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EXPERIMENTAL ANALYSIS OF FREE OSCILLATING LIQUID DROPS

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ABSTRACT

Finite amplitude, axially symmetric oscillations of small liquid drops in a gaseous environment have been studied experimentally. A weak nonlinearity in the fundamental oscillation mode is observed when the oscillation amplitude exceeds 0.1. As the amplitude decreases the initial oscillation frequency of the drop increases (in most of the cases), reaching asymptotically the value predicted by a linear theory. The viscous-damping decay of the oscillation amplitude can be well described with help of the linear theory.

INTRODUCTION

The free oscillations of drops have been studied extensively in the past, both for the sake of basic scientific understanding as well as for various applications in chemical industry, zero-gravity crystal growth technology, or meteorology. Our experimental study is motivated by the desire to develop a non-intrusive method for determining an instantaneous value of surface tension of a flow medium.

Our previous study (Hiller & Kowalewski 1989a) of small-amplitude oscillations of ethanol and water drops has shown that the linear theory of Lamb (1932) yields predictions of the oscillation frequency that are in good agreement with the experimental results. This paper deals with quantitative and qualitative investigations of finite-amplitude oscillation of drops, when the nonlinear effects begin to play a role. The observation of drops during several periods of their oscillations permits us to analyze the viscous damping.

THEORETICAL BACKGROUND

Lamb (1932) presents Rayleigh's (1879) linear model for free vibrations of a liquid drop. The axially symmetric solution for the surface shape has the form of a linear superposition of n modes, which are described by Legendre polynomials $P_n(\cos\Theta)$:

$$r(t,\Theta)=R\{1+\Sigma A_n \sin(\Omega_n t+\Phi_n)P_n(\cos\Theta)\}. \quad (1)$$

R is the unperturbed radius of the drop. A_n is the amplitude of the n-th mode of oscillation, and Θ is the polar angle of the spherical coordinate system with its origin at the centre of the drop. The oscillation frequency Ω_n for the liquid drop surrounded by vacuum or air is given by:

$$\Omega_n^2 = \frac{(n-1)\cdot(n+2)\cdot n\cdot\sigma}{\rho\cdot R^3} \quad (2)$$

where σ is the surface tension of the droplet medium and ρ is its density.

The linear analysis has been extended to include the influence of viscosity, first by Chandrasekhar (1959) for a free oscillating drop and later, for the more general case of a viscous drop in a viscous outer fluid, by Miller & Scriven (1968) and Prosperetti (1980a). It was shown that the initial motion of the viscous drop is just that executed by a damped harmonic oscillator, for which the amplitude A_n decays exponentially:

$$A_n=A_{no}\exp(-t/\tau_n)\ . \quad (3)$$

A_{no} is the initial amplitude. The time constant for the amplitude decay rate τ_n depends only on density and dynamic viscosity, μ, of the fluid according to a simple relation:

$$\tau_n = \frac{\rho\cdot R^2}{\mu\cdot(n-1)\cdot(2n+1)}\ . \quad (4)$$

The viscosity of the fluid lessens the oscillation frequency Ω_n. This effect is of order $\sqrt{1-(\Omega\cdot\tau)^{-2}}$. For low-viscosity liquids (water, ethanol) and relatively large drops (R > 0.1 mm) frequency shift predicted by the linear theory is negligible small.

Slightly nonlinear oscillations of a drop were analyzed by Tsamopoulos & Brown (1983). They have shown that the oscillation frequency decreases with increasing amplitude. The effect of small viscosity was incorporated into the nonlinear numerical study by Lundgren & Mansour (1988). They found that viscosity may have a relatively large effect on the higher modes of oscillation, changing their near-harmonic resonance coupling with the fundamental mode n=2.

The only experimental work dealing with large amplitude oscillations of drops that we could find is that of Trinh & Wang (1982). Their experiment was performed on drops suspended in a neutrally buoyant and immiscible liquid. Their results have confirmed qualitatively the behaviour predicted by the above theories of a decrease in oscillation frequency with increasing amplitude. Lack of available experimental data concerning nonlinear behaviour of drops oscillating in air encouraged us to undertake the present investigations.

