TENTH AUSTRALASIAN FLUID MECHANICS
CONFERENCE - UNIVERSITY OF MELBOURNE
11-15 DECEMBER 1989

MORISON FORCE COEFFICIENTS FOR COMPLIANT CYLINDERS IN WAVES

N. HARITOS

Department of Civil and Agricultural Engineering
University of Melbourne
Parkville, Vic. 3052
AUSTRALIA

ABSTRACT

Force coefficients as used in Morison's equation have been fitted to experimentally recorded support force data from three cylinders of diameters 0.15, 0.10 and 0.15m for a variety of wave input conditions and for both rigid and compliant support states.

Results currently 'in hand' suggest that a mild near linear variation with frequency in the inertia coefficient C_M alone (i.e. where the drag coefficient C_D has been set to zero) provides the best fit to the observed character of the wave forcing, by comparison with fits obtained using 'constant-valued' versions of both C_M and C_D.

INTRODUCTION

Morison's equation has traditionally been used by design engineers to predict the hydrodynamic loads acting on the cylindrical members of offshore structural forms such as channel beacons and jacket style oil rigs.

The form of this equation as it applies to the force per unit length $f(z,t)$ acting on a rigid vertical surface-piercing cylinder of diameter D at position z from the mean water level (MWL) is given by

$$f(z,t) = \alpha\, \dot{u}(z,t) + \beta\, u(z,t)\,|u(z,t)| \qquad (1)$$

where $\alpha = \frac{\pi}{4}\rho\, C_M D^2$, $\beta = \frac{1}{2}\rho\, C_D D$, ρ is the density of sea water, $u(z,t)$ and $\dot{u}(z,t)$ are the local values of water particle velocity and acceleration respectively and C_M and C_D are empirical coefficients for inertia and drag force that are generally recognized as being dependent upon cylinder surface roughness, Reynold's number $\Re_N$ and Keulegen-Carpenter number K_C viz

$$K_C = \frac{U_m\, T}{D} \qquad (2)$$

where U_m is the peak amplitude of water particle velocity at the level of interest and T is the period of dominant wave energy in the sea state under consideration (Sarpkaya and Isaacson, 1981).

Since both K_C and $\Re_N$ would vary with position along a vertical surface-piercing cylinder, C_M and C_D would be expected to also, be position dependent, (Haritos[1], 1989). Nonetheless, lumped values of C_M and C_D as they may apply to the cylinder 'as a whole' are often adopted in design for the purpose of simplicity.

Values for C_M and C_D for a wide range of K_C and $\Re_N$ and for various roughness conditions have been reported by researchers who have studied the forces obtained on horizontal cylinders placed in a U-tube apparatus in which the water column was allowed to oscillate with a pre-defined amplitude and frequency thereby 'simulating' the horizontal component of oscillatory motion to be expected from regular waves.

Results from such tests are used by designers with some trepidation, in the case of vertical surface-piercing cylinders, since they pertain to uniform uni-directional oscillatory fluid motion and so do not include the influence of the vertical component in the 'orbital' style water particle motion associated with real ocean waves kinematics nor their variation with depth, (Moberg, 1988).

In addition, any influence due to the frequency content of real irregular ocean waves, their directional spread or the effects of currents is also not able to be modelled by such tests.

Consequently attention has recently been given by hydrodynamicists to the laboratory study of vertical surface-piercing cylinders in a variety of sea state conditions in order to ascertain the degree of influence of some of the above conditions on the forcing and hence the inferred values of force coefficients C_M and C_D.

More recently still, the question of compliancy and its influence on the hydrodynamic forcing likely to be experienced by such cylinders has also emerged as a topical research issue in this field of interest (Haritos[1], 1989).

This paper reports on some preliminary results obtained from a research programme currently being conducted on vertical bottom-hinged circular cylinders, of various diameters, which have either been made compliant by the introduction of spring mountings at their free end or are 'simply supported'.

DESCRIPTION OF THE TESTS

Figure 1 provides a schematic of the experimental arrangement of the cylinder group used in these tests and located in the wave flume of the Michell laboratory at the University of Melbourne. Three separate cylinders, individually connected to rigid base plates via purpose-built bi-directional smooth hinges have been arranged to be supported at their 'free' end by either springs or rigid links to a pair of force transducers that measure the alongwave and acrosswave component of the attachment force for each. Wave probes have been mounted on brackets adjacent to each cylinder to enable local variations in waveheight measurement at a set water depth of 1m.

