THEORETICAL ANALYSIS OF THE DETACHING TURBULENT THREE-DIMENSIONAL FLOW IN THE STERN REGION OF A HULL

Th. KNAACK, J. KUX and K. WIEGHARDT

Institut für Schiffbau, Universität Hamburg

Lämmersieth 90, 2000 Hamburg 60
WEST GERMANY

ABSTRACT

The three components of the mean velocity vector and the components of the Reynolds tensor have been determined by LDV in the stern region flow of a ship-hull double-model in the wind tunnel. From the measured values and their derivatives details of the complex flow field and the turbulence for this complex flow are derived.

NOTATION

$\boldsymbol{a}$ acceleration
b_{ij} anisotropy tensor
d deformation rate
d_{ij} symmetrical part of v_{ij}
k kinetic energy of turbulence
L characteristic length
$\boldsymbol{L}$ Lamb vector = $\boldsymbol{v} \times \boldsymbol{\omega}$
U_∞ incident velocity
$u = v_1,\ v = v_2,\ w = v_3$ components of velocity vector
$\boldsymbol{v}$ mean velocity vector
v_{ij} shear tensor = $\partial v_i / \partial x_j$
W_k kinematical vorticity number
$x = x_1,\ y = x_2,\ z = x_3$ coordinates
δ_{ij} Kronecker symbol
ρ density
ω magnitude of vorticity vector
$\boldsymbol{\omega}$ mean vorticity vector
ω_{ij} antisymmetric part of v_{ij}
II second invariant of v_{ij}

INTRODUCTION

The flow around a ship hull is a complex three dimensional field. It would be desirable to have a thorough knowledge about its details in order to optimize hull form and propulsion at least for the stationary case. Nevertheless not very much is known about it. Information is obtained from models scaled down by a factor between 20 and 100. Since measurements in towing tanks are expensive and time consuming, really detailed data on the velocity field , appropriate to be used as test cases for prediction methods (Larsson (1980)) have been obtained on double models (without rudder, propeller, appendages) in the wind tunnel: Larsson (1974), Hoffmann (1976), Wieghardt and Kux (1980), Löfdahl (1982), Wieghardt (1982), (1983), (1986), Knaack et al. (1985). This corresponds to the resistance test performed in the towing tank (no interaction with the propeller). With double-models the deformation of the free surface of the water has been suppressed, as it is neglected in most non-potential calculation methods, and so is the influence of all those motions as roll, pitch, heave, etc.

The complexity of the flow in the stern region is elucidated by the visualization shown in Fig. 1. What is seen is merely the directional distribution of the wall shear-stress vector field. Clearly separation is more than only bubble separation, the equivalent to two dimensional separation. This separation seems not to be present here. What we have is a topologically complex pattern (compare Peake and Tobak (1980)) with line separation (where the fluid in regions of converging streamlines escapes away from the wall, the friction not necessarily going down to zero) and point separation (where the fluid spirals away from the wall, the core of the vortex being created, leaving its footprint as a point on the wall). In addition the figure shows areas which are the projections of the domains where in the current investigations measurements are under way. Space available forces us to restrict the data shown here to one single area taken from such a domain in a transverse plane (parallel to the planes of the frames) at a lengthwise coordinate x equal to 8 mm. The origin of the cartesian righthanded coordinate system used, is shown in the same figure, the z axis pointing upward, the y coordinate running horizontally.

Applying Laser Doppler Velocimetry (LDV) now gives the possibility to determine the velocity field for both, the model and the full scale case. The flow field is not disturbed as it is by material probes and if not only the three components of the mean velocity vector but also the six different components of the Reynolds tensor are measured in points of a three-dimensional net, many details of the turbulence emerge too (Knaack et al (1985)). With the three component LDV system now available at the wind tunnel of the Institut für Schiffbau (IfS) domains of the field in the stern region have been scanned to obtain a survey of both fields. Relating to the mean velocity field, these measurements complete a previous pressure probe data set (obtained at almost 14 000 points) in the stern region of this same model. The spacing between the points of the LDV investigation has been reduced to 2 mm in all three coordinate directions. Therefore spatial derivatives can safely be computed by numerical methods from the experimental values. This means that the gradient tensor of the mean velocity vector (the shear tensor) may be calculated and thus vorticity and acceleration, as well as the derivatives of the Reynolds tensor components too and thus the turbulence force on the right hand side of the Reynolds averaged Navier Stokes equation (RANSE).

