A CALCULATION METHOD FOR TURBULENT BOUNDARY LAYERS ON ROUGH SURFACES

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ABSTRACT

A prediction method for rough surfaces has been given that is based on a simple extension of the van Driest (1956) damping function. The method stimulates the turbulent shear stresses near the wall by manipulating the amount of viscous damping applied close to the wall. The method has been applied to a series of test cases of varying complexity and surface roughness showing that the predictions of flows along rough surfaces can be computed with the same degree of accuracy as for flows over smooth surfaces.

INTRODUCTION

A number of turbulence models have been proposed that allow the calculation of turbulent boundary layers on rough surfaces by minor modifications in the formulation of models for smooth surfaces. (Cebeci & Chang (1978), Granville (1985), Hoekstra (1983) and others). Common to most of them is that they are based on the work by Rotta (1962). Rotta argued that the direct influence of surface roughness is only felt very close to the surface and came to the conclusion that this effect on the law of the wall could be obtained by shifting the reference surface plane with respect to the mean surface level. Although this produces the desired effect on the law of the wall it does not seem to have any direct relation to the physical problem since the shift required is frequently much higher than the height of the roughness elements.

The development of boundary layers on rough surfaces is highly dependent of the characteristics of the surface. The way the different surfaces differ in an aerodynamic sense will be in the amounts of local separations and vorticity that the surface produces. The flow close to the surface, where the flow is dominated by the local roughness elements, would then have to be a function of a large number of geometry dependent variables k_1 to k_n defining the surface texture.

$$U = F(\rho,\mu,\tau,y,k_1...k_n) \tag{1}$$

Thus it will not be likely that a unique relationship will exist that may characterize a rough surface aerodynamically from general statistics of the grain size parameters. We will instead assume that the aerodynamic characteristics of the surface may be related to a reference surface which may be characterized by a single roughness length. We take this to be a surface consisting of sand grain roughness.

At distances from the wall that are comparable to the roughness heights the velocity distribution is likely to be strongly dependent on the position relative to the elements so that equation (1) may be written in the following nondimensional form

$$U^+ = F(\frac{y}{k},k^+) \tag{2}$$

(The superscript denotes variables that have been made dimensionless using wall variables. These are the friction velocity U_τ and the viscous length scale ν / U_τ).

At larger distances from the wall where $y/k >> 1$ the flow will be fully turbulent and will not know what kind of surface that generated the turbulence. We may therefore assume that, except for a constant that depends on the inner region, the velocity in this region will show the same dependence as for a smooth surface, i.e.

$$U^+ = \frac{1}{\kappa}\ln y^+ + A(k^+) \tag{3}$$

The only way that equation (2) and (3) can match smoothly is if $A(k^+)$ is a logarithmic function. Thus we see that the two equations may be rewritten as

$$U^+ = \frac{1}{\kappa}\ln y^+ + A - \Delta U^+(k^+) \tag{4}$$

and

$$\Delta U^+ = \frac{1}{\kappa}\ln k^+ + B \tag{5}$$

which must apply in the fully turbulent region. We shall therefore require that our model generates a velocity profile that has a logarithmic region with the same slope for all roughness lengths and that the velocity shift compared to the smooth surface is the shift that is found experimentally for sand roughness.

TURBULENCE MODEL

The main effect of surface roughness is to increase the turbulence close to the roughness elements. The primary goal for a turbulence model is therefore to incorporate this effect in the way the turbulent shear stresses are modelled by relating this increase directly to the surface roughness. The basis of the proposed model is the van Driest (1956) damping function combined with the mixing length formulation of Michel, Quémard & Durant (1969). In the region close to the wall where the shear stress may be assumed to be constant the following relation exists

$$\tau = \tau_w = \mu\frac{dU}{dy} - \rho\,\overline{u'v'} \tag{6}$$

τ_w being the wall shear stress.

