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FLOW PAST A ROTATING CYLINDER

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ABSTRACT

An experimental and numerical study of flow past a rotating cylinder has been performed. These flows may be characterised by two parameters: a Reynolds number, Re, based on stream speed and cylinder diameter, and a pitch, p, which is the ratio of tangential velocity of the cylinder to free stream velocity. A number of different flow regimes exist in regions of the (Re,p) parameter plane. These include, of course, the well-known sequence of flows past a circular cylinder at p = 0 and the Re = 0 case of axisymmetric circulation. Between these limiting cases lies an extremely interesting set of flows in which fluid close to the cylinder is carried completely round the cylinder even at small values of p while free stagnation points appear in the flow; and at Reynolds numbers for which Karman vortex streets appear at p = 0, increasing p results in an increasing region of stable laminar flow and finally in a steady flow with reduced wake strength.

INTRODUCTION

The flow generated by a circular cylinder, of radius a, rotating about its axis at uniform angular velocity, or in a steady uniform stream, U, is of fundamental importance, on the one hand as a revealing example of vorticity dynamics involving the generation, advection, diffusion and annihilation of vorticity, and on the other hand for its effect on the lift and drag on the cylinder and on its wake downstream. Rotating cylinders provide a possible source of rapid lift enhancement or of boundary layer control on aerofoils.

Prandtl (1934) performed experiments on this problem and suggested that there is a regime of flow which is largely steady, has little wake, and appears to be at least quantatively similar to the inviscid flow past a circular cylinder with circulation, e.g. Batchelor(1967). Prandtl suggested that the maximum transverse force (or lift) on the cylinder would occur when the circulation is just sufficient to allow a single stagnation point on the cylinder and on this basis predicted a maximum lift coefficient, $C_L = 4\pi$. This conclusion has been questioned by Swanson (1956), using experimental techniques, Glauert (1957) on a theoretical basis and Ingham (1983) using numerical techniques.

In this paper we present results obtained using a numerical study of the full Navier Stokes equations using a stream function/vorticity formulation. To avoid the difficulties in satisfying the boundary conditions at large distances from the cylinder a new numerical technique has been developed. Further, a series expansion solution has been obtained which is valid at small values of p ($= a\Omega/U$, where a is the radius of the cylinder and Ω is the angular velocity of the cylinder) but which is found to be applicable over an unexpectedly large range of values of p. Three interesting ranges of flow emerge and some of the results obtained from the numerical and experimental investigations are presented here.

SMALL REYNOLDS NUMBER SOLUTIONS ($Re \lesssim 40$)

In this regime numerical solutions have been obtained using the numerical technique which will be reported in full later. When there is no rotation, p = 0, the results obtained by the present method are within a few percent of those obtained by Fornberg (1985) and Dennis and Chang (1970) which are regarded as being the most accurate. For small values of p a solution in powers of p is sought and it is found that the lift and drag coefficients, C_L and C_D respectively, vary as

C_L	~	$2.77\,p + O(p^3)$	Re = 5
	~	$2.54\,p + O(p^3)$	Re = 20
C_D	~	$3.947 - 0.072\,p^2$	Re = 5
	~	$1.995 - 0.109\,p^2$	Re = 20

where $Re = Ud/\nu$, where d is the diameter of the cylinder. Figures 1 and 2 show the variation of C_L as a function of p by solving the full Navier Stokes equations, the series expansion results and those obtained by Lyul'ka (1977), Ta (1975), Ingham (1983) and Badr and Dennis (1986). The agreement between the full numerical solution and the series solution for $p \lesssim 1$ is excellent. There is reasonable agreement with the other results but those of Badr and Dennis most closely agree with the present results. One of the reasons for this is that many of the previous investigators have obtained their results using a coarse grid size.

Despite the many discrepancies between all the previous results for the values of C_L and C_D there is reasonable agreement with other features of the flow, e.g. the surface vorticity, and therefore these results are not presented in this paper. Figure 3 shows the streamlines for Re = 20 at various values of p. The streamlines for p = 0 can be compared with those obtained by Dennis and Chang (1970) and Fornberg (1985) and they are, graphically, indistinguishable. As expected the effects of rotation is to change substantially the streamline pattern near the surface of the cylinder. In the potential flow problem closed streamlines near the cylinder only exist if p > 0.5 but in the case of viscous flow closed streamlines will *always* exist for *all* non zero values of p. These streamlines only exist very close to the cylinder for small values of p