Beside a more complete picture of the flow around ship hulls with these quantities available it is expected that more insight into the basic relationships and the physics of flow fields may be deduced. So the terms of the RANSE may be computed and compared. At the same time further characteristics of the velocity field are emerging and should carefully be checked, a process which requires a thorough evaluation of the accuracy of all figures, specifically statistical errors have to be worked out clearly. Seeking for a deeper understanding, relationships have to be uncovered which possibly have remained hidden up to now, just because we failed in finding the right representation. Vectors are well understood and the way their components vary when the basis of the coordinate system is transformed. But what do we deduce from a complete set of components of a second rank tensor defined with relation to a given basis system? The determination of its components relative to its intrinsic coordinate system, the system based on its eigendirections may contain the information needed to disclose relationships of importance.

REVIEW OF THE EXPERIMENT

The measurements proportionating the data used in this paper were made in the open test section of the (Göttingen type) wind tunnel of the IfS. The double-model, meanwhile being investigated for many years, has become known as HSVA tanker or Hoffmann tanker, a ship never built. Model length is 2.74 m and the incident velocity is 27 m/sec, leading to a Reynolds number (based on the length) of $5 \cdot 10^6$. The model has no appendages, is suspended in a slotted wall by wires (which means that no forces may be measured), has 150 pressure taps and boundary layer transition is forced by tripping. Model beam is .40 m and the draught is .18 m.

The LDV system allows the simultaneous measurement of all three velocity components by the crossed beam (or differential Doppler) technique in backscattering mode with three pairs of beams of different colour (514.5 , 488 and 476 nm). The front optics is composed of two lense systems of constant focal length, their axes forming an angle of 25 degrees, one emitting two pairs of incident rays and receiving the scattered radiation of the third incident beam pair, which is emitted by the other lense system, where the scattered radiation of the first two colours is detected. The frame supporting the two lense systems is positioned with high precision under computer control by stepping motors. The Ar ion laser and the main optics are arranged on an optical table and connected by fiber optics (length 10 m) to the front optics. The detectors (photomultipliers) are situated directly on the front optics lens systems. The optical arrangement implies that the three components measured are not at right angles. By shifting the frequency by acusto-optics means for all three components the sign is determined. Counters were chosen as signal processors for the three channels. A specially developed circuit cares that only coincident signals are accepted for further processing. A PC monitors the system, processes the data, controls the positioning. Seeding was used: water droplets with a slight addition of glycerine of a mean diameter of 1 µm.

A tradeoff always has to be found between time available for the measurements and statistical accuracy. For a series of measurements 1000 (coincident) bursts per point were taken. This figure was later reduced to 500. Mean averaging time per point came out to be 1 minute. Ideally a probability density function (over the three-dimensional manifold of the velocity components) should be fitted to the data of each point. The accuracy with which the parameters of this distribution can be determined is ruled by statistics. First a gaussian distribution is fitted. The statistical errors will set narrow bounds to the derivation of more general distribution functions precluding the determination of further coefficients in the sense of a Gram-Charlier expansion. The error margins for the components of the mean velocity vector and of the Reynolds tensor in the oblicue, in the cartesian and for the latter in its intrinsic coordinate system, have to be derived. Since spatial derivatives have to be computed herefrom, these errors limit the range of validity of any statement on relationships drawn from the experimental data.

DETAILS OF MEAN VELOCITY FIELD

As indicated, the details of the field shall be explained with a small area chosen to exemplify matters. In Fig. 2 the mean velocity field is plotted, the transverse components as arrows, the x-component as isoline chart. Included are lines of constant kinematical vorticity number as introduced by Truesdell (1953). This number is a dimensionless measure of vorticity, defined as $W_k = \omega / d$, with ω as the magnitude of the vorticity vector and d a scalar representing the deformation rate. Decomposing the shear tensor $v_{ij} = \partial v_i / \partial x_j$ ($x_1 = x$, $x_2 = y$, $x_3 = z$) into its symmetric (d_{ij}) and its antisymmetric (ω_{ij}) part, as a sum: $v_{ij} = d_{ij} + \omega_{ij}$, we obtain $d^2 = d_{ij} d_{ij}$ and $\omega^2 = \omega_{ij} \omega_{ij}$ (summation over repeated indices). To scale vorticity we could of course divide by U_∞ / L (with the incident velocity U_∞ and a suitable length L), but there seems to be no appropriate L. Nondimensionalizing with d mixes deformation with vorticity, but is a genuine local scaling.

