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FLOW PAST NORMAL FLAT PLATES

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ABSTRACT

The results of a numerical and experimental study are presented for both steady and unsteady two-dimensional flow through a uniform cascade of normal flat plates. The Navier Stokes equations are written in terms of the stream function and vorticity and are solved numerically using a second-order-accurate finite-difference scheme which is based on a modified procedure to preserve accuracy and iterative convergence at higher Reynolds numbers. For improved accuracy, asymptotic solutions are employed both upstream and downstream with appropriate upstream and downstream boundary conditions. A frequently used method for dealing with corner singularities is found to be inaccurate and a method of overcoming this deficiency is described. Numerical solutions have been obtained for blockage ratio 50% and Reynolds numbers in the range $0 < Re < 500$, and results both for the lengths of attached eddies and for drag coefficients are presented. The calculations indicate that the eddy length, L, increases linearly with Re at least up to Re = 500 and that the constant of proportionality is in very good agreement with the theoretical predictions of Smith (1985a), who considered a related problem. Numerical results for simple harmonic oscillatory motion of the same uniform cascade of normal flat plates, perpendicular to the plane of the plates, has also been obtained for $0 < Re < 25$ and flow streamlines have been obtained and the length of eddy investigated. An experimental investigation has been performed at small and moderate values of Re for both steady and oscillatory motion of the cascade and there is excellent agreement between the experimental and numerical results both for flow streamlines and eddy lengths.

INTRODUCTION

The problem of steady viscous incompressible flow past bluff bodies has over a long time received much attention,both theoretically and numerically. In spite of the many numerical methods and calculations on flow past a circular cylinder, accurate results have been obtained only for Re ($=Ud/\nu$, where U is the uniform speed relative to the cylinder at large distance, d the diameter of the cylinder and ν the kinematic viscosity of the fluid) up to about 600, see Fornberg (1985). He found that the wake bubble (region of attached or recirculating flow) has eddy length $L \alpha Re$ but with width $W \alpha Re$ up to Re = 300 and $W \alpha Re^{1/2}$ for Re > 300. Smith developed an asymptotic theory, which is based on an extension of Kirchhoff's free – streamline solution, and which agrees with Fornberg's results up to $Re \approx 300$. A fresh interpretation of Fornberg's results have been given by Smith (1985b) and Peregrine (1985).There are several differences between these theories, some of which are a matter of interpretation, and these are unlikely to be resolved without further analysis and computational work.

Hudson and Dennis (1985) obtained numerical solutions of the Navier–Stokes equations for steady laminar flow of a viscous incompressible fluid past a single normal flat plate. They were able to obtain accurate numerical results for Re up to 20 only, because of the difficulty in resolving the singularity at each edge of the plate. Their results for L were in excellent agreement with the observations. However, although the eddy length increases linearly with Re, in agreement with Smith (1985b) and Peregrine (1985), the constants of proportionality they obtained were significantly different. Acrivos et al (1968) performed experiments on flow past a variety of isolated bluff bodies and confirmed that L increases linearly with Re. However, they reported also that bubble widths attain a limit 0(1) as Re increases, in accordance with their earlier predictions. This result has been disputed by Smith (1985b), Fornberg (1985) and Peregrine (1985), all of whom suggest that $W \alpha Re$. In a series of papers (see Milos et al, 1987), Acrivos et al have studied the related problem of laminar flow over a backward facing step on one side of a two dimensional channel. They have studied both the boundary – layer equations and the full Navier Stokes equations for the simulated problem in which entry velocity profiles over the step are specified for the cases : parabolic, uniform and uniform flow above a simulated boundary layer; in each case taking free slip boundary conditions at the ceiling and floor of the channel. Results have been obtained over a wide range of values for λ, the ratio of the channel height above the step to the step height. The general conclusions they reached are: first that steady solutions are not always possible; where they are, $L \alpha Re$ for small values of λ but for larger values of λ then L is 0(1) for large values of Re. In contrast, Smith (1985a) studied a related problem and found that for all values of λ that $L \alpha Re$.