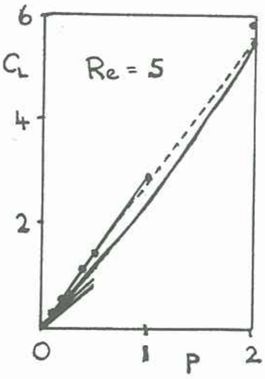


Figure 1

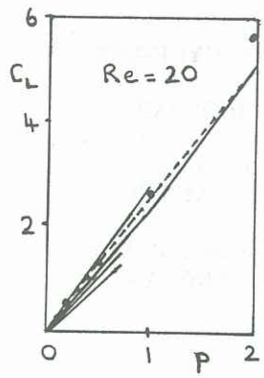
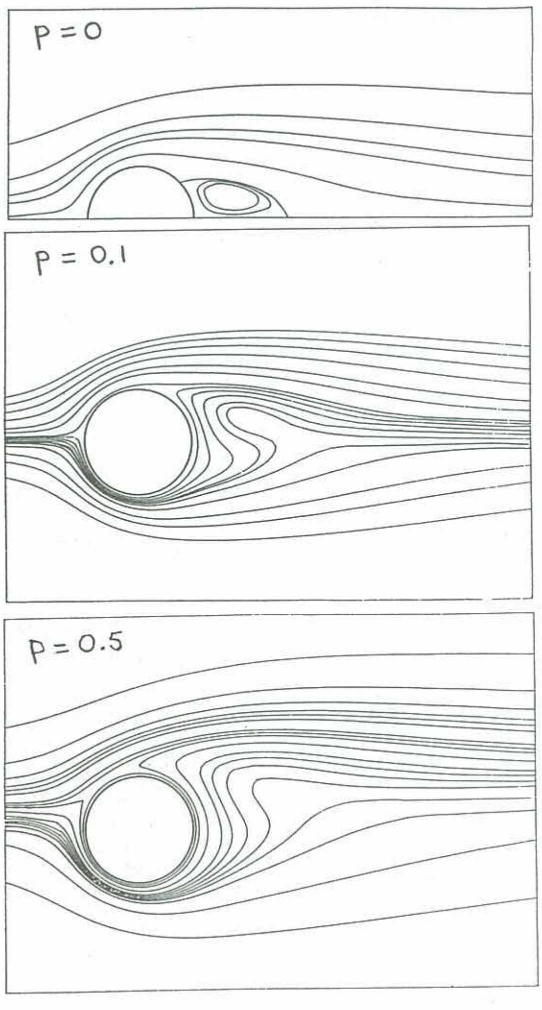


Figure 2

Variation C_L with p; • numerical ; — — — series; —— reading downwards, Badr and Dennis, Ingham, Ta and Lyul'ka.

but as p increases they exist in larger and larger regions as illustrated in figure 3. Further as p increases an increasing volume of fluid is rotating with the cylinder. For Re = 5 similar streamline patterns exist as for the case Re = 20 except that at small values of p where no closed streamlines exist behind the cylinder and hence these results are not presented in this paper. For Re = 20 the effects of rotation annihilates the closed streamline region behind the cylinder and this contrasts with the work of Ta (1975) who finds closed streamlines



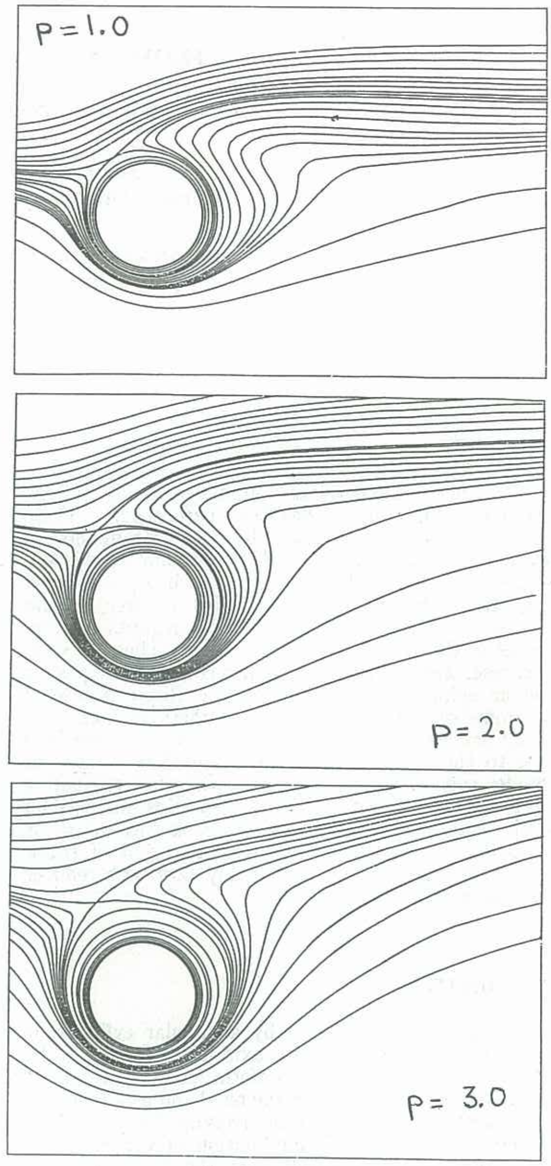


Figure 3 Streamlines for Re = 20 and various values of p.