Investigating vector fields means looking at its gradient tensor, here the shear tensor. The acceleration, in the stationary case the convective term on the left hand side of the RANSE, is but for the factor density (ρ) the product of this tensor with the mean velocity. A decomposition of this tensor as sum (as we did above) or as product (a very interesting alternative) may give enhanced insight. Anyhow invariant characteristics of this tensor should bear valuable information. So it will be worthwhile to inspect its invariants, its eigenvalues and its eigendirections. The left hand side of the RANSE being dominated by this tensor, the right hand side is dominated by the Reynolds tensor if we discard

viscous terms and assume the pressure gradient playing a secondary role. Again, it is not the tensor itself but its divergence which represents the vector of turbulence-force. Inspection of the same invariant characteristics of the Reynolds tensor should thus provide insight too. What are the relations between all these eigenvectors and the vectors of mean velocity and vorticity?
Though some of these quantities have been determined over large parts of the regions scanned, no answer can be given yet.

Several vectoranalytical identities exist, which may be exploited in addition. One of the most interesting is $div\ \boldsymbol{a} = -2\ II = 1/2\ (d^2 - \omega^2)$, valid for incompressible fluids, where II is the second invariant of the shear tensor, an identity which allows to compute $div\ \boldsymbol{a}$, clearly a quantity containing second derivatives of the velocity $\boldsymbol{v}$, solely out of first derivatives of $\boldsymbol{v}$.

Taking the curl of the RANSE yields

$$curl\ \boldsymbol{a} = curl\ \boldsymbol{L} = -\frac{1}{\rho}\ curl\ T$$

with the Lamb vector $\boldsymbol{L} = \boldsymbol{v} \times \boldsymbol{\omega}$ on the left hand side and the turbulence force $\boldsymbol{T}$, the divergence of the Reynolds tensor, on the right hand side. There is experimental evidence that $\boldsymbol{v} \cdot curl\ \boldsymbol{L} = 0$, implying $\boldsymbol{v} \cdot curl\ \boldsymbol{T} = 0$. A consequence of $\boldsymbol{v} \cdot curl\ \boldsymbol{L} = 0$ is $div\ [\boldsymbol{v} \times \boldsymbol{L}] = 0$. In Fig. 3 we compare this divergence (normalized by the sum of the magnitudes of its three terms) with $div\ \boldsymbol{v}$, which should be zero (same normalization). The experimental errors preclude $div\ \boldsymbol{v}$, derived from the measurements, from vanishing. The bahaviour of $div\ [\boldsymbol{v} \times \boldsymbol{L}]$ is not different. So there seems to be some evidence that in fact $\boldsymbol{v} \cdot curl\ \boldsymbol{L} = 0$, implying $\boldsymbol{v} \cdot curl\ \boldsymbol{T} = 0$.

THE REYNOLDS TENSOR

The Reynolds tensor has been determined over specific regions of the separating flow. Again only a small area in the plane indicated is shown in Fig. 4. The kinetic energy k of the turbulence is a quantity modeled in several prediction methods. A detailed survey is now available from these experiments which may serve for comparison. In Fig. 4 we have an isoline plot of k, essentially the trace of the Reynolds tensor. The symmetry in the y-coordinate is evident and the slight misalignment known from the mean velocity field representations is found again. Similar plots could be given for all the components of the tensor, but this is not very informative. Much more information about turbulence can be extracted from the Reynolds tensor as a whole. Though no correlations or spectra have been measured, there is a possibility to furtzher caracterize turbulence with the aid of this tensor. First the (real) eigenvalues and eigendirections of this symmetric tensor may be used for this purpose. Second, we can determine the anisotropy of the turbulence following Lumley (1978).