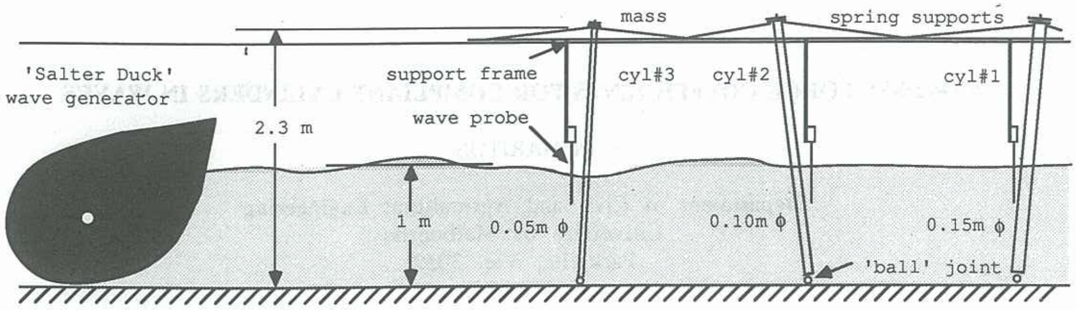

Figure 1 Schematic of Compliant Cylinders for Experimental Determination of Force Coefficients

An IBM® compatible PC-XT computer with an 16 channel AD and single channel DA data acquisition/control card has been used to control the 'Salter duck' style of wave generator and the consequent production of selected wave configurations and to simultaneously acquire data from the total of 9 channels of instrumentation (2 forces and 1 wave height for each of three cylinders) at the rate of 4Hz for 1000 seconds per test for each wave configuration.

Some 8 separate wave trains are being used in these tests, viz

(i) Three sets of Swept Sine Waves (SSW), (Haritos[2], 1989) varying linearly in frequency from 1.75 Hz down to 0.25 Hz in the testing period of 1000 seconds with amplitudes of 20mm, 14mm and 10mm respectively.

(ii) Four full-amplitude irregular Pierson-Moskowitz wave trains with peak wave frequencies f_p of 0.8, 0.7, 0.6 and 0.5Hz respectively.

(iii) A half-amplitude irregular Pierson-Moskowitz sea state with f_p = 0.4Hz.

Each wave train has been repeated four times and results averaged for each of these sets.

The three cylinders that are being tested have been chosen from four hollow smooth aluminium tube sections with external diameters of 0.15, 0.10, 0.05 and 0.025m. Results currently in hand are presented for the larger diameter cylinders for which inertia force has been observed to dominate.

FORCE COEFFICIENTS FOR COMPLIANT CYLINDERS

In the case of a bottom-pivoted vertical surface-piercing cylinder, the hydrodynamic force per unit length can be obtained via Morison's equation and a 'relative velocity' assumption (Sarpkaya & Isaacson, 1981), and is given by

$$f(z,t) = \alpha\,(\dot{u} - \frac{C_M-1}{C_M}\ddot{x}) + \beta(u-\dot{x})|u-\dot{x}| \quad (3)$$

where $\dot{x}$ and $\ddot{x}$ are the cylinder velocity and acceleration respectively at the level under consideration, and the equation of motion is represented by that of a single degree of freedom (SDOF) oscillator with a mode shape $\psi(z)$ given by

$$\psi(z) = (\frac{z}{h_p}) \quad (4)$$

where h_p is the distance below the MWL of the pivot point.

If $x_o(t)$ is the alongwave response at the MWL of this cylinder, then

$$\ddot{x}_o + 2\omega_o\zeta_o\,\dot{x}_o + \omega_o^2 x_o = \frac{F(t)}{M_o+M'} \quad (5)$$

where M' is the so-called 'added mass', M_o, ζ_o and ω_o are the equivalent mass, critical damping ratio and natural circular frequency for the SDOF model of cylinder response. F(t) is the 'equivalent' alongwave hydrodynamic forcing at the MWL given by

$$F(t) = \int_{-h_p}^{0} \{\alpha\dot{u} + \beta(u-\dot{x}_o\psi(z))|u-\dot{x}_o\psi(z)|\}\psi(z)\,dz \quad (6)$$

The alongwave restraint force, R(t), is given by

$$R(t) = (M_o + M')\omega_o^2\, x_o(t) \quad (7)$$

The spectral variation of R(t), $S_R(f)$, can be related to that of the equivalent hydrodynamic forcing at the MWL, $S_F(f)$, via

$$S_R(f) = S_F(f)\,\chi_m^2(f) \quad (8)$$

where the structure magnification function $\chi_m(f)$ is given by

$$\chi_m^2(f) = \frac{1}{(1-(\frac{f}{f_o})^2)^2+(2\zeta_o(\frac{f}{f_o}))^2} \quad (9)$$

in which f_o is the natural frequency, $\omega_o/2\pi$.

