TENTH AUSTRALASIAN FLUID MECHANICS
CONFERENCE - UNIVERSITY OF MELBOURNE
11-15 DECEMBER 1989

RESONANT COLLAPSE IN A PRECESSING TANK

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ABSTRACT

A contained rotating fluid subjected to a forced precession is an extraordinarily rich system, exhibiting a wide variety of complex nonlinearities. The most dramatic of these involve a catastrophic breakdown of the flow and subsequent chaotic behaviour, and are called the *resonant collapse* phenomenon. An experimental study of fluid behaviour in a filled right cylindrical tank is described. An analytic linear solution based on normal modes can be found for the flow in this geometry. The tank was subjected to a forced precession at various forcing frequencies and nutation angles. The aim was to test the linear inviscid theory, and to identify and catalogue the nonlinear phenomena when the fluid is forced near the eigenfrequency of a low order mode. The fluid behaviour near resonant eigenfrequencies is dominated by the resonant collapse phenomenon, first described by McEwan (1970). The flow develops a modal waveform, becomes unstable and there is a transition to disorder, or to fine length-scale turbulence if the forcing is strong. However the term "resonant collapse" is merely a generic term for the wide variety of routes to chaos, or *collapse régimes*, which can occur in this system.

INTRODUCTION

This work is motivated by the instability problems experienced with spin-stabilised spacecraft carrying large quantities of fluids. The liquid fuel in the tanks of spinning spacecraft can be excited by precession of the spacecraft, to resonate *inertia wave modes*. Dynamic coupling of these wave modes with the spacecraft motion has been proposed as a mechanism by which the whole spacecraft becomes unstable. A major numerical and experimental project is underway. The ultimate aim is to predict of the behaviour of the spacecraft-fluid combination. Our initial studies seek an understanding of the fundamentals of the fluid behaviour in this environment. The initial experiment described here involved flow visualisation only, and was intended to study the response of the fluid to steady precessional forcing.

There have been several previous studies of breakdowns in a rotating fluid. Malkus (1968) found instabilities leading to turbulence in spheriod precessing through large angle. A thorough and definitive study was conducted by McEwan (1970). His experiment forced inertia waves in a rotating cylindrical tank by means of an angled top rotating relative to the tank. Stergiopolous & Aldridge (1982) noted breakdowns in a cylindrical tank where a free surface provided the forcing. Malkus (1988) observed breakdowns of azimuthal wavenumber-two modes excited by tidal distortion of a cylindrical tank.

FORMULATION

Referring to Figure 1, the absolute spin rate of the tank in inertial space is $\omega_1 + \omega_2$ for θ small, the frequency at which the fluid is forced is the spin rate of the turntable relative to the tank, ω_1, and the forcing amplitude is the *nutation angle*, θ. There are two parameters in the linear inviscid incompressible problem; the frequency parameter $\omega = 2(1 + \omega_2/\omega_1)$ and θ. In the experiments reported here most of the investigations explored a space spanned by these two parameters. The homogeneous boundary value problem, based on cylindrical polars fixed in the tank, is

$$\frac{\partial \mathbf{u}}{\partial t} + \omega \hat{\mathbf{k}} \times \mathbf{u} + \nabla p = 0, \qquad (1.1)$$

$$\nabla \cdot \mathbf{u} = 0, \qquad (1.2)$$

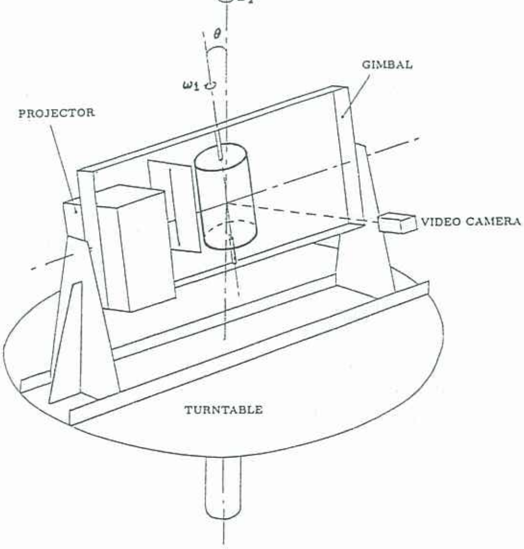


Figure 1 Apparatus

$$\mathbf{u} \cdot \hat{\mathbf{n}} = 0, \qquad (1.3)$$

where $\mathbf{u}$ is the fluid velocity, p is a reduced pressure, $\hat{\mathbf{k}}$ is aligned along the tank axis of symmetry and $\hat{\mathbf{n}}$ is a unit normal at the tank wall. For a specified time dependence we get Poincaré's classical eigenvalue problem

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial Q}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 Q}{\partial \phi^2} + (1-\omega^2)\frac{\partial^2 Q}{\partial z^2} = 0, \qquad (2)$$

where Q is the pressure amplitude. For right cylindrical boundary conditions and $\omega > 1$ we can find a separation of variables solution for a densely spaced set of inertia wave modes.