The distribution of one set of eigendirections is shown in Fig. 5. There is a peculiar difficulty when solving the (third order) secular equation to find the eigenvalues all over the domain scanned. The numerical procedure used to find the first root will yield a value, but it is not sure whether this corresponds to the same root that was determined as the first in the neighbouring point. It is difficult to track the identity of the eigenvalues as function of the coordinates. If one plots the three eigenvalues against a coordinate, say y, one obtains intersecting curves and it is not without ambiguity which is the right branch to follow after the intersection. An alternative is to identify the eigenvalues according to their eigendirections. One starts numbering eigendirections at a certain point, giving the label 1 to say the one with the smallest angle against the x -direction and so on and proceeds along one coordinate tracking this particular eigendirection. Going back to the eigenvalues plotted against this coordinate one may be led along another branch than the one chosen at first sight. Again it should not be forgotten, that each quantity has a certain incertitude since it was derived from measured magnitudes. Nevertheless a field of eigendirections is shown in Fig. 5.

By defining $b_{ij} = \frac{1}{2k} v_{ij} - \frac{1}{3} \delta_{ij}$ as dimensionless anisotropy tensor, we may depict anisotropy in the following way: After determining the eigenvalues of this traceless tensor, two of them may be chosen as components of a vector which we may attach to the point under consideration in our turbulence field. The direction of this arrow caracterizes the anisotropy, its length the degree of anisotropy: we have isotropy if its length is zero, i.e. the two eigenvalues chosen vanish (and therefore so does the third too). Again if the identitiy of the eigenvalues chosen could not be tracked safely, the picture will appear spoiled. Though this may still be the case with our data, in Fig. 6 we show such an anisotropy chart.

CONCLUSION

With all components of the Reynolds- and the shear-tensor determined from experiment in the complex flow in the stern region of a ship-hull double-model, interesting facts about the mean velocity field and the turbulence and its anisotropy can be deduced, even without correlation or spectral measurements of turbulence. Certain relationships show up and it is suggested that the characteristics of the tensors mentioned are best studied by investigating eigenvalues and eigendirections of the mentioned tensors.

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FIGURES

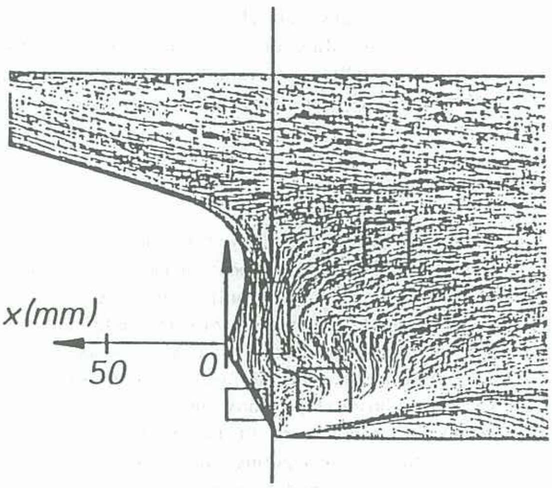

Fig. 1 Visualization of wall shear-stress directional field in he stern region of double-model in wind tunnel

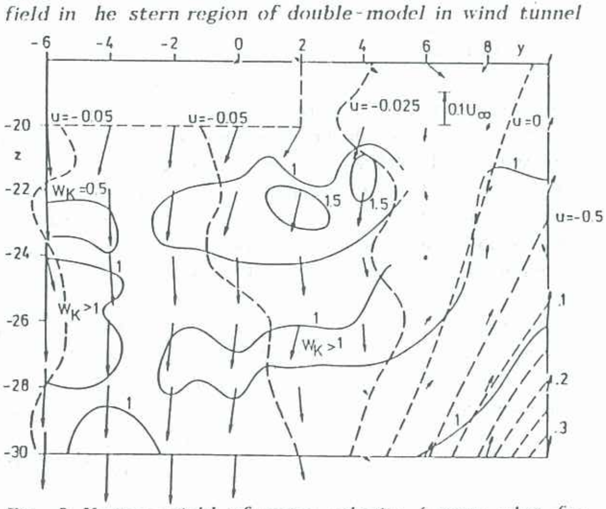

Fig. 2 Vector - field of mean velocity (arrow plot for transverse, isolines for longitudinal component) and isolines of kinematical vorticity number in the transverse plane x = - 8 mm