Assuming that the turbulent stress may be written

$$-\rho\,\overline{u'v'} = \rho\,(lF)^2 \tag{7}$$

equation (6) may be integrated to give the well known van Driest version of the law of the wall

$$U^+ = \int_0^{y^+} \frac{2dy^+}{1+\sqrt{1+\left(2l^+F\right)^2}} \tag{8}$$

This shows that if the product l^+F grows linearly when it is significantly larger than 1 we are guaranteed that U^+ will have a region of logarithmic growth. The slope of this region is given by the inverse of the constant that relates l^+F to y^+. According to what was stated above this constant, κ, must remain independent of surface roughness. l^+ is the dimensionless mixing length which when using the model of Michel & al. becomes

$$l^+ = \frac{0.085\delta U_\tau}{\nu}\tanh\left(\frac{\kappa y}{0.085\delta}\right) \tag{9}$$

and F is the van Driest damping function which for a smooth surface is written

$$F = 1 - \exp\left(-\frac{y^+}{A^+}\right) \tag{10}$$

The effect of the damping function is to reduce the contribution of the turbulent stress for y^+ distances less than about $3A^+$. With the commonly accepted value of $A^+ = 26$ it is seen that the flow will be fully turbulent from about $A^+ = 75$.

In the case of a rough surface the extent of the viscous layer close to the wall is reduced since the turbulent mixing is more vigorous than for a smooth surface. It was proposed by van Driest that this effect could be obtained by reducing the amount of damping. He proposed an additional term to equation (10) to account for roughness

$$F = 1 - \exp\left(-\frac{y^+}{A^+}\right) + \exp\left(-\frac{y^+R^+}{A^+k^+}\right) \tag{11}$$

R^+ was given the value of 60 which is roughly where the damping disappears. Thus we see that for $k^+ = R^+$ there is no damping so this would be the definition of a fully rough surface. Obviously the same cancellation could be obtained by taking $\left(\frac{R^+}{k^+}\right)$ to any power. Sivykh (1984) pointed out that this formulation underpredicts the ΔU^+ shift in the law of the wall for high roughness numbers as shown in figure 1. This is because the formulation of equation (11) limits $F \le 1$ whereas for large roughness elements it may even be necessary to increase the estimated shear stress above the corresponding undamped value for a smooth surface.

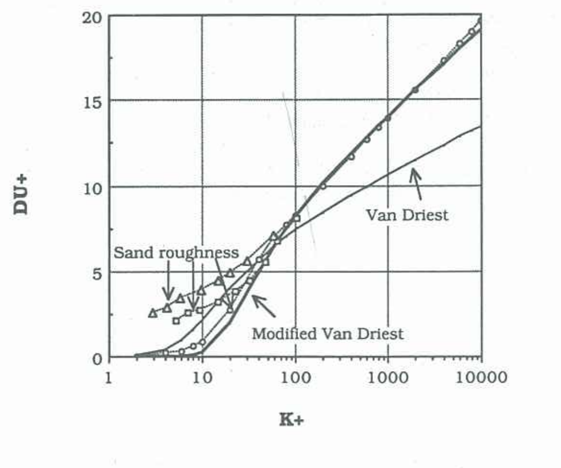

Figure 1 Shift in the law of the wall due to roughness. (Predictions using equation (11) or (12) compared to experiments).

We therefore propose the following formulation

$$F = 1 - \exp\left(-\frac{y^+}{A^+}\right) + \exp\left\{-\frac{y^+}{A^+}\left(\frac{R^+}{k^+}\right)^{\frac{3}{2}}\right\}\sqrt{1+\exp\left(-\frac{R^+}{k^+}\right)} \tag{12}$$

Here $R^+ = 70$ which produces a logarithmic increase in ΔU^+ for $k^+ > 100$ as observed experimentally. By adding the square root term the slope is increased by the necessary amount. As seen from figure 1 this modified formulation gives a considerable improvement to the original van Driest formulation as it produces a velocity shift that is identical to the shift found from experiments with sand roughness. For $k^+ \le 100$ the experimental results are no longer unique since they will depend on the fraction of elements that are sufficiently small to be considered as aerodynamically smooth ($k^+ \le 5$). No effort has therefore been made to match any particular distribution. In the region where ΔU^+ grows logarithmically the following equation is obtained

$$\Delta U^+ = 2.50\ln k^+ - 3.37 \tag{13}$$

Figure 2 shows the law of the wall that the model produces. As the roughness increases, the onset of the logarithmic region shifts to higher values of y^+.

