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# THIN UNCAMBERED AEROFOILS WITH A LEADING-EDGE SEPARATION BUBBLE

B.G. NEWMAN and M-C. TSE

Department of Mechanical Engineering
McGill University
Montreal H3A 2K6
CANADA

## INTRODUCTION

A simple theory has been developed which predicts the length of the leading-edge separation bubble and the lift and drag of a flat-plate aerofoil in incompressible flow. The bubble typically exists for angles of incidence less than 8°. Predictions of this sort are of interest for the related, but more complicated, problems of flow about thin cambered aerofoils such as those in compressors and turbines, and around jib sails.

The method of singularities in irrotational flow is used. The choice of a vortex, or an array of vortices, to represent the bubble runs into the difficulty that the theoretical drag is zero while at the same time the real flow has no leading-edge suction (Saffman & Sheffield (1977)). This dilemma is resolved by using source-sink singularities of finite total strength.

The development of the theory is supported by NACA measurements (Rose & Altman (1949, 50)) and by new measurements on a geometrically singular $1/5^{th}$ scale double-wedge aerofoil of 4.2% thickness. Bubble size, pressure distribution, lift and drag were measured at three Reynolds numbers between 0.25 and $0.90\times10^6$. The Reynolds number for the NACA measurements was $5.8 \times 10^6$.

In this paper two aspects of the theory are examined:

i. That the measured momentum-loss thickness of the boundary layer just downstream of reattachment is approximately equal to the calculated loss-thickness in the mixing layer outside the reattaching (dividing) streamline (see fig.1).
ii. That the displacement and momentum thicknesses at the trailing edge due to the upstream bubble are approximately equal.

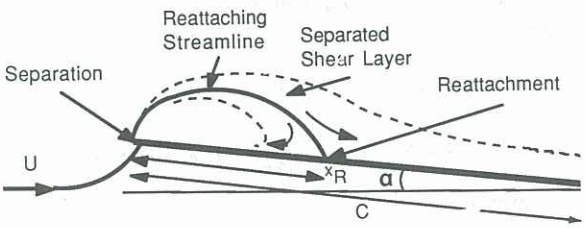



Figure 1 Time-averaged two-dimensional separation bubble.

## THEORY

A more complete description of the theory will be found in Newman and Tse (1990).

At high Reynolds numbers the mixing layer which forms when the flow separates at the leading edge is effectively turbulent over the complete bubble. If the curvature of the bubble, the pressure rise to separation and the backflow within the bubble near the surface are neglected, the separated shear layer is similar to a classical mixing layer with uniform velocity $U$ on one side and zero velocity on the other. This mixing layer, being independent of viscosity, grows linearly and the volumetric rate of fluid entrained per unit distance $x$ on the zero velocity side, which is on the wall side in the bubble application, is constant. If this is denoted by $m$, dimensional analysis shows that

$$\frac{m}{U} = k \qquad (1)$$

where $k$ should be a universal constant.

However $k$ has been shown experimentally to be dependent on upstream turbulence and unsteadiness (Oster & Wygnanski (1982)). Calculated values of $k$ based on their measurements of the rates of growth (Abramowich (1963)) range from 0.03 to 0.045.

This entrainment per unit $x$ into the separated shear layer would be represented in irrotational flow for thin aerofoils by an array of sources of strength $m$ per unit $x$ extending over the length of the bubble $x_R$. Neglecting any change of $U$ over the bubble, the displacement thickness at the trailing edge due to the sources is

$$\delta^* = \frac{m x_R}{U} \qquad (2)$$

Since the shear layer flow near the trailing edge has a small deficit, the momentum-loss thickness $\theta$ very nearly equals the displacement thickness $\frac{m x_R}{U}$. In making this statement the boundary layer growth downstream of reattachment is properly ignored. The same assumption is made when determining wake blockage in a wind tunnel and when calculating the streamlines and the drag for flow through a very porous strip (Taylor (1948)).