'Constant-valued' Force Coefficients

Equation (8) can be used to obtain a 'least squares' fit to α and β (and hence C_M and C_D) via a numerical integration of Eq. (6) and the observed variations of waveheight $\eta(t)$ and response $x_o(t)$ recorded for each cylinder. The technique as it applies to the context of an instrumented segment of cylinder has been described by Haritos[1] (1989) and requires as an intermediate step the definition of the wave kinematics from the recorded $\eta(t)$ for each cylinder via application of a suitable wave theory.

Here, Eq. (8) is re-cast in the form of

$$S_F(f) = S_{R'}(f) = \frac{S_R(f)}{\chi_m^2(f)} \quad (10)$$

and the least squares fitting procedure is conducted between the 'theoretical' form of $S_F(f)$ (inferred from wave kinematics derived from $\eta(t)$ via Airy wave theory with C_M and C_D assumed constant-valued for the cylinder 'as a whole') and the observed form of R(t) from the support force measurements.

f_o and ζ_o as they pertain to each individual cylinder support spring stiffness/added mass configuration are used in the definition of $\chi_m(f)$ via Eq. (9) in this fitting procedure. ($\chi_m(f)$ is effectively 1 for a 'rigid' support at the free end).

'Frequency-dependent' Force Coefficients

C_M and C_D can alternatively be considered to be frequency-dependent ($C_M(f)$, $C_D(f)$) and their values ascertained at each frequency component from the Fourier coefficients of F(t) and R(t) of Eqs. (6) and (7) respectively where due account of the phase shifts associated with a SDOF model for the vibration of each cylinder has been considered in the evaluation (Haritos[1], 1989).

RESULTS CURRENTLY 'IN HAND'

Tests conforming to the description outlined above have been performed on the three larger diameter cylinders (ϕ = 0.15, 0.10 and 0.05m) for a 'simply' supported condition and, separately, for a single spring setting.

A range of six sets of mass combinations have been used with the single spring setting producing natural frequencies for the cylinders spanning from 0.4 to 1.4 Hz which effectively encompasses the energetic range of the waves used in the tests.

Results for Rigid Supports

"Constant-valued" force coefficients.
Because the three larger diameter cylinders are inertia dominant throughout the range of wave types used in these tests, estimates of C_D for any small contributions made by drag force using the least squares spectral fitting procedure described above become unreliable.

Figure 2 compares the observed wave spectra for one of the SSW wave trains with its corresponding constant-valued target spectrum whilst Figure 3 conducts a similar comparison for one of the irregular Pierson-Moskowitz sea states with f_p = 0.6Hz.

The simulated waves for both of these wave types are considered to be reasonably good representations of the target wave types they attempt to model.

Figure 4 compares the alongwave force spectrum fitted using the least squares procedure for C_M and C_D with the observed force spectrum for the case of the SSW wave train of Figure 2 and the cylinder with diameter ϕ = 0.10m whilst Figure 5 depicts a similar comparison for the case C_D deliberately set to zero and C_M alone optimised to the observed force spectrum.

It is clear that the adoption of a non-zero C_D coefficient has little influence on "goodness of fit" between observed and theoretical forms of the force spectra as evidenced by inspection of Figs. 4 and 5.

The influence on the resultant value of C_M, however, does become 'appreciable' under these circumstances with C_M being larger for the case of C_D set to zero than for the case of C_M, C_D jointly optimised.

Although Figs. 4 and 5 apply to a particular cylinder and wave train, the comments made above would appear to be typical of all three cylinders tested and for all wave types adopted in these tests.

