the control actuators. The optimal achievable noise reduction level does not give any indication of the required control force or the resulting panel response at optimum. In practice, the control force is constrained by the output capacity of the actuators, while the panel velocity at the driving point is directly related to how easily the actuator can drive the panel. The total panel vibration level corresponding to the optimally controlled situation should also be taken into account, because high panel vibration amplitudes may be associated with strong local reactive power flow close to the panel surface.

Figures 3(a) and 3(b) show the amplitudes of the optimal control force at the actuator locations when the optimal noise reductions (as shown in Figures 2(a) and 2(b)) are achieved. It is shown that, in the low frequency range, greater force amplitudes are needed to achieve optimal noise reduction when the actuator location is away from the panel centre and the corresponding maximum achievable noise reduction levels are reduced. At higher frequencies, the highest noise reduction is not necessarily associated with lowest actuator force amplitude. For example, at frequencies of 151 Hz and 171 Hz the optimal force amplitude for the actuator at the centre is lower than the amplitudes corresponding to the other two actuator locations, and results in a higher NR level. However, at frequencies around 98 Hz in Figures 2(b) and 3(b), the large optimal force amplitude for the actuator location at $(L_x/8, L_y/8)$ results in a corresponding higher NR level.

To obtain the entire picture of the dependence of the optimal noise reduction level (first optimization result) upon the actuator location, the dependence of the acoustical qualities (such as noise reduction level, amplitude of the optimal control force and the total vibration level) on the actuator locations is investigated in some detail at a few frequencies (such as 171 Hz, 181 Hz), where the noise reduction levels are low in the uncontrolled case. For each frequency, a set of contour plots are obtained as functions of the actuator location in Figures 4 and 5. Figure (a) in each set is a plot of the optimal noise reduction (above the uncontrolled NR level) as a function of actuator location. A discontinuity exist at the boundaries of the plot, which is represented by a jump from 0 dB (for the actuator at the boundary) to a finite value (which may not be achievable in practice by the limit of the optimal control force). Figure (b) represents the amplitude of the optimal force represented in dB ($20 \log_{10} |F_{opt}|$) plot-

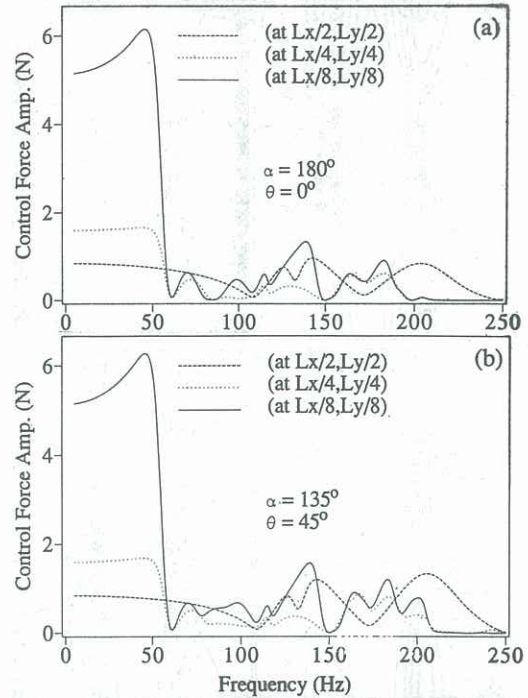


Figure 3. Optimal control force amplitude. (a) for $(\alpha = 180^\circ, \theta = 0^\circ)$; (b) for $(\alpha = 135^\circ, \theta = 45^\circ)$.

ted as a function of actuator location. Figure (c) represents the average panel velocity level difference ($10 \log_{10}(\frac{\langle v v^* \rangle}{\langle v_0 v_0^* \rangle})$), and Figure (d) represents the reactive power level difference ($10 \log_{10}(P^{Im})$, where P^{Im} is the amplitude of the imaginary power on the panel internal surface for the controlled and uncontrolled situations.

Reviewing Figures 4 and 5, the effect of the actuator location on the first optimal noise reduction level, the control force amplitude, the average velocity level and the reactive power flow at the panel internal surface is summarized as follows:

1. There are usually multiple locations or regions on the panel, where the actuator can achieve the maximum NR level. Each region has its properties. For the case illustrated in Figure 4, the optimal actuator location regions (the contours with 32 dB NR level) away from the panel boundary are characterized by minimum control force amplitudes and minimum reactive power levels. For those regions close to the panel boundary, although the maximum achievable NR level is attractive, the requirement for the optimal control force amplitude is too large to be of practical use.
2. For the case illustrated in Figure 5, there exists secondary maximum NR level regions, where the NR level is 4 dB smaller than the NR level corresponding to the actuator located at the panel centre. These secondary maximum NR regions correspond to an increase in the average panel velocity level and the reactive power level. There is also evidence to show that an ordinary optimization search may fail to find the optimal actuator location, and instead find one of the secondary optimal regions.