EXPERIMENTAL

Experimental technique

The drops are generated by the break-up of a small-diameter laminar jet discharging into a gaseous environment from a convergent nozzle. The diameter of the nozzle can be chosen to be in the range from 100 to 300 µm so that droplets with diameters in the range from 200 to 500 µm can be generated. The pressure inside the plenum chamber of the nozzle is modulated by a piezoceramic device. By a proper choice of the modulation frequency one achieves nearly monodispersed droplets which are oscillating in axially symmetrical modes. The low velocity of the drops, which is between 1 and 2 m/s, and their small sizes allow us to neglect the influence of aerodynamic forces acting on their surfaces.

The droplets are observed through a microscope in bright field illumination. Their images are registered by a CCD camera (Sony XC77CE), digitized and stored in a computer for further processing. A detailed description of the apparatus and multi-exposure registration technique has been given elsewhere (Hiller & Kowalewski 1989 a,b). Fig.1 illustrates this technique, showing a multi-exposed image of a single ethanol drop of 230µm radius. This observation technique allows one to obtain high temporary resolution of the oscillations but only during a short time, of the order of one period of oscillation. Therefore, in the present study the registration technique was supplemented by a beat-frequency stroboscopic method of observation. A digital logic circuit is used so that the frequency of illumination is infinitesimally shifted relatively to the frequency modulating the liquid jet. In effect the periodic phenomena are no longer observed as stationary but change their phase slowly. Depending on the sign of the frequency difference one obtains forward or backward slow-motion "replays" of the observed droplets. Moving the nozzle slowly, one can follow the drop along its way. Due to the high reproducibility of the controlled break-up of the jet, the oscillations of the generated drops could be observed from the tip of the jet to the distance of about 5 cm using such a "false time" method. At 5 cm the drops practically reach an equilibrium spherical shape. The images of the drop were recorded periodically by an image processor and stored on a hard disk of the computer. Usually a series of 200 images were taken using a time of 3 seconds per image (the time needed to save an image on the disk). The "real time" resolution, controlled by the beat frequency, was kept in a range of 10 -30 µsec. During a 200-image run this allowed observation of several periods of drop oscillations.

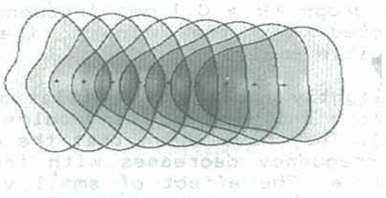

Figure 1 Multiexposure of an oscillating drop observed at short time after its generation. Time interval between single exposures - 40µs. Solid line - function (7) fitted to the drop boundaries. The amplitudes a_n found for the first drop (left one): $a_2 = -0.34$, $a_3 = 0.076$, $a_4 = 0.19$, $a_5 = -0.064$.

Analysis of drop images

We assume that deformation of the drop surface does not change its volume. The axially symmetric description of the drop surface by (1) fulfilled this condition only to first order in amplitude A_n. Therefore in the present study, where finite values of amplitudes are expected, we have to describe the drop surface using the modified equation:

$$r(t,\Theta)=R\{1+\delta(t)+\Sigma A_n \sin(\Omega_n t+\Phi_n)P_n(\cos\Theta)\}. \quad (5)$$

The additional volume correction term, $\delta(t)$, is independent on the position on the surface. Its value is calculated for each time t from the following integral equation:

$$\frac{4\cdot\pi\cdot R^3}{3} - \iiint_V r(t,\Theta)dV = 0 . \quad (6)$$

The small time-dependent term $\delta(t)$ in the equation of the drop radius in practice modifies only slightly its linear description given by (1). In our case, however, where the volume of the drop observed in the experiment is not known a priori, the above formula allowed us to evaluate its correct value.

With the help of computer-aided image analyses the boundaries of the droplet at corresponding moments of exposure are traced using from 100 to 400 points. In a second step, these points are matched to the function

$$F = R[1+\delta+\Sigma a_n P_n(\cos\Theta)], \quad (7)$$

which describes the momentary shape of the drop. The analysis of the oscillations is limited to n=5, whereas the first mode of interest is n=2. Therefore, F depends on eight fitting parameters: four momentary amplitudes ($a_2 \div a_5$), the unknown equilibrium radius of a drop R, two coordinates and the rotation angle of a coordinate system connected with an image of the drop. The least-square fitting of (7) to the previously found points of the drop boundary is based on quasi-Newton optimization method. The vectorized computer code allows fitting of one drop image with typically 100 points during ca. 0.5 sec of CPU time on an IBM 3090-300E.