Frequency-dependent force coefficients.
When frequency dependence is considered for both C_M and C_D and the pertinent fitting procedure described earlier in this paper is in fact adopted, a very interesting observation is made, viz: $C_D(f)$ appears to be insignificant for all three cylinders and for all wave types tested in these experiments and a much closer fit than for the constant-valued C_M and C_D assumption is obtained to the observed force spectrum from $C_M(f)$ alone.

Figure 6 depicts the fit based upon $C_M(f)$ for the same cylinder and its associated SSW wave treated in Figs. 4 and 5 compared with the observed force spectrum. The corresponding variation for $C_M(f)$ for this case is depicted in Fig. 7 as the 'solid' line.

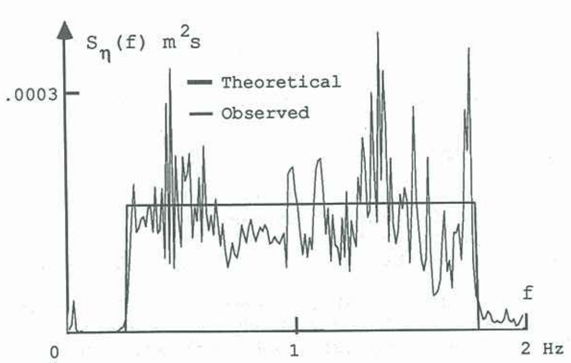

Figure 2 Observed/Target Wave Spectra (SSW)

