TENTH AUSTRALASIAN FLUID MECHANICS
CONFERENCE - UNIVERSITY OF MELBOURNE
11-15 DECEMBER 1989

A UNIFIED APPROACH TO THE PREDICTION OF SOUND ATTENUATION IN LINED DUCTS WITH FLOW

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ABSTRACT

A generalized theory is presented for sound propagation in lined ducts of arbitrary cross section where acoustic wave propagation in the lining is also taken into account. The effects of a mean fluid flow in the duct airway, an anisotropic bulk reacting liner and a limp, impervious membrane covering the liner are all taken into account. Isotropic and locally reacting liners are treated as limiting cases. The general analysis is applied to ducts of both rectangular and circular section, taking into account higher order modes as well as plane wave sound propagation. Some design charts for duct attenuation in octave frequency band averages and in terms of dimensionless parameters are presented.

INTRODUCTION

The work described here extends previous analytical work (Cummings, 1976) to include the practical case of a duct liner separated from the duct airway by a limp impervious membrane. A general analysis for any duct section is followed by an evaluation of ther particular case of a rectangular section. Even and odd higher order modes and plane waves are considered for ducts with flow and both isotropic and anisotropic, bulk reacting liners. The results are also applicable for liners protected with a perforated panel, provided the panel has an open area in excess of 40% (Cummings, 1976) and is separated by an open mesh spacer (25mm mesh minimum) from any membrane covering the liner.

THEORY

The geometry for a general section duct is shown in Figure 1. Note that advantage is taken of symmetry and only half the duct section is analysed.

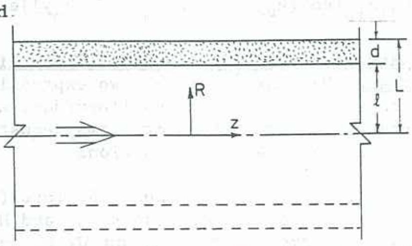

Figure 1. Geometry for a general duct section

Propagation in the Duct Airway.

The wave equation for the velocity potential ϕ in an arbitrary cross section airway with a mean axial flow U_o in the z direction may be written as

$$c_o^2 \nabla^2 \phi = \left[\frac{\partial}{\partial t} + U_o \frac{\partial}{\partial z}\right]^2 \phi \tag{1}$$

It is assumed that the z coordinate is separable. Thus

$$\nabla^2 = \frac{\partial^2}{\partial z^2} + \nabla_R^2 \tag{2}$$

where the R subscript refers to the direction normal to the duct wall(s) which contain(s) the liner. Using eq.(2), eq.(1) may be written as

$$c_o^2 \left[\frac{\partial^2}{\partial z^2} + \nabla_R^2\right] \phi = \left[\frac{\partial}{\partial t} + U_o \frac{\partial}{\partial z}\right]^2 \phi \tag{3}$$

The following relations for the acoustic pressure p and particle velocity v follow from the definition of the potential function implied by eq.(3).

$$v = -\nabla\phi \quad \text{and} \quad p = \rho_o\left[\frac{\partial}{\partial t} + U_o \frac{\partial}{\partial z}\right]\phi \tag{4}$$

The variable u is introduced; it is defined in terms of the complex phase speed c_N in the z direction in the duct airway as follows:

$$u = z - c_N t \tag{5}$$

Making use of eq.(5) and assuming a separated solution for the potential function ϕ, a solution for eq.(3) is

$$\phi = GH(u) \tag{6}$$

Substituting eq.(6) into eq.(3), and dividing through by $c_o^2 GH$, the following is obtained

$$\frac{\nabla_R^2 G}{G} + \frac{1}{H}\frac{\partial^2 H}{\partial u^2}\left[1 - \left(\frac{U_o - c_N}{c_o}\right)^2\right] = 0 \tag{7}$$

Eq.(7) is separable and can be solved in terms of axial and radial wavenumbers k_N and k_M respectively. Thus,

$$\nabla_R^2 G + k_R^2 G = 0 \tag{8}$$

$$\frac{\partial^2 H}{\partial u^2} + k_N^2 H = 0 \tag{9}$$

Using the relation $k_N = \omega/c_N$ and eq.(5) a solution to eq.(9) may be written as

$$H = H_o e^{i(\omega t - k_N z)} \tag{10}$$

Defining the Mach number M as U_o/c_o, substituting eq's (8) and (9) into (7) and using the relation $k_o = \omega/c_o$ the following is obtained:

$$k_R = \left[(k_o - Mk_N)^2 - k_N^2\right]^{1/2} \tag{11}$$

Propagation in the Duct Liner.