behind the cylinder for p up to 0.2. Although numerical results have only been presented for values of p up to 3 there is no difficulty in obtaining results for much larger values of p.

INTERMEDIATE REYNOLDS NUMBER SOLUTION ($60 \lesssim Re \lesssim 100$)

In this parameter range Badr and Dennis (1986) were unable to obtain any steady state results for $p \neq 0$ using their unsteady numerical procedure. The values of C_L and C_D oscillate about a mean value after a large interval of time and these values are quoted for several values of p and Re. Using the numerical technique developed by the present authors results have been obtained for the solution of the full Navier Stokes equations and for the series in p method. Figures 4 and 5 show the variation of C_L as obtained by solving the full Navier Stokes equations and the series expansion results for Re = 60 and 100,

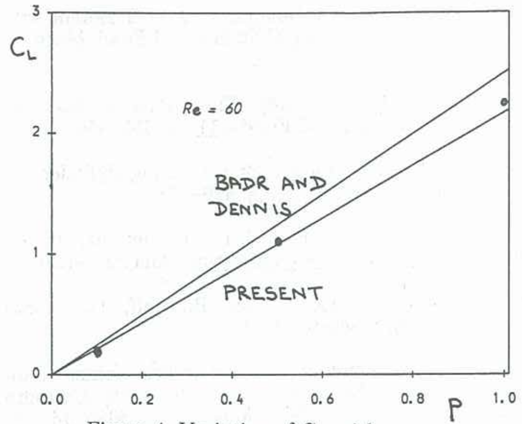


Figure 4 Variation of C_L with p

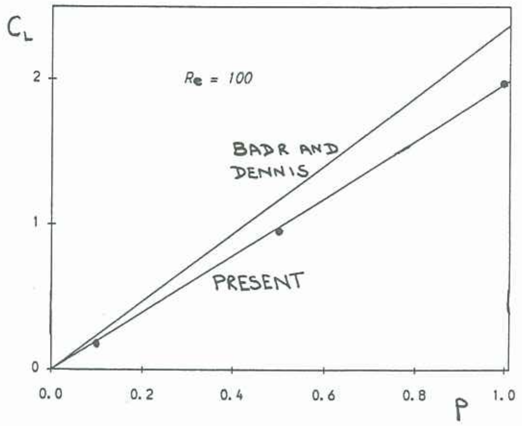


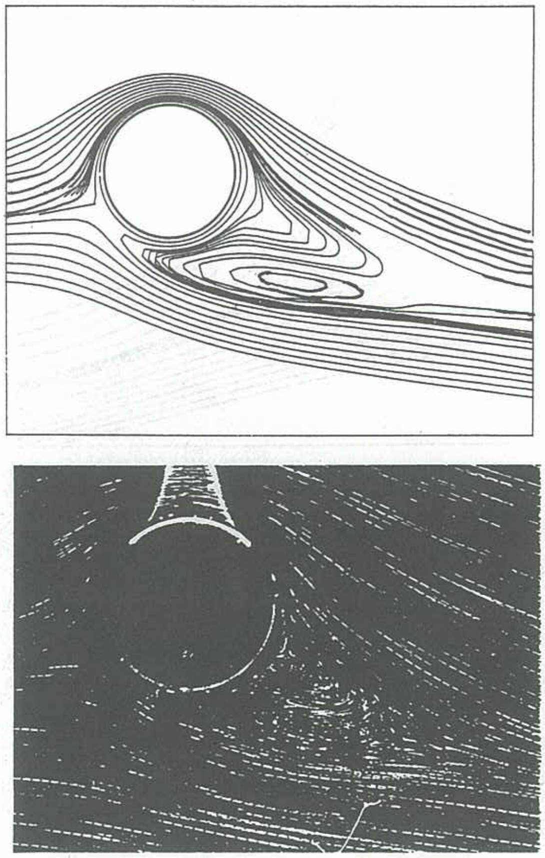
Figure 5 Variation of C_L with p

respectively and the agreement is very good. The results of Badr and Dennis are a little higher than those given by the present calculations and it is perhaps relevant to point out that their results for Re = 5 and 20 are also a little higher than all the previous investigators and it does not seem possible to state a reason for these discrepancies.