EXPERIMENT AND VISUALISATION METHOD

With $\theta = 0$ initially, the tank and turntable are spun up to speeds whose combinations yield a resonant eigenfrequency. Forcing is begun by increasing θ to a small preset value.

The tank is viewed from the *turntable* frame of reference by both video and still cameras. A sheet of white light illuminates a plane through the cylinder. Fine reflective flakes, called pearlescence, are suspended in the fluid. These flakes take a preferred orientation relative to the *direction* of the local vorticity vector. Thus we have a very sensitive indicator of the direction of shear in the flow, but not of its magnitude. Linear inertia waves are travelling waves in the tank frame of reference. With a plane of light rotating in the turntable frame, we are effectively following the wave pattern, which appears fixed to us.

SOME COLLAPSE RÉGIMES

In this brief report we will only be able to give some examples of the variety of ways that contained inertia waves can break.

Figures 2(a-c) show a breakdown of the fundamental or (1,1,1) mode. The axial wavenumber can be identified in these visualisations by counting the number of axial half-wavelengths in the pattern. The tank is spinning with $\omega_1 = 10.47$ rad s^{-1}. The nutation angle $\theta = 3°$. Here $\omega = 2.720$, which is within about 2% of the resonant eigenfrequency. Times after the forcing is begun are (a) 2.7, (b) 3.3 and (c) 4.5 seconds, or 4.5, 5.5 and 7.5 revolutions of the tank relative to the turntable. The flow reaches a state which could be described as turbulent, about 7 tank revolutions after the commencement of forcing. In this "Type A" collapse régime, turbulence is maintained as long as the forcing is maintained.

However the Type A collapse is just one of many collapse régimes. Apparently different régimes are observed when each mode is forced, and furthermore different régimes are consistently observed when a given mode is forced in different regions of parameter space.

Figures 3(a-c) show a resonant collapse of the (3,1,1) mode, where $\omega = 1.20$, $\theta = 3°$. It is characterised as a "Type D" collapse. Note the three axial half-wavelengths. At about 5 revolutions after the impulsive tilt, unsteadiness appears in the wavy pattern. Mod-

(a)

(b)

(c)

Figure 2

ulations of smaller wavelength appear. Fine-grain turbulence is produced, as shown in Fig. 3(b), about 10 to 20 revolutions after the commencement of forcing. The turbulence then becomes less fine-grain and by about 30 revolutions some order can once more be discerned in the flow. The modal waveform re-appears superimposed on the general disorder, as in Fig. 3(c).

It is common to many collapse régimes that the behaviour after collapse does not remain completely disordered. Even if forcing is maintained, the disordered flow usually recovers some modal structure after the collapse. In many regions of parameter space, the flow after an initial collapse becomes quite ordered and then undergoes further recurrent collapses, the flow regaining some order after each recurring collapse. This laminar "intermittency" or recovery of order from chaos is very unusual and has only been observed in a few fluid systems, such as that described by Mullin and Price (1989).

In these flow visualisation experiments only a subjective judgement can be made of when collapse has occurred, but careful estimates can still provide us with useful information. McEwan (1970) found the times for collapses to occur in his system were quite consistent. This was also found to be the case in these experiments. Figure 4 shows the (1,1,1) mode collapse time as a function of ω. The collapse times t_c are in the number of tank revolutions relative to the turntable after the forcing began. Here we see the behaviour in a small bandwidth in ω space. At small θ, there is much more sensitivity in selecting collapse behaviour as we vary ω. The Type B collapses are a very different régime to the Type A collapses; they are characterised by wavy patterns with a higher axial wavenumber increasing in intensity until the whole flow becomes disordered. Here we see records of the Type C recurrent collapse behaviour, joined by vertical dotted lines.