The wave equation in terms of the sound pressure p_ℓ, axial phase speed c_z and radial phase speed c_M in the porous liner, may be written (assuming no mean axial flow in the liner) as:

$$c_z^2 \frac{\partial^2}{\partial z^2} p_\ell + c_M^2 \nabla_M^2 p_\ell = \frac{\partial^2}{\partial t^2} p_\ell \quad (12)$$

For pressure balance at the duct liner boundary, the solution for p_ℓ must have the following form

$$p_\ell = G_\ell H_\ell (u) \quad (13)$$

where u is defined by eq.(5). Substituting eq's (5) and (13) into (12), the following is obtained:

$$c_z^2 G_\ell \frac{\partial^2 H_\ell}{\partial u^2} + c_M{}^2 H_\ell \nabla_M{}^2 G_\ell = c_N{}^2 G_\ell \frac{\partial^2 H_\ell}{\partial u^2} \quad (14)$$

Eq.(14) is separable and may be rearranged to give

$$\frac{\nabla_M^2 G_\ell}{G_\ell} + k_M^2 = 0 \quad \text{where } k_M = \psi/c_M \quad (15)$$

and

$$\frac{(c_z^2 - c_N^2)}{H_\ell} \frac{\partial^2 H_\ell}{\partial u^2} - \psi^2 = 0 \quad (16)$$

Solutions to eq.(15) will be considered in subsequent sections. The solution to eq.(16) may be written as

$$H_\ell = H_{\ell o} e^{-i\left(\frac{\psi (z - c_N t)}{(c_N^2 - c_z^2)^{1/2}}\right)} \quad (17)$$

Consideration of eq.(17) shows that

$$\omega = c_N \psi/(c_N^2 - c_z^2)^{1/2} \quad (18)$$

Then, because $k_N = \omega/c_N$

$$\psi = k_N (c_N^2 - c_z^2)^{1/2} \quad (19)$$

and eq.(17) becomes

$$H_\ell = H_{\ell o} e^{i(\omega t - k_N z)} \quad (20)$$

Boundary Condition at the Duct Airway/Liner Interface.

Consider a thin limp sheet, of surface density σ, separating the liner from the duct airway. The displacement of the sheet is given by

$$\zeta = \zeta_o e^{i(\omega t - k_N z)} \quad (21)$$

At the inner surface of the sheet (next to the moving medium in the duct airway) the resulting normal acoustic velocity in the fluid is

$$v = \frac{\partial \zeta}{\partial t} + U_o \frac{\partial \zeta}{\partial z} \quad (22)$$

Using eq's (2) and (4) the linearized momentum equation becomes,

$$\rho_o \left[\frac{\partial v}{\partial t} + U_o \frac{\partial v}{\partial z}\right] = -\nabla_R p \quad (23)$$

Introducing eq's (21) and (22) into (23) and rearranging gives

$$\zeta = \nabla_R p / [\rho_o (\omega - k_N U_o)^2] \quad (24)$$

In the liner, ρ_ℓ is the complex gas density and the linearized momentum equation in the liner at the surface is

$$\rho_\ell \omega^2 \zeta = \nabla_\ell p_\ell \quad (25)$$

$$\text{or} \quad \zeta = \nabla_\ell p_\ell / (\rho_\ell \omega^2) \quad (26)$$

The acoustic pressure p in the duct airway and the acoustic pressure p_ℓ in the liner must satisfy the following equation at the sheet

$$[p - p_\ell]_{surface} = -\sigma \omega^2 \zeta \quad (27)$$

Substitution of eq.(24) into eq.(27) gives

$$[p - p_\ell]_{surface} = \frac{-\sigma \omega^2 \nabla_R p}{\rho_o (\omega - k_N U_o)^2} \quad (28)$$

Evaluation of the Theory for a Rectangular Duct Lined on Two Opposite Sides

Acoustic pressure distribution in the duct airway

Eq.(9) is separable into x and y coordinates so that $G = G_x G_y$. For a duct lined on two opposite sides (those in the y direction),

$$G_x = \cos(m\pi x/\ell_x) \text{ if m is even} \quad (29)$$

$$G_x = \sin(m\pi x/\ell_x) \text{ if m is odd} \quad (30)$$

$$G_y = G_1 \cos(k_{Ry}^n y) + G_2 \sin(k_{Ry}^n y) \quad (31)$$

where m and n are the mode orders in the x and y directions respectively. (The mode order superscript will be omitted from now on but it is implied that the expressions to follow refer to a single mode). For even values of n, the acoustic pressure gradient at the duct centre line (y = 0) must be zero. Thus,

