TENTH AUSTRALASIAN FLUID MECHANICS
CONFERENCE - UNIVERSITY OF MELBOURNE
11-15 DECEMBER 1989

LAMINAR FLUID CONVECTION OF LOW PRANDTL NUMBER FLUID IN THE ANNULI OF ROTATING CYLINDERS

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ABSTRACT

Convective fluid flow pattern and heat transfer characteristics in the annular space between horizontal cylinders when the inner cylinder is heated and rotating, has been studied numerically for a fluid with Prandtl number of 0.02 (ie liquid mercury). The local and overall equivalent thermal conductivity are obtained for Rayleigh number up to 10^6 with rotational Reynolds number up to 10^3. For a radius ratio of 2.6, studies show that for a fluid with Prandtl number 0.02 and a Rayleigh number of the order of 10^5, a multi-cells flow pattern on one side of the cylinder is suppressed, giving an observer the impression that the thermal plume is moving in a direction opposite to that of the rotation of the inner cylinder. For a particular Rayleigh number, the overall thermal equivalent conductivity appears almost constant with respect to the rotational Reynolds number, though the local thermal conductivity exhibits very different features. Streamlines and isotherms are presented for this study.

INTRODUCTION

Convective fluid motion in region bounded by two horizontal cylinders with parallel axes has been the subject of many studies in recent years. In most of these studies, attention has been focused on fluids with large Prandtl numbers of order one and larger. Kuehn & Goldstein (1976; 1978) present results of experimental and numerical studies of the motion of air and water within a horizontal annulus. A comprehensive review of the work involving convective fluid motion and heat transfer in cylindrical annuli have also been collated. The reported studies, however, were concerned with stationary cylinders with high Prandtl number fluids and few consider the rotation of the inner cylinder with low Prandtl number fluids. Because of the excellent heat transfer characteristics of the low Prandtl number fluids, such as liquid mercury, such fluids are being considered for used increasingly as the working fluid in several power generating cycles. It is therefore of considerable important to understand the convective fluid flow motion and heat transfer characteristics when such fluids are in used.

The present study is to investigate the fluid motion and heat transfer characteristics of a fluid with a Prandtl number of 0.02 contained within a horizontal cylindrical annulus of radius ratio 2.6. The inner cylinder is assumed heated and rotating. The flow is assumed to be steady, two-dimensional and laminar. End effects of the rotational cylinder is assumed negligible.

GOVERNING EQUATIONS

A schematic configuration of the annulus is shown in Figure 1. The dimensionless governing equations that describe the motion of the incompressible fluid within the annulus subject to the Boussinesq approximation are the vorticity transport equation, the energy transport equation, a vorticity vector defined by

$$\zeta^* = -\nabla^2 \psi^* \tag{1}$$

and the velocity vector

$$u^* = \nabla \times \psi^* \tag{2}$$

where $r^*=r/L$, $t^*=t/(L^2/\alpha)$, $u^*=u/(\alpha/L)$, $v^*=v/(\alpha/L)$, $\zeta^*=\zeta/(\alpha/L^2)$, $\psi^*=\psi/\alpha$,

and a dimensionless temperature $\Theta = (T-T_R)/T_m$. (From here on the asterisk indicating a dimensionless quantity is dropped for simplicity)

NUMERICAL METHOD

The finite difference solutions of the governing equations with their appropriate boundary conditions are obtained at the nodal points of a