$\rho U^2 \theta$ can be thought of as the drag due to the
If there were no bubble there would be a leaa.
suction force

$$S = (\text{Lift})\ \alpha = L\alpha$$

where

$$\frac{L}{\frac{1}{2}\rho U^2 c} = C_L = 2\pi\alpha \tag{4}$$

It is therefore consistent to take

$$S = \rho U^2 \theta = \rho U m x_R \tag{5}$$

Using equations (1) to (5)

$$\alpha\ 2\pi\alpha \frac{1}{2} \rho U^2 c = S = \rho U x_R\ m = \rho U x_R\ Uk$$

$$\frac{\pi\alpha^2}{k} = \frac{x_R}{c} \tag{6}$$

The total drag coefficient $C_D$ is $C_S$ due to the bubble combined with the skin friction coefficient due to the attached boundary layers downstream of the bubble and on the underside of the aerofoil. A constant skin friction $C_F$ is chosen for the latter.

Thus

$$C_D = 2\pi\alpha^2 + C_F\,(2 - \frac{\pi\alpha^2}{k}) \tag{7}$$

where $C_F$ is chosen to be 0.003 at high Reynolds numbers (Ruderich & Fernholz (1986)).

The sources corresponding to the constant skin friction $C_F$ (Newman & Tse (1990)) are

$$\frac{m_F}{U} = \frac{C_F}{2} \tag{8}$$

The effect on the lift $C_L$ of the sources $m$ and $m_F$ per unit $x$ is readily obtained either by the Joukouski transformation of a circle or by thin aerofoil theory.

With

$$\phi_R = \cos^{-1}\left[1 - \frac{2x_R}{c}\right]$$

$$C_L = 2\pi \left[\alpha - \frac{(k - \frac{C_F}{2})}{2\pi}(\phi_R - \sin\phi_R)\right] \tag{9}$$

when the Kutta condition is applied at the trailing edge of the plate.

Since $k$ is greater than $\frac{C_F}{2}$ ; $C_L$ is reduced by the presence of the source singularities.

## EXPERIMENT

Experiments were made on a thin double-wedge aerofoil (4.2% thick) which was a $1/5^{th}$ scale model of that used by Rose and Altman (1949, 1950). Being a wedge it was possible to install pressure taps in the model and to determine the lift from the pressure distribution. The drag was determined from wake traverses near, and downstream of, the trailing edge using Jones method.

The bubble length $x_R$ for the present measurements was determined using both two dimensional (flag-like) and three dimensional tufts attached to the surface. At reattachment the nearby tuft would spend as much time pointing downstream as it did pointing upstream. For the NACA measurements $\frac{x_R}{c}$ was obtained from a survey of total and static pressure near the surface.

## THEORY COMPARED WITH EXPERIMENT

In fig. 2 it is seen that with $k = 0.08$, $\frac{x_R}{c}$ is quite well predicted for $\alpha < 6^o$ for the NACA measurements and $\alpha < 7^o$ for the present measurements. This is for chordwise Reynolds numbers exceeding about $0.50 \times 10^6$. Predictions using the extreme mixing values for $k$, 0.03 and 0.045, are also shown for comparative purposes. The need to use a higher value of $k$ than is appropriate for a classical mixing layer is attributed to the backflow within the bubble and the pressure rise to reattachment.

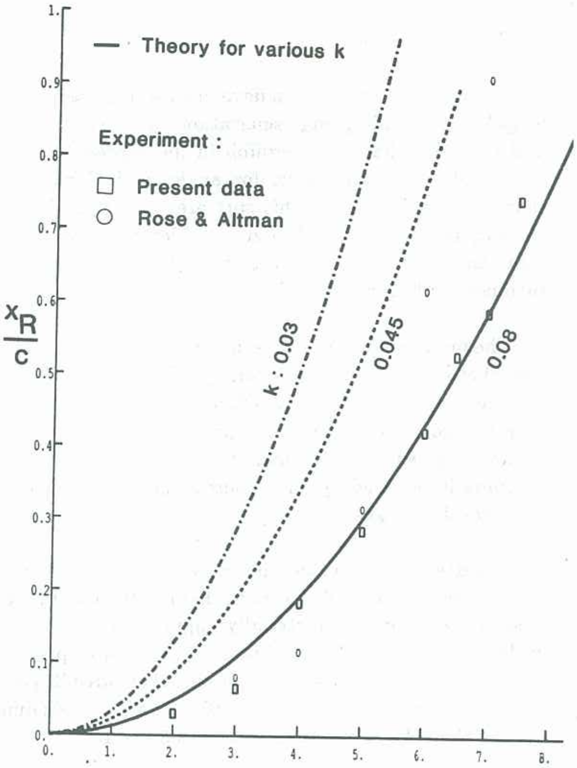



Figure 2 Reattachment length $x_R$ for the leading-edge bubble. Theory and experiment.