In order to quantitatively compare the experimental results for drop oscillations with theory, the correct value of drop radius must be known. The accuracy of measuring drop shapes depends straightforwardly on the CCD camera resolution. The coordinates of a single point on the image can be measured within one pixel accuracy. In our case the drop images have mean diameter of about 250 pixel on the 756 x 581 pixels CCD-sensor. Thus a single point of the image is defined with an accuracy of better then 0.5%. The description of the drop surface given by (7) with four modes was found to be very good (compare Fig.1). The mean fitting error is below 0.3 pixel for oscillation amplitudes having values below 0.4. Hence, we may estimate the error of the measured drop radius to be smaller than 0.1%. The other source of possible error is the presence of non-axially symmetric modes of oscillation. These will turn out as fluctuations of values of the mean drop radius R obtained from the fitting procedure. In such cases the doubtful data are not used for later analysis.

RESULTS

Experiments were performed with water and 95% ethanol (denaturated with methyl-ketone mixture with water) droplets in air. Figs. 2a,b display a time sequences of the dimensionless amplitude a_n for the modes n=2 and n=3, obtained for an ethanol drop. The mean radius of the drop is 177µm. Each square on the figures displays a value of the momentary amplitude a_n obtained by the fitting procedure. It can be seen that the initial value of the amplitude a_3 for third mode is about four times lower than that one of the fundamental mode n=2. Initial amplitudes a_4 and a_5 of the fourth and fifth oscillation mode have values of 0.05 and 0.03, respectively. The viscous damping appears as an amplitude decay, which, in accordance with (4), is largest for higher oscillation modes. As a result only the second and third modes of oscillation could be analyzed in the chosen time scale, whereas the higher oscillation modes became very small quickly disappearing in a "noise" of the fitting method (see Fig.3).

Displayed in the Fig.2 the waveforms seem, on the first view, to be that of a damped harmonic oscillator More careful analysis shows, however, small variations of the oscillation period and asymmetry between negative and positive displacements. This type of oscillation we may try to describe using an equation of a damped harmonic oscillator with a restoring force having a symmetrical component (i.e. depending on even power of displacement). Using an asymptotic expansion for small amplitude we obtain the following approximation, which we introduce to describe the observed periodic variation of the amplitudes a_n:

$$a_n(t) = A_n \sin\{\Omega_n[\sqrt{1-(\tau_n\Omega_n)^{-2}}+A_n^2\alpha_n]t + \Phi_n\} + A_n^2\beta_n \tag{8}$$

Here, the term with α_n takes into account the amplitude dependence of the oscillation frequency Ω_n and the term with β_n demonstrates the existence of the nonsymmetry of the restoring force between prolate and oblate deformation of the droplet. A_n is given by (3). The six free parameters α, β, τ, Ω, Φ, and A_o of the equations (8) & (3) were used to fit the experimental points for oscillation amplitudes a_2 and a_3 obtained from the drop image analysis. The solid lines in Fig. 2 gives the best fitting function (8) calculated for the displayed experimental points. The resultant values of the fitting parameters are:
Ω_2= 6121sec^{-1}, τ_2=0.00397, α_2=1.1, β_2=0.3,
Ω_3=12279sec^{-1}, τ_3=0.00134, α_3=-30, β_3=5.1.
The values calculated from (2) and (4) are:
Ω_2 = 6298sec^{-1}, τ_2 = 0.00412,
Ω_3 = 12197sec^{-1}, τ_3 = 0.00147.

The following conclusions can be drawn from the analyzed experimental runs:

1. There is a weak dependence of the frequency on the amplitude. The "best fitting" value of α_2 is O(1). For initial amplitude a_2 = 0.4, which is about the highest value we observe in our experiments, the frequency of the second mode deviates from that calculated according to (2) by about 11 %. This deviation falls to below 0.5 % for a_2 = 0.1.