Although some agreement between theoretical, numerical and experimental results exists, there is a need for further work in all these aspects of this fundamental and classical problem. In this paper we investigate the flow of a uniform stream past a geometrically simple cascade consisting of an infinite array of identical flat plates of finite width which are normal to the flow direction, see figure 1. The steady state Navier–Stokes equations are solved using a finite–difference technique in an infinite region with boundary conditions of uniform flow being applied at infinity in both the upstream and downstream directions. It should be noted particularly that in most previous numerical work the boundary conditions have been applied at the station where the flow leaves the cascade and at large distance downstream, and that the flow leaving the cascade has been assumed to be : (i) parabolic, (ii) uniform, or (iii) uniform above a simulated boundary – layer of specified thickness discharging over the step. As will be seen in the next section, none of these models fully represent the flow through a cascade of plates normal to the stream because they fail totally to represent the pressure field in the plane of the cascade, and in this paper we illustrate some of the inadequacies.

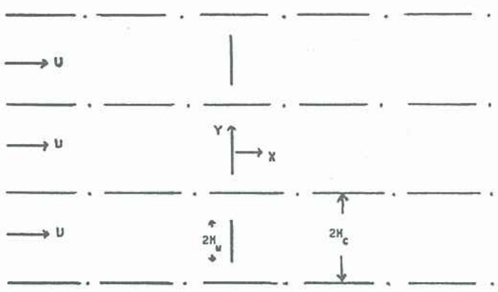

Figure 1 The overall cascade structure.

The fluid flow through a cascade of bluff bodies is of fundamental importance for two reasons. First, the resultant predictions relate to the performance of the cascade itself, providing a model for flow past compressor and turbine blades, for many types of heat exchanger and for other interactive body configurations. In practice, the attached flow behind the cascade may have severe consequences for heat transfer and drag, and an understanding of the flow and of the dependence of L on Re is of immediate concern. Secondly, the cascade configuration provides valuable insight into quite fundamental properties of flow interaction with bluff bodies, exhibiting strong plate interaction at large blockage ratios and tending asymptotically to the flow past a single plate as the blockage ratio is reduced. Smith (1985a) established a theoretical basis for understanding the separated flow structure produced by the cascade configuration and there is special interest in relating experimental and numerical studies to his theory.

In a numerical treatment of flow past a cascade of normal plates, difficulties arise in applying boundary conditions near the plate edges. The problem has been discussed by several authors (see Ingham 1983). This discontinuity at the edge of the plate gives rise to a singularity in the vorticity and this is handled in our treatment by using the appropriate form of the analytical solution in this vicinity following Moffatt (1964). In dealing with the edge singularity the commonly used method of Dennis and Smith (1980) can be shown mathematically to be inaccurate for this problem. The method involves rotating the finite difference grid through 45° at points which would normally involve evaluating the vorticity at the singular point. It is essential that the solution near the singularity is accurate, as errors introduced are swept downstream and may severely affect the calculated value of L. Also we adopt asymptotic solutions both upstream and downstream so that the numerical integrations need be carried out over a restricted region only.

A laboratory investigation has been performed for steady flow through a cascade of normal flat plates with equal plate and gap widths. At small and moderate values of Re the flow is laminar and symmetrical, and the experimental results compare well with both present numerical predictions and the theoretical results of Smith (1985a) for a related problem.

Flows produced by harmonic oscillation of isolated bluff bodies in incompressible fluids otherwise at rest have also been studied extensively, both theoretically and numerically, and a good review of this may be found in Kim and Troesch (1989). Most work has been restricted to circular cylinders and spheres oscillated transversely with very small ratio, ϵ, of oscillation amplitude to cylinder radius. In this paper we investigate both numerically and experimentally the flow which is induced when a cascade of normal flat plates is oscillated harmonically in the direction perpendicular to the plane of plates in an unbounded incompressible fluid which is otherwise at rest. Numerical solutions of the Navier Stokes equations are obtained by solving the corresponding finite–difference equations using an implicit method . Interest is concentrated mainly on small and moderate values of Re (taking the typical speed as the maximum speed of oscillation) and $\epsilon = 0(1)$, implying an amplitude of oscillation comparable to the plate width and a small frequency of oscillation. A periodic asymptotic appromixation has been developed in order to deal with the boundary conditions at large distances from the body. The governing partial differential equations are written in finite–difference form using a technique similar to that employed for the steady flow so that accuracy and the rate of convergence of the iterative scheme is improved. All the numerical results show reasonable agreement with our laboratory experiments.

STEADY STATE RESULTS

The steady state Navier Stokes equations under the assumption of constant fluid properties have been solved numerically for the geometry shown in figure 1. A corresponding series of laboratory experiments have also been carried out as a test of the numerical solutions and as a means of distinguishing these from other solutions obtained for this class of problems. Full details of both the numerical and experimental technique will be published later.