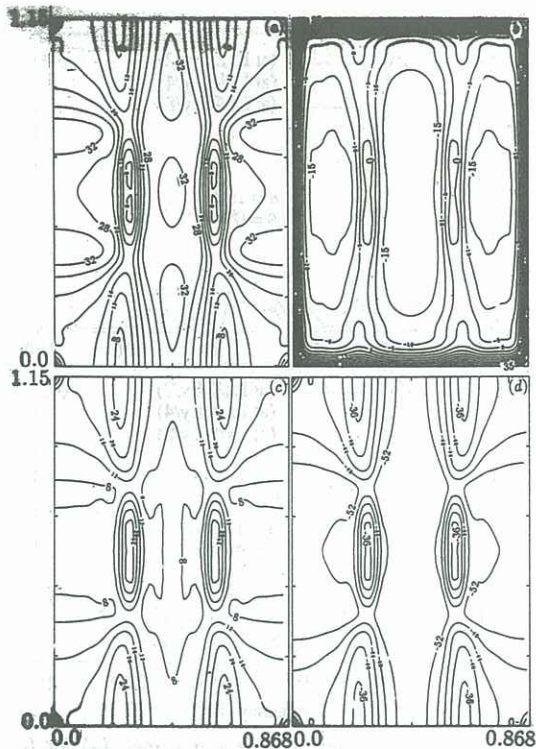


Figure 4. Contour plots of the (a) optimal NR level, (b) optimal control force amplitude, (c) average panel velocity level (ref: -76.7dB the average panel velocity level in the uncontrolled case) and (d) the reactive power level (ref: -62.8dB the reactive power in the uncontrolled case). $f = 172$ Hz. ($\alpha = 180^\circ$, $\theta = 0^\circ$).

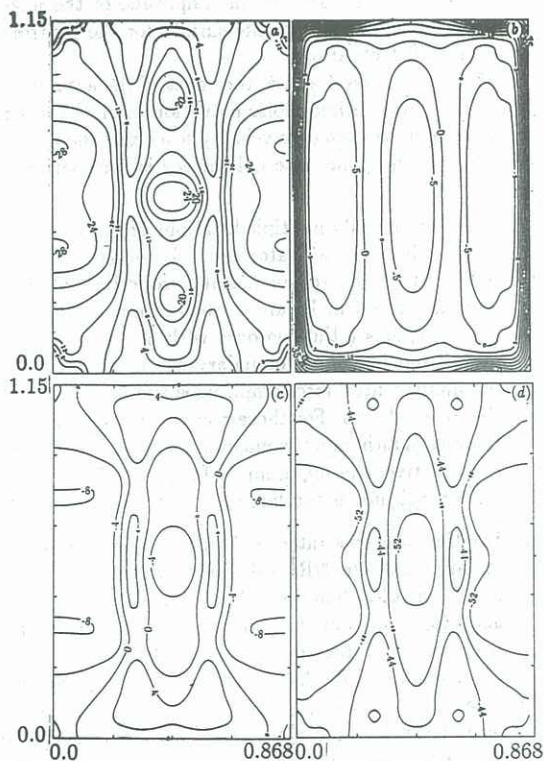


Figure 5. Contour plots of the (a) optimal NR level, (b) optimal control force amplitude, (c) average panel velocity level (ref: -61.7dB the average panel velocity level in the uncontrolled case) and (d) the reactive power level (ref: 40.7dB the reactive power in the uncontrolled case). $f = 181$ Hz. ($\alpha = 180^\circ$, $\theta = 0^\circ$).

4. Conclusions

When sound transmission through a panel into an enclosure is considered, the maximum possible noise reduction achievable using an active control system driving a vibration actuator on the panel is dependent upon actuator location.

The optimal location for the actuator is not necessarily the location which yields the maximum noise reduction. Other constraints such as the control force amplitude, average panel velocity and reactive sound field generation must be taken into account. When this is done, the optimal location may be found to be that which produces 1 or 2 dB less noise reduction than the maximum possible. It was also found that locations requiring larger control forces do not necessarily result in higher noise reductions.

Results also indicate that standard optimization algorithms may not converge to the best solution; that is, a global search must be incorporated in any algorithm for it to be reliable.

It can be seen that when multiple actuators are used, the problem becomes more complex and that simple contour plots may not be appropriate. A suitable global search technique for the multiple actuator case is the subject of on-going research.

References

1. Pan Jie, D.A. Bies and C.H. Hansen, "Active control of noise transmission through a panel into a cavity: I. analytical study", submitted to J. Acoust. Soc. Am.

Acknowledgments

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