$$G_y = G_1 \cos(k_{Ry} y) \quad (32)$$

For odd values of n (odd modes) the sound pressure at the centre line is zero and

$$G_y = G_2 \sin(k_{Ry} y) \quad (33)$$

Using eq's (4), (6), (17), (29), (30), (32) and (33) the following expressions may be derived for the acoustic pressure in the duct. For even modes in the y-direction (and even modes in the x-direction),

$$p = i\rho_o G_1 \cos(k_{Ry} y)(\omega - U_o k_N)\cos(m\pi x/\ell_x) H_o e^{i(\omega t - k_N z)} \quad (34)$$

and for odd modes in the y-direction (and even modes in the x-direction),

$$p = i\rho_o G_2 \sin(k_{Ry} y)(\omega - U_o k_N)\cos(m\pi x/\ell_x) H_o e^{i(\omega t - k_N z)} \quad (35)$$

To simplify matters, only the analysis for the even n modes will be shown in detail, although the final result for the odd n modes will also be given.

Acoustic pressure distribution in the liner. For a rectangular section duct, solutions to Eq.(15) may be written as

$$G_\ell(y) = G_{\ell 1} \cos(k_M y) + G_{\ell 2} \sin(k_M y) \quad (36)$$

At the duct wall ($y = \ell + d = L$), the boundary condition is that the pressure gradient in the liner be zero. Thus,

$$G_{\ell 2} = G_{\ell 1} \tan(k_M L) \quad (37)$$

Combining eq's (13), (20), (36) and (37) the acoustic pressure in the liner may be written as

$$p_\ell = G_{\ell 1} H_{\ell o} [\cos(k_M y) + \tan(k_M L)\sin(k_M y)] e^{i(\omega t - k_N z)} \quad (38)$$

Boundary conditions at the duct airway liner interface. Next are obtained two expressions for the pressure gradient at the liner/duct airway interface; one from the duct airway equations and the other from the liner equations

Substitution of eq's (34) and (38) into (27) gives an expression for the constants $G_{\ell 1}$ and $H_{\ell o}$ of eq.(38) in terms of the constant G_1 of eq.(34). That is,

$$G_{\ell 1} H_{\ell o} = \frac{i G_1 H_o \cos(m\pi x/\ell_x)}{(\omega - U_o k_N)} \times$$

$$\times \left[\frac{\rho_o (\omega - U_o k_N)^2 \cos(k_{Ry}\ell) - \sigma \omega^2 k_{Ry} \sin(k_{Ry}\ell)}{\cos(k_M \ell) + \tan(k_M L)\sin(k_M \ell)}\right] \quad (39)$$

Subst. eq.(39) into (38) and differentiating gives

$$\left.\frac{\partial p_\ell}{\partial y}\right|_{y=\ell} = \frac{iG_1 H_o k_M \cos(m\pi x/\ell_x)}{(\omega - U_o k_N)} \times$$

$$\times \left[\rho_o(\omega - U_o k_N)^2 \cos(k_{Ry}\ell) - \sigma\omega^2 k_{Ry} \sin k_{Ry}\ell\right] \times$$

$$\times \left[\frac{-\sin(k_M\ell) + \tan(k_M L)\cos k_M\ell}{\cos(k_M\ell) + \tan(k_M L)\sin k_M\ell}\right] e^{i(\omega t - k_N z)} \qquad (40)$$

From eq.(34)

$$\left.\frac{\partial p}{\partial y}\right|_{y=\ell} = -i\rho_o G_1 H_o \cos(m\pi x/\ell_x) \times$$

$$\times (\omega - U_o k_N) k_{Ry} \sin(k_{Ry}\ell)\, e^{i(\omega t - k_N z)} \qquad (41)$$

The gradients in p and p_ℓ can be matched at the liner surface using the continuity of displacement through the thin impervious membrane. Thus from eq's (26) and (28)

$$\left.\frac{\partial p}{\partial y}\right|_{y=\ell} = \frac{\rho_o}{\rho_\ell}\left[1 - M\frac{k_N}{k_o}\right]^2 \left.\frac{\partial p_\ell}{\partial y}\right|_{y=\ell} \qquad (42)$$

Equating (41) and (42), substituting eq.(40) into (42), using $L = \ell + d$ and rearranging gives

$$k_{Ry}\tan(k_{Ry}\ell) = \frac{-(\rho_o/\rho_\ell)(1 - Mk_N/k_o)\, k_M^2 \tan(k_M d)}{1 - (\sigma/\rho_\ell) k_M \tan(k_M d)} \qquad (43)$$

The dependence of the mode order m in the x-direction is included in the expression for k_{Ry}. Successive solutions correspond to successive even values of mode order n in the y-direction. n=0, the first solution, corresponds to a plane wave.