Figure 2 shows a set of experimental photographs and streamline fields from the numerical solutions for corresponding values of Re (based on plate width and stream velocity). Because of symmetry, the streamlines need be shown near one plate only. It may be seen that the numerical and experimental realisations are in excellent agreement despite additional constraints in the laboratory experiment due to side walls, floor of the channel and free surface of the liquid. It should be noted that the laboratory flow is illuminated with a thin sheet of light normal to the plates, which themselves produce the diagonal shadows for a pair of light sources slanting rearwards to the right and forwards to the left; the marker particles are very small plastic spheres used in emulsion paint, and the motion field is recorded by a short duration time exposure. Although the illumination is inelegant it is sufficient to resolve the pattern of the flow. A better system of illumination should be used in future experiments.

Figure 3 shows the variation of L with Re. The laboratory measurements inevitably exhibit some scatter, but lie systematically a little under the numerical and theoretical solutions, although taking into account the difficulty in identifying the stagnation point marking the end of the attached flow our laboratory results provide striking support for our numerical solutions. The numerical results suggest for $Re >> 1$ that $L \sim 0.21$ Re in comparison with the theoretical results $L \sim 0.2$ Re of Smith (1985a) and $L \sim 0.18$ Re of Milos et al (1987) for the related problems.

Figure 4 shows the x and y components of the velocity, $u(0,y)$ and $v(0,y)$, at $x=0$ between plate edges as a function of y for Re=1,100 and 500. It is observed that as Re increases $u(0,y)$ approaches the asymptotic profile used by Smith (1985a) but $v(0,y)$ does not. In a symmetric (inviscid) flow $v(0,y)$ would be zero, but when (viscous) flow separates from the plate edges there are significant changes in pressure gradient, especially as the flow passes the plates, with corresponding changes in flow acceleration leading to the transverse velocity components $v(0,y)$ comparable to the

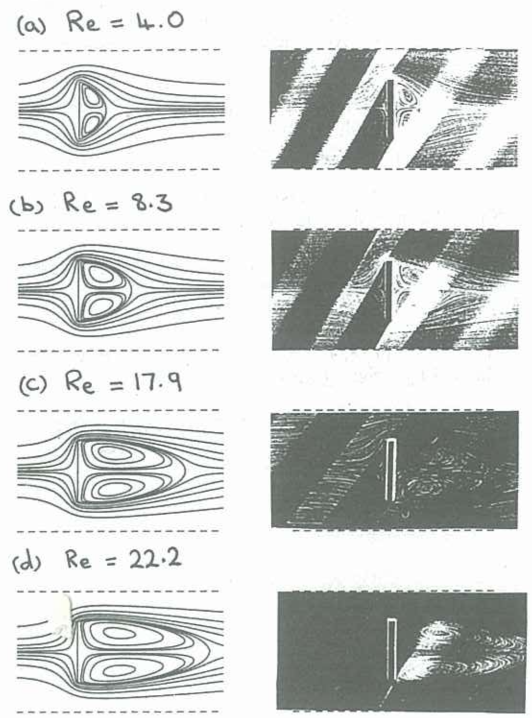

Figure 2 Comparison of calculated and experimental streamlines.

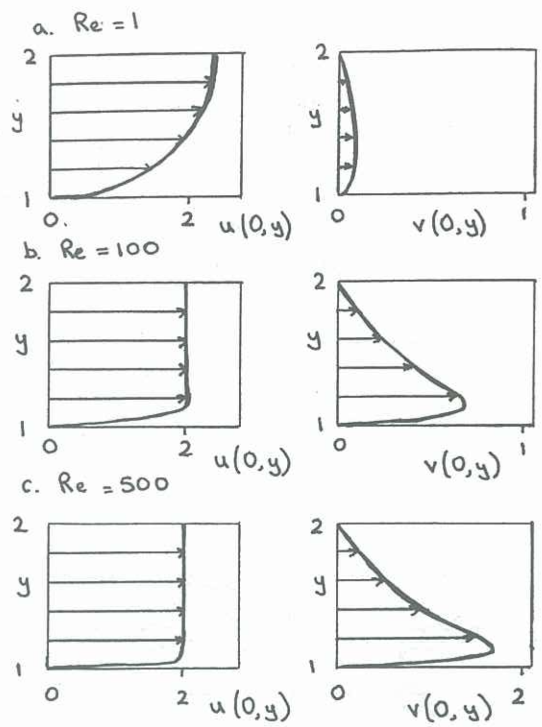

Figure 4 The x– and y– components of velocity at x=0.