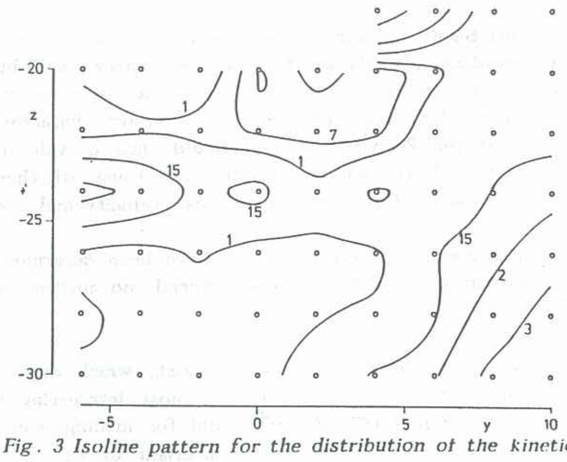

Fig. 3 Isoline pattern for the distribution of the kinetic energy of turbulence, plane x = - 8 mm

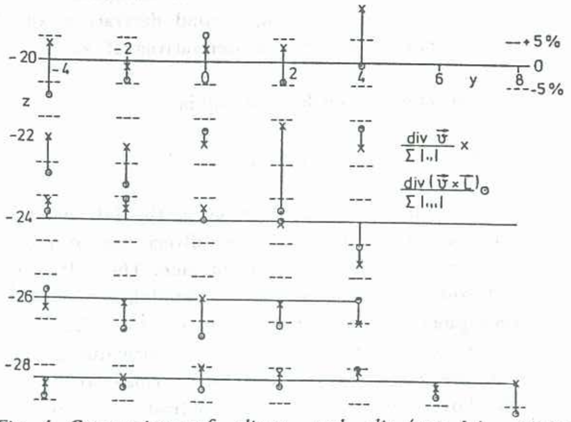

Fig. 4 Comparison of div v and div (v x L), plane x = - 8 mm

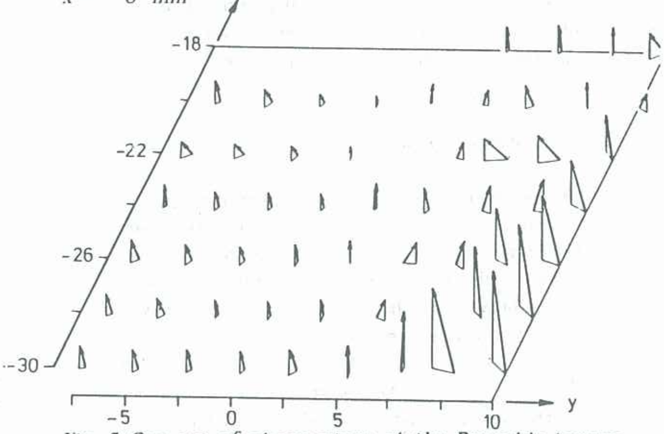

Fig. 5 One set of eigenvectors of the Reynolds tensor, plane x = - 8 mm

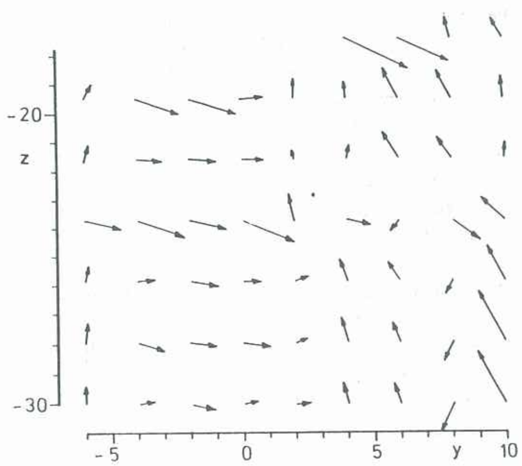

Fig. 6 Arrow plot of anisotropy characteristic of turbulence, plane x = - 8 mm