A similar analysis may be undertaken for odd order n modes with the following final result

$$k_{Ry}\cot(k_{Ry}\ell) = \frac{(\rho_o/\rho_\ell)(1 - Mk_N/k_o)\, k_M^2 \tan(k_M d)}{1 - (\sigma/\rho_\ell)\, k_M \tan(k_M d)} \qquad (44)$$

SOLUTION PROCEDURE

Equation (43) is solved for the propagation constant k_N, the imaginary part of which (when multiplied by -8.69 and the duct airway width ℓ) gives the attenuation in dB per duct width. Successive solutions for k_N correspond to successive even modes n in the y-direction. For the rectangular duct, the results presented here correspond to a mode order of m=0 in the x-direction, as this is the most important case.

To solve equation (43) it is necessary to use as input, the following quantities:

- normalized flow resistances $R_{1y}d/\rho_o c_o$ and $R_{1z}d/\rho_o c_o$ for a length or thickness d of porous liner material in the radial and axial directions respectively;
- normalised surface density $\sigma/\rho_o\ell$ of the impervious membrane;
- ratio of the liner thickness to the duct half width d/ℓ; and
- Mach number M of the mean flow through the duct.

To find a solution for k_N and k_M of equation (43) it was necessary to begin with a reasonably close first estimate. Wassilieff (1987) showed that as $k_o \to 0$ a reasonable first estimate (for M=0 and m=0) is given by

$$k_{Ry} = [\tfrac{1}{2}n\pi/(1 + d/\ell)] + i0$$

k_N can be found from k_{Ry} using Eq.(11), the relation $k_R^2 = k_{Ry}^2 + (m\pi/\ell_x)^2$ and the porous material axial propagation parameter (ω/c_z). This latter quantity is derived from the normalized flow resistance of the liner material (Bies & Hansen, 1988). The complex density ρ_ℓ of the liner material can be calculated from the normal impedance and radial propagation parameter (ω/c_M) (Bies and Hansen, 1988). k_M can be found from k_N using Eq's. (15) and (19).

Solutions for higher frequencies are obtained by incrementing the frequency by sufficiently small amounts and using the solution for the previous frequency as a first estimate of the solution for the next higher frequency.

DESIGN CHARTS

Design charts are presented for various combinations of the dimensionless input variables, $R_{1y}d/\rho_o c_o$, $R_{1z}d/\rho_o c_o$, $\sigma/\rho_o\ell$, d/ℓ and M. Figures 2 to 6 contain curves covering a range of commonly used liner configurations. Figures 7 to 9 contain curves demonstrating the effect on the attenuation rate of varying particular parameters.

To simplify their use, the charts represent octave band frequency averages. To find a particular octave band averaged attenuation for a given configuration, an estimate of the frequency parameter $2\ell/\lambda$ is required, where λ is the wavelength of the octave band centre frequency. The chart is then entered at this abscissa value and the attenuation corresponding to the appropriate curve is read directly from the ordinate in decibels of attenuation for a length of duct equal to the duct width, 2ℓ.

CONCLUSIONS

The theoretical work reported here demonstrates that when calculating the attenuation of sound propagating down a lined duct, the effects of a mean flow in the duct and the effects of any protective covering on the liner are important and must be taken into account. The generalized theory presented here was evaluated for ducts of rectangular cross-section. However, the theory is amenable to evaluation for ducts of arbitrary cross section.

The results of the theory for rectangular ducts are presented here in the form of design charts for a range of commonly encountered configurations. For a duct lined on all four sides, the total attenuation is the sum of the attenuations obtained by considering each pair of sides independently.

ACKNOWLEDGEMENTS

Support for this work from the University of Adelaide and the Australian Research Council is gratefully acknowledged.

REFERENCES

1. A. CUMMINGS 1976 *Journal of Sound and Vibration* 49, 9-35. Sound attenuation in ducts lined on two opposite walls with porous material, with some applications to splitters.

2. C. WASSILIEFF 1987 *Journal of Sound and Vibration* 114, 239-251. Experimental verification of duct attenuation models with bulk reacting linings.