The measured ratio of the displacement thickness $\delta^*$ to the momentum thickness $\theta$ for the upper-surface boundary layer near the trailing edge is shown in fig. 3. It is seen to be fairly constant between $\alpha = 1.5^o$ and $\alpha = 6.5^o$, the average value being 1.5. This appears to be inconsistent with the assumption that was made when calculating the bubble sources $m$, namely that $\delta^* = \theta$ for the shear layer flow in the wake. However, most of the increase of $\delta^*$ occurs near the wall which can be attributed to the boundary layer flow.

Thus equation (9) would become

$$C_L = 2\pi \left[\alpha - \frac{(k - \frac{HC_F}{2})}{2\pi}(\phi_R - \sin\phi_R)\right] \tag{10}$$

where

$$H = \frac{\delta^*}{\theta}$$

and equals approximately 1.5.

This modification to equation (9) leads to insignificant changes in the values of $C_L$ when $C_F = .003$ and $k = .08$.

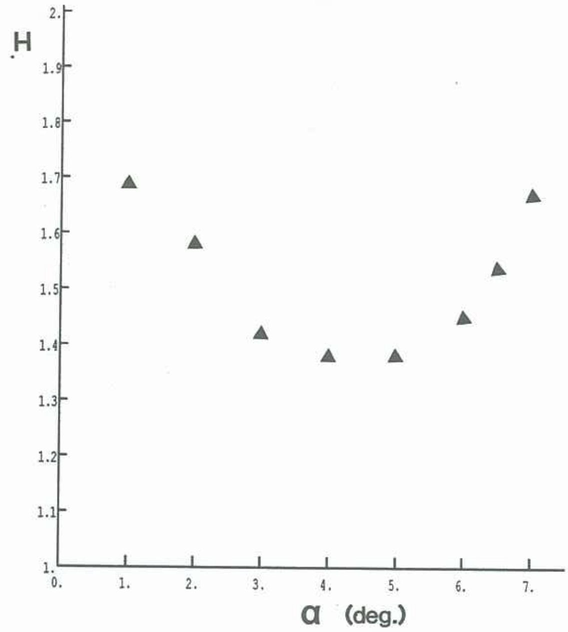



Figure 3 Shape factor $H = \dfrac{\delta^*}{\theta}$ at the trailing edge on the bubble side of the aerofoil.

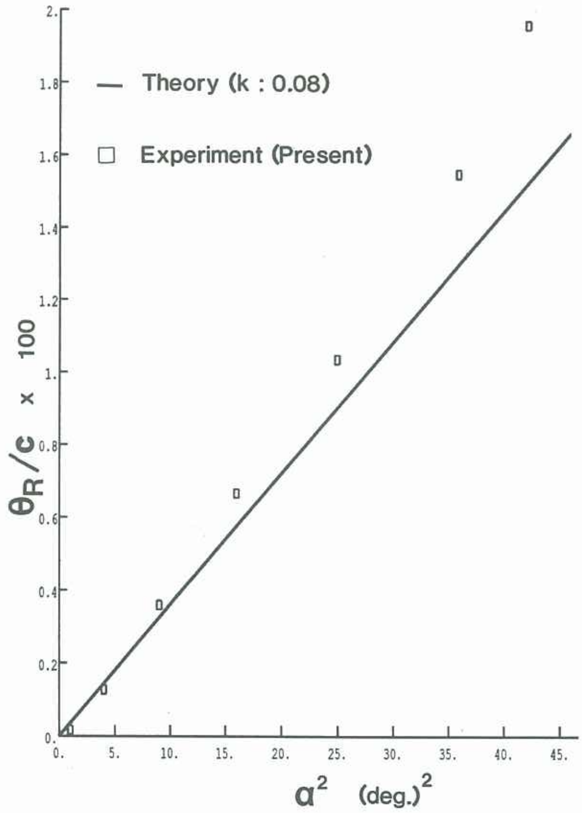
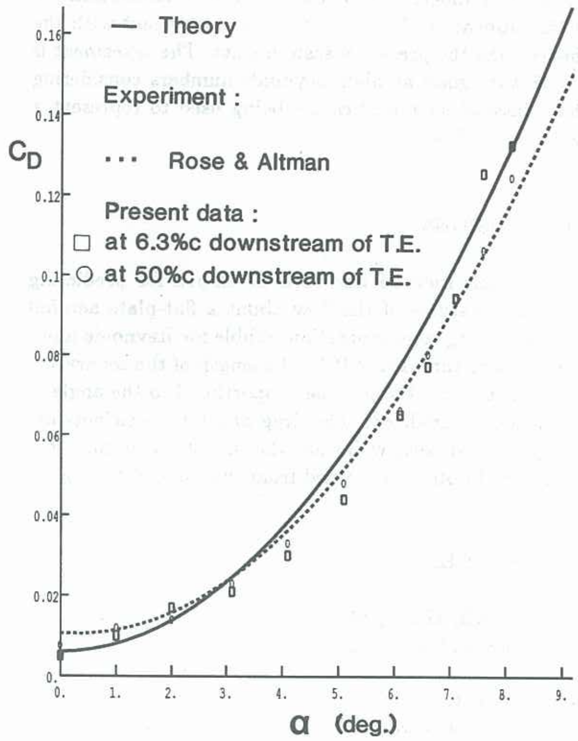