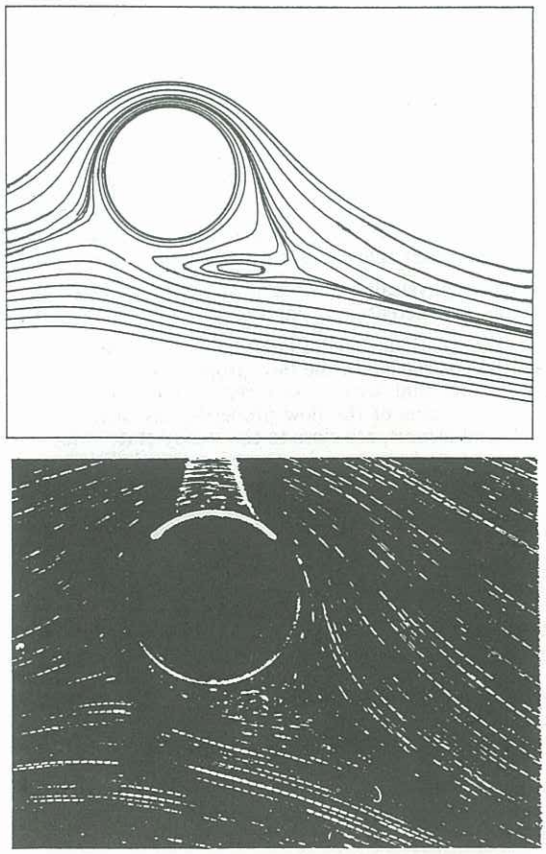
LARGE REYNOLDS NUMBER SOLUTIONS ($Re \gtrsim 100$)

As the Reynolds number increases it becomes more difficult to obtain accurate solutions of the full Navier Stokes equations when p=0, see Fornberg (1985). This is mainly due to the large recirculating zone that occurs behind the cylinder. However as the rotation of the cylinder increases this annihilates the wake. Figure 6 shows the results obtained using both experimental and numerical techniques for Re = 254 and p = 1.75, 2.54 and 2.71. Thus as p increases the symmetrical pair of eddies become asymmetric with the upper eddy eventually disappears, figure 6c.

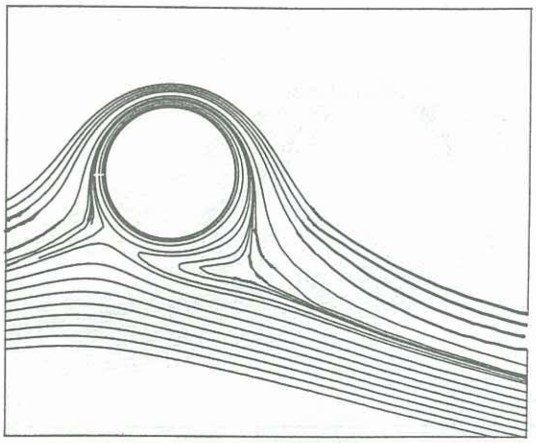
p = 1.75



p = 2.42



p = 2.71



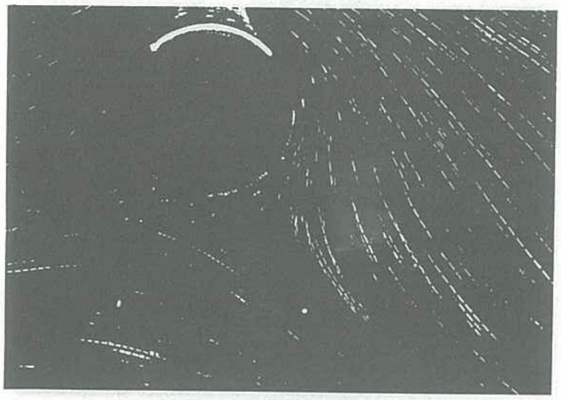


Figure 6 Streamlines for Re = 254 and various values of p.

CONCLUSION

For small values of the Reynolds numbers ($\lesssim 40$)steady state flows exist for all values of p and these observations have been confirmed numerically in this paper. As the Reynolds number increases, above about 40, then there exists a value of p, say p_c, below which no physically observable steady state flows exist and as Re increases p_c increases. In this regime of parameters Badr and Dennis found, numerically, a perioodic variation in the flow properties. However we have found that steady state results do exist and that the mean value of the flow properties, as calculated by Badr and Dennis, are close to the steady state values.

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