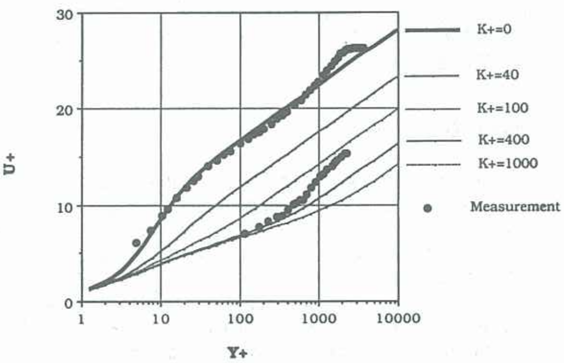

Figure 2 Law of the wall as function of surface roughness.

RESULTS

The proposed model was incorporated into the prediction method of Krogstad (1985) which is a fully three-dimensional prediction method for incompressible boundary layers using the ADI (Alternating-Direction-Implicit) solution scheme. In order to verify the model it is necessary to apply it to a number of test cases of different complexities. An overview of rough surface experiments has been given by Uram (1981).

Flat plate flows

It is apparent that the model must predict the flow along flat plates with different degrees of roughness in order to be acceptable. We have computed the flat plate test case of Lewkowicz & Das (1986) who did measurements on a replica of a full scale ship surface. From the measured velocity profiles the shift in the logarithmic region could be measured. Comparing this to equation (13) the equivalent sand roughness was estimated to be k=3.10mm giving $k^+ = 350$ at the initial station. The results are shown in figure 3. Lewkowicz & Das computed c_f from the mean velocity profile and a momentum integral balance. In addition they measured the turbulent shear stress profiles from which an estimate of c_f may be obtained. Although the different estimates deviate somewhat, the calculations are in close agreement with the experimental data. The zero pressure gradient experiment of Scottron & Power (1965) was also calculated with the same level of accuracy. In their experiment, which was performed on a wire mesh surface, the equivalent sand roughness was estimated to be k=11.3mm which gives a dimensionless roughness height of $k^+ = 1375$ at the position where the calculations were started.

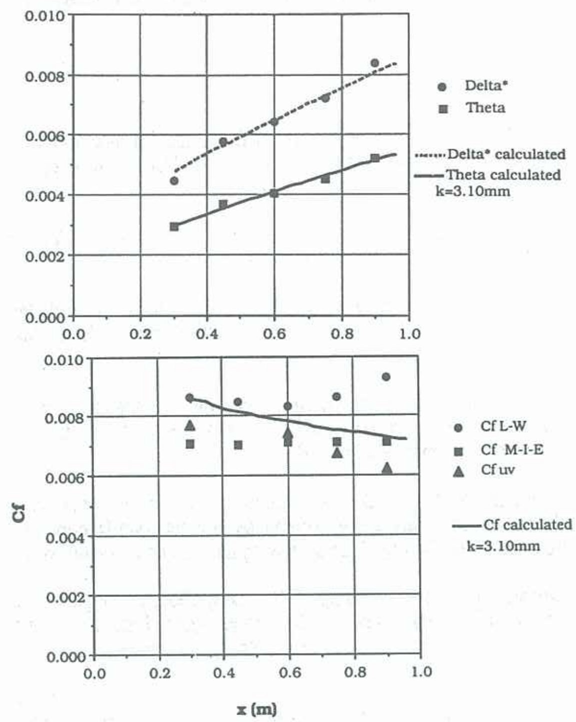

Figure 3 Constant pressure flow of Lewkowicz and Das. Skin friction was estimated from the law of the wall (Cf L-W), momentum integral balance (Cf M-I-E) and turbulent shear stresses (Cf uv).