Figure 4 Momentum thickness $\theta_R$ near reattachment : measurement compared with theory.

In fig. 4 the momentum thickness just after reattachment is compared with the predicted value for the flow outside the dividing streamline in the shear layer. This is calculated to be $\theta_R = 0.03\, x_R$ when $k = 0.08$.

It may be noted that the measured $\theta_R$ is about 10% bigger than the theoretical value. This may be attributed partly to the fact that the measured values were made a small distance, .01 to .02 $c$, downstream of reattachment.



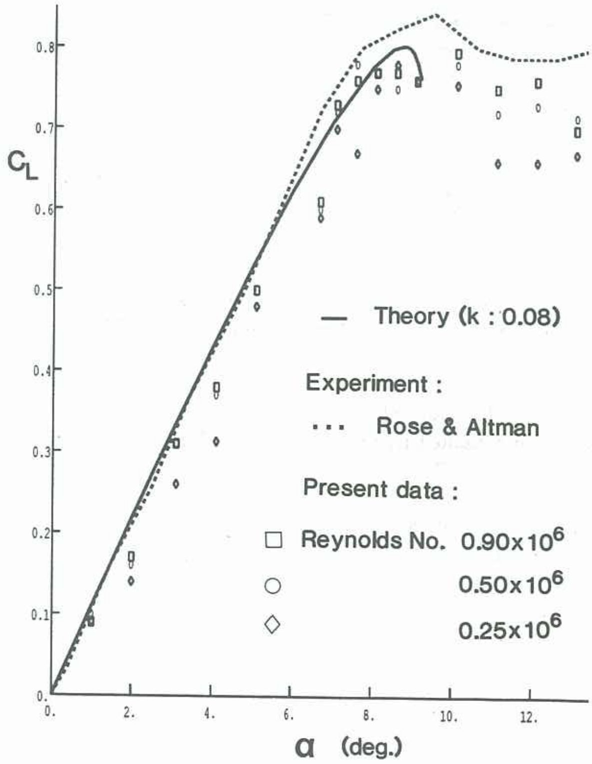




Figure 6 Lift coefficient for various $\alpha$. Theory and experiment.

The drag and lift coefficients are compared with experiments in figs. 5 and 6. Drag is dominated by the loss of leading-edge suction and is relatively insensitive to Reynolds number. The lift measured by Rose and Altman (1949) appears to be in much better agreement with the theory than the present measurements. The agreement is remarkably good at high Reynolds numbers considering that classical singularities are being used to represent a real viscous flow.

## CONCLUSIONS

A simple method has been developed for predicting the main features of the flow about a flat-plate aerofoil with a leading edge separation bubble for Reynolds numbers greater than $1/2 \times 10^6$. The length of the separation bubble, $x_R$ , is shown to be proportional to the angle of incidence squared, $\alpha^2$. The drag and lift coefficients are also predicted fairly well when the rate of entrainment beneath the bubble is obtained from the slope of the $x_R/\alpha^2$ curve.

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