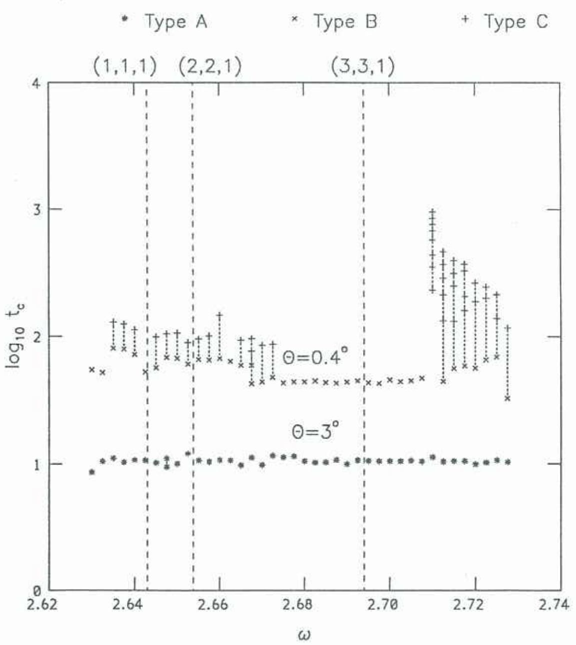
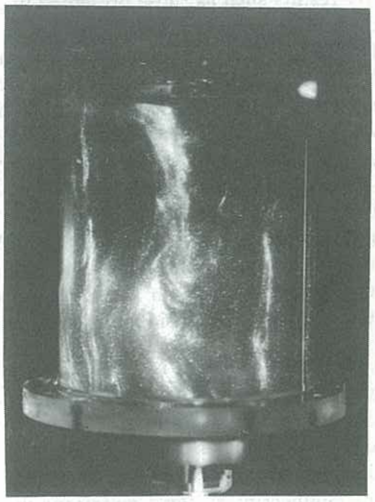


Figure 4 (1,1,1)—Mode Collapse Times

(a)

(b)



(c)

Figure 3

Figure 5 shows the variation in Type A collapse times with the Ekman number, defined as $E = \frac{\nu}{(\omega_1+\omega_2)D^2}$ with ν the kinematic viscosity and D the tank diameter. It can be varied by scaling ω_1 and ω_2 appropriately while maintaining their ratio constant. Vertical bars indicate 90% statistical confidence limits. These results show that variations in the top and bottom boundary layer thicknesses, which scale as $E^{\frac{1}{2}}$, have a minimal influence on the Type A collapse behaviour.

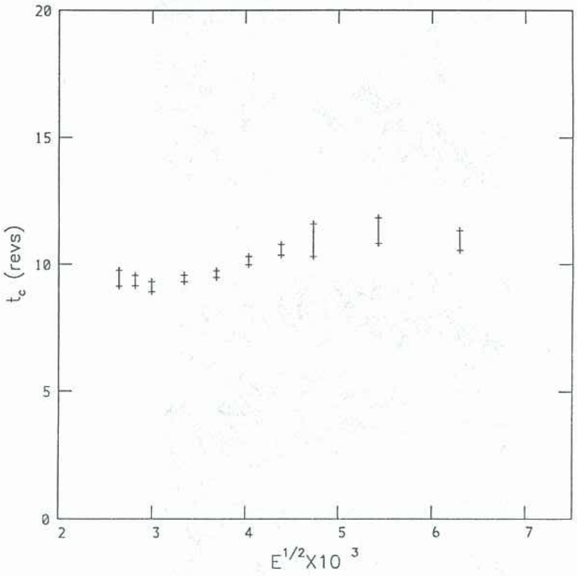


Figure 5 (1,1,1)–Mode Collapse Times

ω=2.64, Θ=5°

DISCUSSION & CONCLUSION

The variety of different collapse régimes indicate that different instability mechanisms are at work. Boundary layers instabilities were originally proposed by Malkus (1968), however results such as those presented in Figure 5 and observations of the "global" nature of the collapses make this an unlikely mechanism. The modification of the basic rotation by an azimuthal circulation has been proposed by Aldridge (1989, personal communication) as a collapse mechanism and implicated by McEwan (1970) in his observations of resonant collapse. Weakly nonlinear interactions of modes in groups of three is possible, and can be modelled by means of evolution equations. It is anticipated that image processing of the flow visualisations of collapse types B and C will provide data on the wavenumbers of the interacting modes, for inclusion in such a model.

In conclusion, "resonant collapse" is a generic term. It covers a rich variety of instabilities occurring in rotating fluids where inertia waves are forced. Much work has to be done to model the behaviour of contained inertia waves, which have an unexpectedly powerful influence on rotating engineering systems containing fluids. No single approach is likely to provide the solution.

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