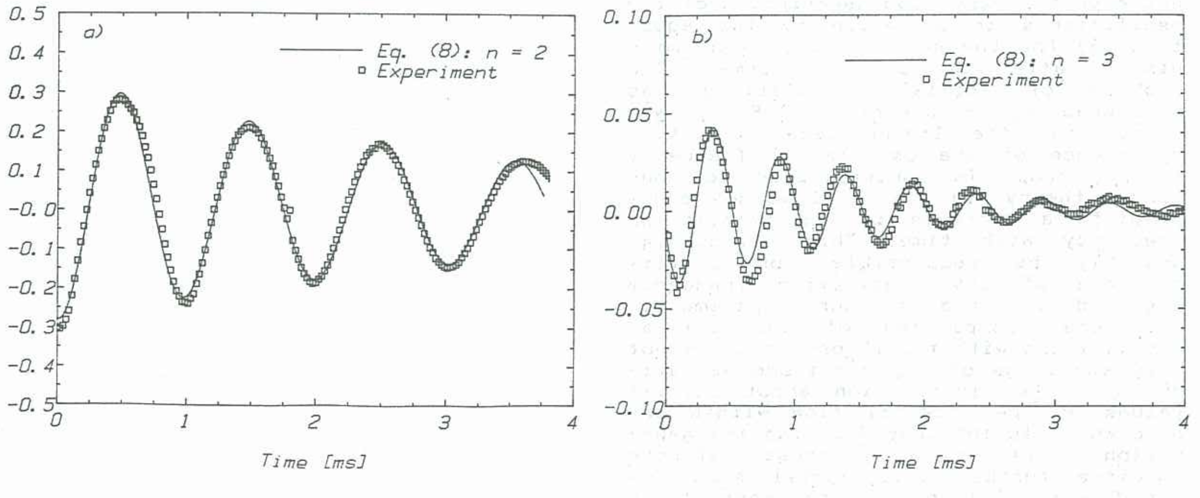

Figure 2 The variation of the amplitude of the second (a) and third (b) mode for an oscillating ethanol drop correlated with the function given by (8).

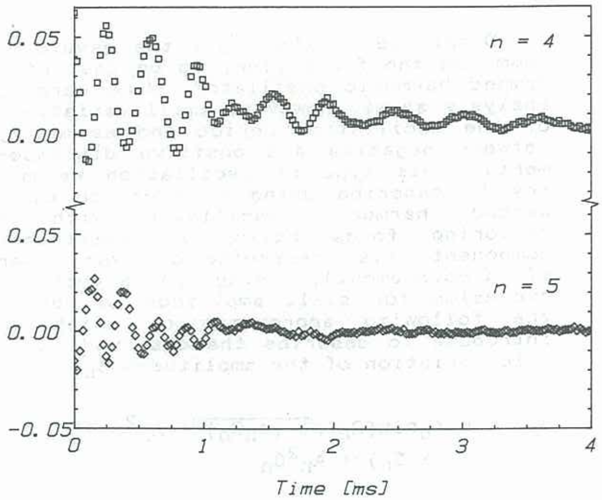

Figure 3 The variation of the amplitude of the forth and fifth mode for the drop of the figure 2.

2. In most cases observed up to now, the oscillations speed up for small amplitudes. Sometimes, however, the frequency goes down.

3. The decay rate of the amplitude is very well described by the exponential relation (3). There seems to be no significant excitation of higher modes from the base mode n = 2 if its amplitude does not exceed 0.3. This is inferred from the observation that the decay rate τ, measured in the above way, yields a value very close (within a few per cent error) to that one calculated from (4). If there were a strong excitation of the higher modes, one should expect a larger damping decrement of the fundamental mode.

FINAL REMARKS

From the experiments performed to date it seems possible for practical purposes to make corrections for the nonlinear effects using the simplified equation (8) and, in such a way, to obtain an asymptotic value of the oscillation frequency given by (2). However, one should not expect a very good description of the oscillations to be given by the equation (8) for the short time period immediately after the drop is formed. The problem of initial oscillations was discussed by Prosperetti (1980b), who found, for the linear case, the time dependence of the oscillation frequency of the drop. In contradiction to nonlinear theory, his "initial value" model predicts a decrease in the oscillation frequency with time. This effect may possibly be responsible for initial increase of the oscillation frequency observed in some of our experiments. Therefore, comparison of the initial oscillations with the theory requires not only knowledge of the amplitude variations but also information about initial values of the internal flow within the drop which is introduced during its generation. Analysis of these effects requires further experimental study of the drop oscillation with the possibility of controlling conditions of their initial excitation.

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