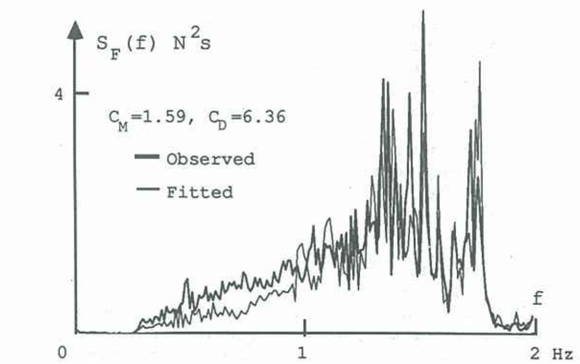

Figure 4 Observed/Fitted Wave Force Spectra

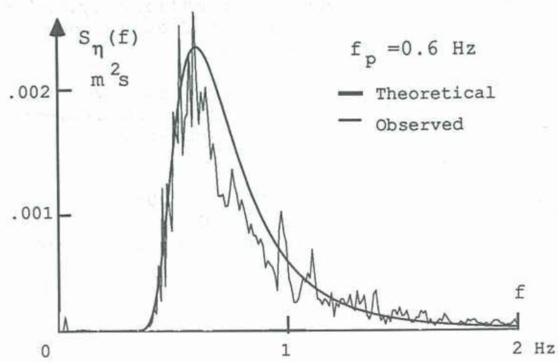

Figure 3 Observed/Target P/M Wave Spectra

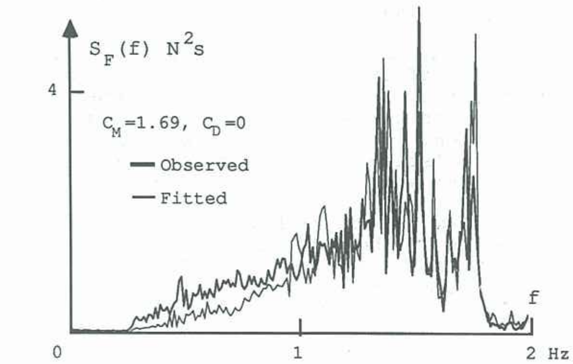

Figure 5 Observed/Fitted Wave Force Spectra

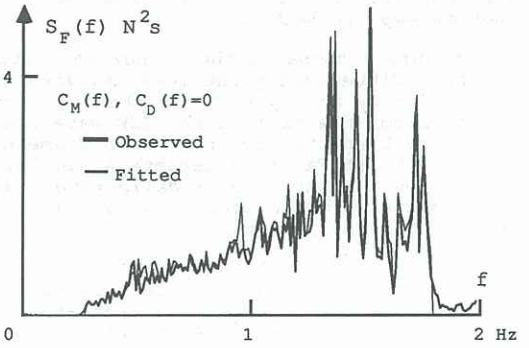

Figure 6 Observed/Fitted Wave Force Spectra

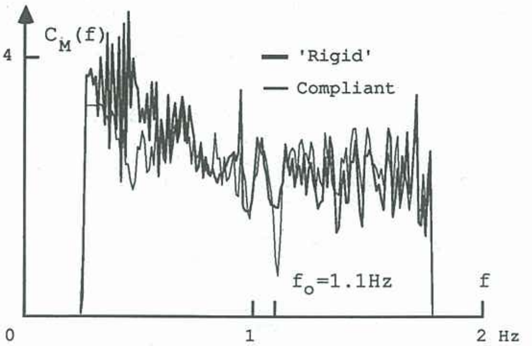

Figure 7 $C_M(f)$ Variations for SSW Waves

It is clear that the near linear variation in $C_M(f)$ with frequency observed in Fig. 7 is sufficient to not only provide a better fit to the observed force spectrum but to also effectively eliminate the influence of drag that was apparent via the 'constant-valued' force coefficient fitting procedure.

Results for Compliant Tests

Force coefficient fitting procedures described earlier in this paper have been adopted for the range of wave types and natural frequencies for each of the three cylinders tested in the wave tank.

The observations made have been, in nearly every sense, directly similar to those made for the rigid support conditions reported in some detail above.

The value of ζ_o: the effective damping ratio, has been taken to be fixed at 0.002 in the definition of $\chi_m(f)$ of Eq. (9). This rather low value is representative of the contribution to damping made by the 'smooth' base hinge of each cylinder and has been verified through the conduct of 'pluck tests' (Haritos[3], 1989).

The variation in $S_{R'}(f)$ of Eq. (10) in the region of the natural frequency of a particular cylinder is, however, quite sensitive to the value of ζ_o adopted in its description. (Too high a value of ζ_o would produce a 'spike' in this region whereas too low a value would produce a 'dip').

A 'dip' in the variation of $S_{R'}(f)$ near the region of the natural frequency of a compliant cylinder would therefore be indicative of the presence of additional damping which must be resultant from the dynamic response of the cylinder in water.

However, such damping would necessarily not be associated with drag-dependent origins since the drag terms are already accounted for in the model of $S_F(f)$ of Eq. (10). Nonetheless, additional 'hydrodynamic' damping would be possible from 'radiation' damping where the mechanism is that of wave dispersion from the dynamically responding cylinder (Haritos[3], 1989).

Such 'dips' in the variation of $S_{R'}(f)$ translate as corresponding 'dips' in the variation of $C_M(f)$ which have in fact been observed in these variations for the larger diameter cylinders and for moderate to high natural frequencies.

A comparison with the $C_M(f)$ fit obtained for the rigid cylinder in Fig. 7 for the same conditions but for compliant supports producing a natural frequency of 1.1Hz is also depicted thereon.

The 'dip' in the region of the natural frequency for the $C_M(f)$ variation of the compliant cylinder is obvious and is indicative of additional 'radiation' style hydrodynamic damping here estimated to be of the order 0.003. Elsewhere in this $C_M(f)$ variation, the agreement between the two observed forms is considered to be excellent.

CONCLUSIONS

Tests conducted in the wave tank on vertical surface-piercing cylinders with both rigid and compliant support conditions and for a variety of wave inputs have verified an inertia dominant forcing regime in which the underlying character of the inertia coefficient variation is largely similar between support types.

The use of constant-valued C_M and C_D coefficients does not provide as good a fit to the observed cylinder forcing as would be obtained from an frequency-dependent inertia coefficient acting on its own.

Measureable radiation damping is evident in the compliant cylinders with natural frequencies in the higher region of the range of 0.4 to 1.4Hz observed in these tests.

REFERENCES

HARITOS[1], N. (1989) The determination of force coefficients from tests on a compliant segmented cylinder in waves. Proc. Aust. Conf. on Coastal and Ocean Engin., Adelaide.

HARITOS[2], N. (1989) The Swept Sine Wave - an invaluable tool for hydrodynamic research. Proc. XXIII Congress, IAHR, Ottawa, Canada.

HARITOS[3], N. (1989) The determination of hydrodynamic damping from compliant cylinder tests. Proc. Aust. Conf. on Coastal and Ocean Engineering, Adelaide.

MOBERG, G. (1988) Wave Forces on a Vertical Slender Cylinder. Ph.D. Thesis, Department of Hydraulics, Chalmer's University of Technology, Sweden.

SARPKAYA, T. and ISAACSON, M. (1981). Mechanics of Wave Forces on Offshore Structures. Van Nostrand Reinhold, New York.