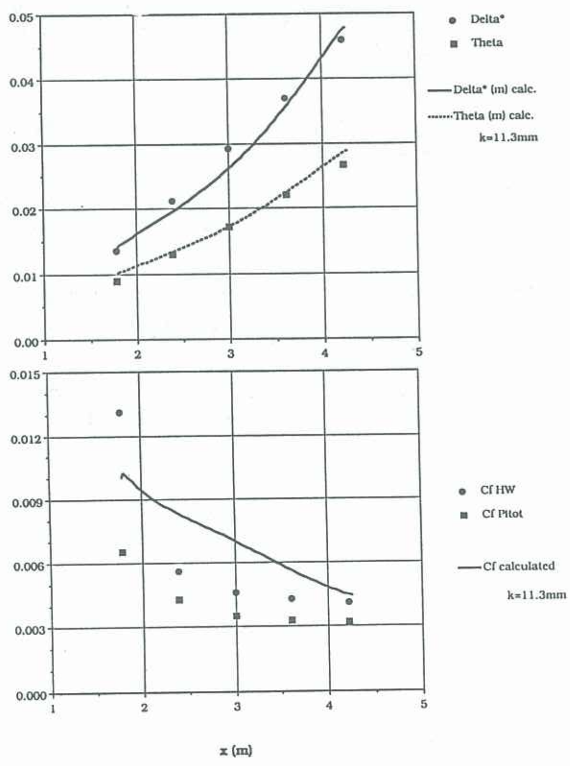

Figure 4 Mild adverse pressure gradient flow of Scottron and Power (geometry 1). Skin friction was estimated from the law of the wall using mean velocity profiles measured with Pitot tube (Cf Pitot), and hot wire (Cf HW).

Adverse pressure gradient flows

Scottron & Power (1965) also made experiments with adverse pressure gradients. The results for their mild adverse pressure gradient is shown in figure 4. The agreement is not satisfactory in particular is the predicted value of c_f too high. However, the presented results are almost identical to the results presented by Cebeci & Chang (1978). As the estimates of c_f from the mean velocity profiles taken by Pitot-static tube and hot wire deviate considerably it is reasonable to believe that these data may be somewhat unreliable.

For the strong adverse pressure gradient (figure 5) the agreement is only fair. This is particularly the case for the displacement thickness. The tendency of a much too low prediction of the displacement thickness and a somewhat high c_f was also obtained in the calculations of Cebeci & Chang (1978) who explained the difference by possible three-dimensionality in the flow. Also the extent of the logarithmic region in the velocity profiles, which is the basis for the evaluation of c_f, is very limited close to separation making the evaluation of c_f difficult.

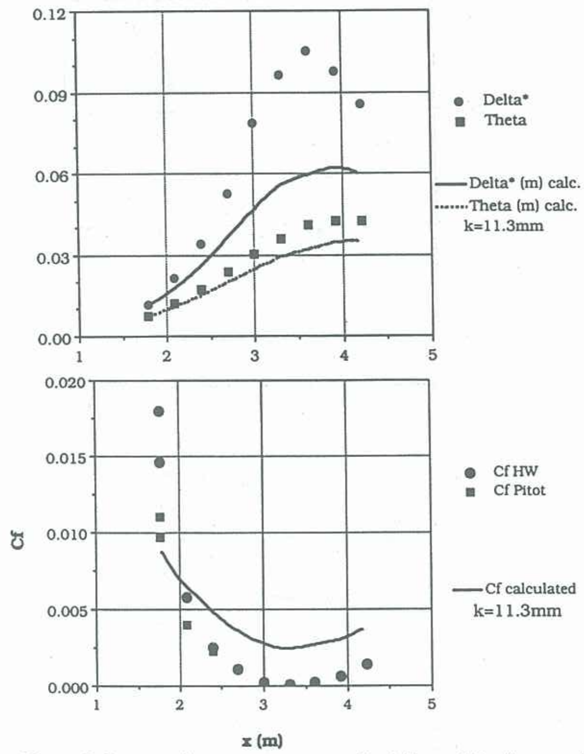

Figure 5 Strong adverse pressure gradient flow of Scottron and Power (geometry 2). Skin friction was estimated from the law of the wall using mean velocity profiles measured with Pitot tube (Cf Pitot), and hot wire (Cf HW).