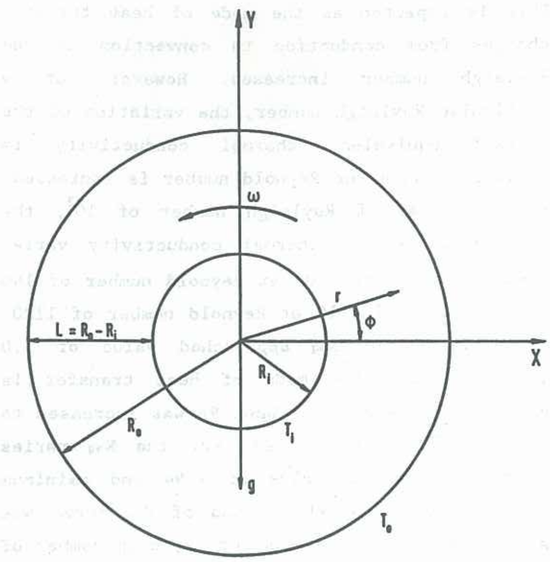
20 x 60 uniform mesh superimposed on the solution region of Figure 1. Second-order central differencing approach was used for all expressions in the equations except the convective terms for which a second order upwind differencing method was used. The Alternating Direction Implicit method of Peaceman & Rachford(1955) was used to solve the vorticity transport equation and the energy transport equation in a time marching manner. For the stream- function vorticity equation (1), the Wachspress parameters, which is a sequence of convergent acceleration parameters, was used to accelerate the convergence of the solution by Successive Over Relaxation method. The stream function ψ_o on the outer cylinder wall is arbitrarily set to zero. At the inner rotating cylinder wall, ψ_i cannot be pre-assigned. The use of $\frac{\partial\psi}{\partial n}$ = wall velocity, gives solution for which $\frac{\partial\psi}{\partial s} \neq 0$ along the wall of the inner cylinder. This implies that fluid was numerically 'leaked' through the moving inner cylinder wall. In the present study, ψ_i is determined using the criterion that the pressure distribution in the solution region is a single-valued function. Mathematically, this criterion implies that the line integral of the pressure gradient $\frac{\partial P}{\partial s}$ along any closed loop circumscribing the inner cylinder is zero i.e. $\oint \frac{\partial P}{\partial s}\, ds = 0$. $\frac{\partial P}{\partial s}$ can be evaluated from the momentum conservation equations. From the stream function solution, the vorticity along the solid walls are then evaluated from equation (1). A convergence criterion of the form $(\Theta - \Theta_{ref})/\Theta_{ref} \leq 0.001$ is used for the temperature field and the overall thermal conductivity to indicate steady state convergence. The stream function, velocity and vorticity fields are noted as steady when the temperature field and the K_{eq} are steady.

RESULTS AND CONCLUSIONS

The flow and temperature fields of high Prandtl number fluid in the horizontal cylindrical annulli have been extensively studied by Kuehn & Goldstein(1976) and many other investigators, and will not be discussed here. For low Prandtl number fluid flow in horizontal cylindrical annulli, the existence of secondary cells creating two thermal plumes spreading at an angle of 45° on both side of the cylinder upward vertical axis has also created much interest(Huetz,1974). However, all of these studies were on stationary cylinders and none consider the rotation of the inner cylinders. For low Prandtl number fluid (say Pr = 0.02) and when the inner cylinder is made to rotate, there is no experimental nor analytical/numerical results available. It is well known that for Rayleigh number in the range of 10^5, with R_o/R_i=2.6 (Kuehn et al, 1976, 1978), the flow pattern of low Prandtl number fluid is distinctly different from those of high Prandtl number. For fluid with low Prandtl number, the results shows a multicellular flow pattern which contrasts with the usual monocellular flow field obtained for high Prandtl number fluids. As a consequence of this multicellular flow pattern, the heat transfer profiles for fluids with low Prandtl number were found to have points of maximum and minimum at the interior nodes, instead of the top and bottom nodes which is known for high Prandtl number. When the inner cylinder is made to rotate, the flow pattern becomes more complex. The multicellular flow pattern on one side of the cylinder is suppressed, giving the observer the impression that the thermal plume is moving in the opposite direction to that of the rotational direction of the inner cylinder. This thermal plume movement is opposite to that obersever for the high Prandtl number fluid flow where the mono-thermal plume on top of the inner cylinder moves in the same direction as the inner rotating cylinder.