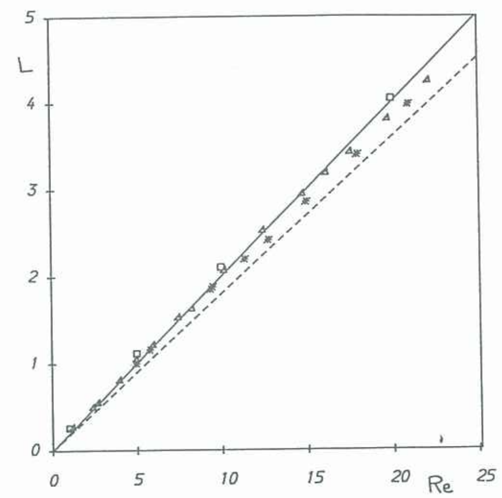

Figure 3 Eddy length as a function of Reynolds number. □ present numerical results; Δ and * experimental results; —— Smith; – – – Milos et al.

streamwise component u(0,y) at quite moderate values of Re. Other authors, including Smith and Acrivos, have taken no account whatsoever of the transverse velocities and pressure gradients at entry. Thus the relevance of much of the earlier work to the physical problem of flow through a cascade is at best uncertain; the similarity of predictions of L/Re suggests that this result may be insensitive to the differences noted above, but this result could not have been assumed without further analysis.

SIMPLE HARMONIC OSCILLATING RESULTS

The unsteady Navier Stokes equations for a fluid of constant properties have been solved for the geometry of figure 1 but with the cascade of plates performing oscillatory motion in a fluid otherwise at rest. Matching laboratory experiments have again been carried out and the details of these and the numerical techniques will be published in full later. We will concentrate here on the parameter range $\epsilon=1$ and Re=1,5,10 and 22.6 in a preliminary comparison of numerical and experimental data. In steady motion $L \simeq 0.2$ Re but in the unsteady motion L varies with time. Its value at the instant that the cascade reaches its maximum speed is shown in figure 5 as a function of Re. As might have been expected, the length is less than that for the steady case, even though the instantaneous speed is the same as that for the steady case. It is also observed for Re = 22.6 that the value of L obtained numerically is smaller than that predicted experimentally; this is possibly due to the difficulty in interpreting the experimental results at the higher values of Re. Further results are being obtained.

If the experimental motion is observed in a laboratory frame, in contrast to a frame moving with the cascade, there are secondary circulations. These are shown by the streamline patterns of figure 6: (a) when the plate is in the middle of its oscillation and moving from right to left, showing the primary circulation caused by the plate and the much weaker secondary circulation to the left of the plate; (b) just before the plate comes to rest at the left extremity of its oscillation; and (c) just after the reversed motion starts from the left, where a new primary circulation is being generated. The old primary circulation becomes a new secondary one, while the old secondary becomes a weakening third circulation which rapidly disappears.

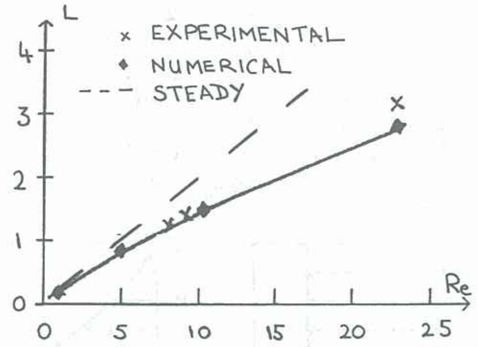

Figure 5 Maximum length of bubble.

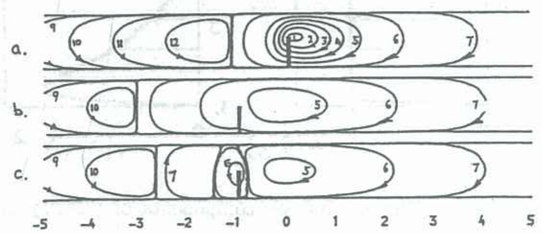

Fig. 6 The streamline pattern for Re=10, ϵ=1 and plate moving
a. right to left at centre of oscillation.
b. right to left just before coming to rest.
c. left to right just after coming to rest.
The numerical lines 1–13 have streamfunction values -0.5, -0.4, -0.3, -0.2, -0.1, -0.01, -10^{-4}, 0, 10^{-6}, 10^{-5}, 10^{-4} and 10^{-3}, respectively.

Further comparisons of experimental and theoretical results will be presented at the conference, where the substantial differences between the oscillating flows generated by single and multiple bodies will be discussed.

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