Three-dimensional flow

Finally we calculated the three-dimensional boundary layer of the wing body junction experiment of Krogstad (1979, 1983) where the inviscid flow is given analytically. Krogstad repeated the same experiment twice, once with a smooth surface and once with the surface covered with commercial #24 sand paper. In this way the effects of roughness should be isolated since any possible imperfections in the experiment should be the same for both cases. Both the smooth and rough surface flows were predicted and results are presented along the streamline denoted 3 in the experiment. This passes close to, but outside the horse shoe vortex formed at the base of the junction. In the way described earlier the equivalent sand roughness was estimated to be k=1.1 mm giving $k^+ = 72$ at the initial station.

The results are shown in figure-6. The agreement with the experimental points is seen to be very good and to be about the same for both cases. The predicted c_f along the rough surface responds however somewhat too strongly to the favorable pressure pressure gradient in the range $-0.1 < x < 0.0$.

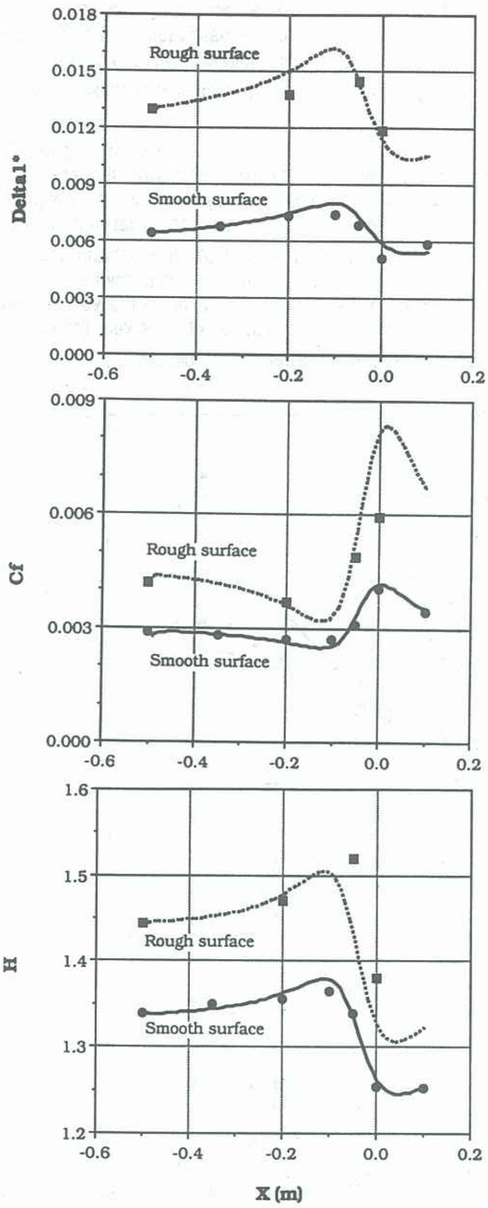

Figure 6 Results for streamline 3 for the wing body junction experiment of Krogstad .

CONCLUSIONS

A prediction method for rough surfaces has been given that is based on a simple extension of the van Driest (1956) damping function. The method differs from previous methods as it stimulates the turbulent shear stresses near the wall by manipulating the amount of viscous damping rather than introducing a shift in the location of wall. The method has been applied to a series of test cases of varying complexity and surface roughness showing that the predictions of flows along rough surfaces can be computed with the same degree of accuracy as for flows over smooth surfaces. The imperfections in the predictions of flows with strong adverse pressure gradients must be attributed to experimental errors or a general inability of the boundary layer method to cope with strong pressure gradient flows, since the method behaves well for zero pressure gradient flows of different surface roughness heights and for the much more complicated three-dimensional flow.

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