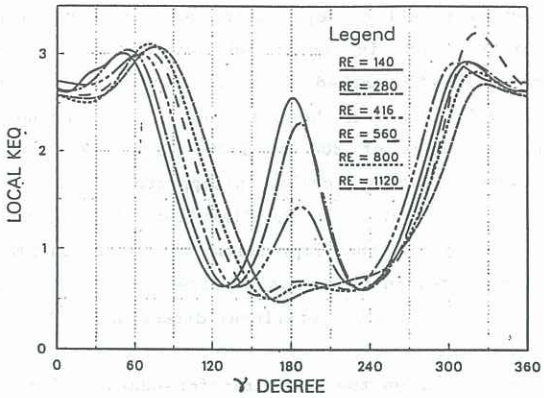
The development of these counter rotating thermal plume for low Prandtl number fluid at a Rayleigh number of 5.5 x 10^4 is shown in figure 2. At a low Reynold number of 140 (Figure 2a), two counter-rotating cells of different strength are formed above the inner cylinder. Because of the formation of these counter-rotating cells, a pair of thermal plumes are formed above the inner cylinder at an approximate angle of 45° on both side of the upward vertical axis. It is noted that the left counter-rotating cell is weaker than the right counter-rotating cell. As the Reynold number is gradually increased, the left counter-rotating cell decreases its strength further. The zero-value streamline gradually enveloped the right counter-rotating cell. At the same time, it is noted that the thermal plume on the left side of the inner cylinder is gradually suppressed.

Increasing the angular speed to the Reynold number of 416 (Figure 2b),it is seen that the left-hand cell decreased in size while there is now only a single counter-roting cell formed above the inner cylinder. The thermal plume on the left hand side continued to be suppressed further. As the Reynold number increases, the



COORDINATE SYSTEM IN THE ANNULAR REGION

FIGURE 1

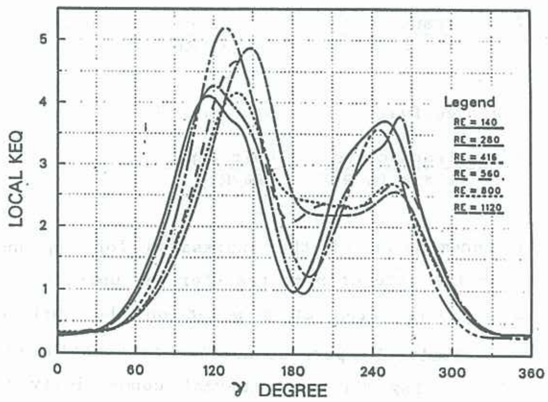


LOCAL EQUIVALENT THERMAL CONDUCTIVITY

OF INNER ROTATING CYLINDER

$R_o/R_i = 2.6$, $Ra = 5.5 \times 10^4$, $Pr = 0.02$

FIGURE 3

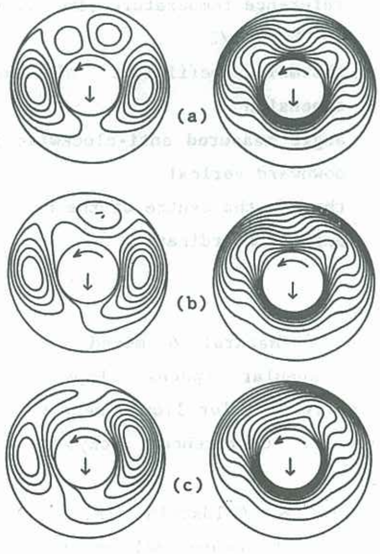


LOCAL EQUIVALENT THERMAL CONDUCTIVITY

OF OUTER STATIONARY CYLINDER

$R_o/R_i = 2.6$, $Ra = 5.5 \times 10^4$, $Pr = 0.02$

FIGURE 4

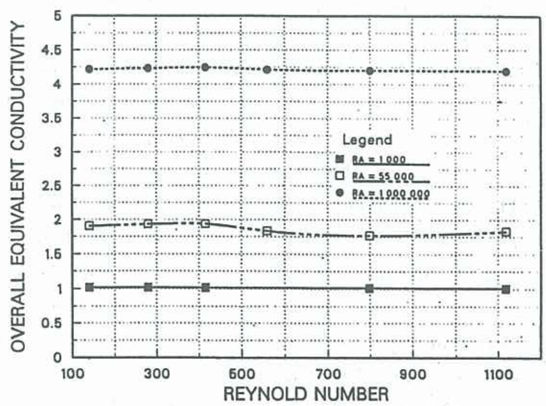


STREAMLINES AND ISOTHERMS

$R_o/R_i = 2.6$, $Ra = 5.5 \times 10^4$, $r = 0.02$

(a) Re = 140, (b) Re = 416, (c) Re = 1120

FIGURE 2



OVERALL EQUIVALENT THERMAL CONDUCTIVITY

AT VARIOUS RAYLEIGH NUMBER

$R_o/R_i = 2.6$, $Pr = 0.02$

FIGURE 5

counter-rotating cell decreases its strength further and is enveloped by the left-hand main-cell. A marked decrease in the left-hand thermal plume can be observed. At the Reynold number range of 800 to 1120 (Figure 2c), the counter-rotating cells disappeared and the left-hand thermal plume "levelled off", giving the observer the impression that the thermal plume moves in the opposite direction to that of the inner cylinder rotational direction.

The effects on the heat transfer characteristics of the low Prandtl number fluid due to the above complex fluid flow pattern when the inner cylinder is rotated, can be described by the equivalent thermal coductivities as defined below.

Local equivalent thermal conductivity, K_{eql}

$$= \frac{\text{(Heat transferred per unit area at a point)}}{k\,(\Delta\,\Theta)/\,(r\,\ln\,(R_o/R_i))}$$

Overall equivalent thermal conductivity, K_{eq}

$$= \frac{\text{(Heat transferred per unit length}}{2\,\pi\,k\,(\Delta\,\Theta)/\,\ln\,(R_o/R_i)}$$

The denominator of the expression for K_{eq} and K_{eql} is the rate of heat transfer per unit length and per unit area at a point on the surface respectively, by pure conduction in a motionless medium having the same thermal conductivity as the fluid. For K_{eql} in equation (11), the radial distance r is either R_o or R_i depending on the surface considered.

As shown in Figures 3 and 4, at a Rayleigh number of 5.5 x 10^4, the distribution of the local equivalent thermal conductivity at the inner and outer cylinders are greatly influnced by the development and modification of the multi-cellular flow pattern due to the rotation of the inner cylinder. Arising from the suppression of the left thermal plume, the peak heat flux on the outer cylinder takes on two different values. The peak value for the right-hand thermal plume being higher than that of the left-hand thermal plume. As the Reynold number is increased, the peak local heat flux near the left-hand thermal plume graudually levelled off. For the inner cylinder, the local peak value near the top of the cylinder gradually levelled off as the Reynold number is increased. For the case of low Prandtl number of 0.02, figure 5 shows that as the Rayleigh number increases, the overall equivalent thermal conductivity increases.

This is expected as the mode of heat transfer changes from conduction to convection as the Rayleigh number increases. However, at a particular Rayleigh number, the variation of the overall equivalent thermal conductivity is negligible when the Reynold number is increased. For the case of Rayleigh number of 10^3, the overall equivalent thermal conductivity varies from a minimum of 1.02 at Reynold number of 140 to a maximum of 1.10 at Reynold number of 1120. These values of K_{eq} approached value of 1.0 indicates that the mode of heat transfer is essentially conduction. When Ra was increased to 5.5 x 10^4, it was noted that the K_{eq} varies between a maximum value of 1.94 and aminimum value of 1.77. Similar trend of K_{eq} curve was also observed for the case of Rayleigh number of 10^6. It varies between a maximum value of 4.23 at Re=416 and a minimum value of 4.19 at Re=1120.

NOMENCLATURE

L	characteristics length, $L=(R_o - R_i)$
Pr	Prandtl number, $Pr=\nu/\alpha$
Ra	Rayleigh number, $Ra=\beta g L^3 T_m/\alpha\nu$
t	time
T	Temperature
T_R	reference temperature ,$T_R= (T_i + T_o)/2$
T_M	$T_M=(T_i - T_o)/2$
β	thermal coefficient of volumetric expension
γ	angle measured anti-clockwise from the downward verical through the centre of the heated cylinder.
ϕ	angular coordinate

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