3. D.A. BIES and C.H. HANSEN 1988 *Engineering Noise Control: Theory and Practice*. App. 3. London: Unwin Hyman.

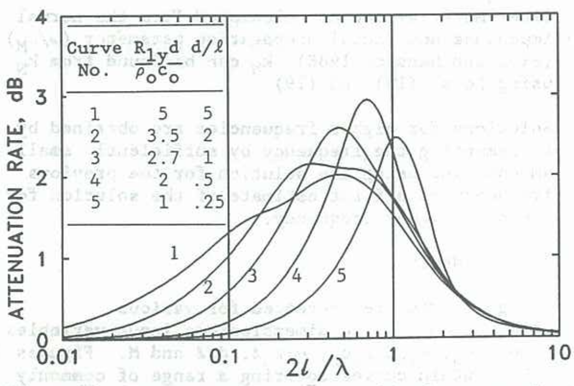

Figure 2 Sound attenuation in a lined rectangular duct. $M = 0$, $R_{1z} = \frac{1}{2}R_{1y}$, $\sigma/\rho_o\ell = 0.01$.

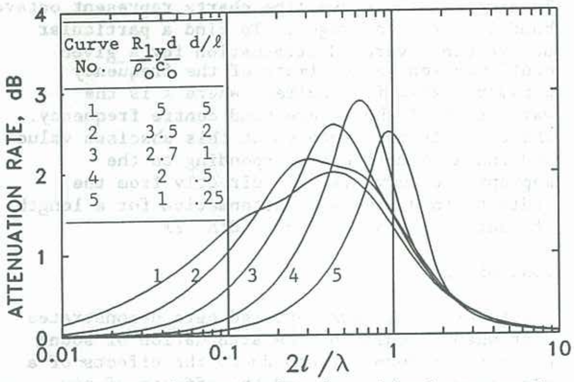

Figure 3 Sound attenuation in a lined rectangular duct. $M = 0$, $R_{1z} = \frac{1}{2}R_{1y}$, $\sigma/\rho_o\ell = 0.1$.

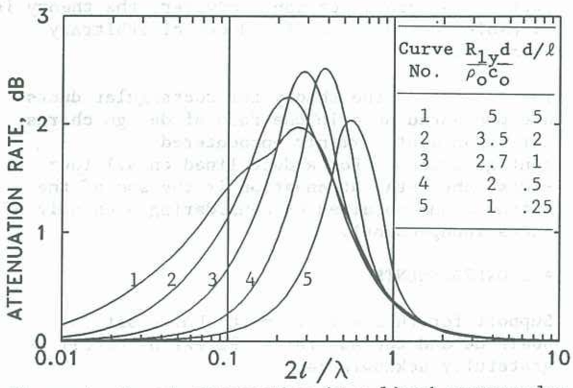

Figure 4 Sound attenuation in a lined rectangular duct. $M = 0$, $R_{1z} = \frac{1}{2}R_{1y}$, $\sigma/\rho_o\ell = 1.0$.

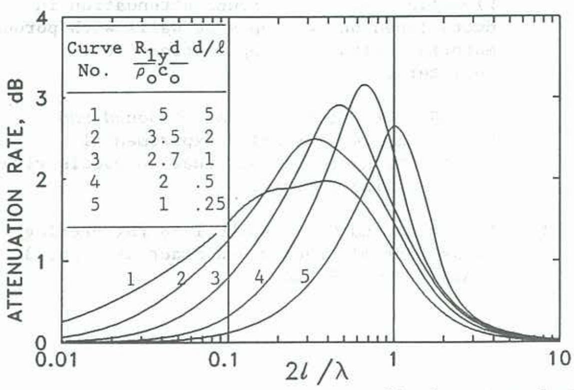

Figure 5 Sound attenuation in a lined rectangular duct. $M = -0.1$, $R_{1z} = \frac{1}{2}R_{1y}$, $\sigma/\rho_o\ell = 0.01$.

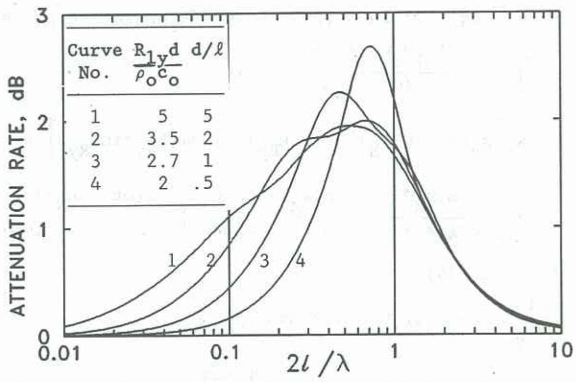

Figure 6 Sound attenuation in a lined rectangular duct. $M = 0.1$, $R_{1z} = \frac{1}{2}R_{1y}$, $\sigma/\rho_o\ell = 0.01$.

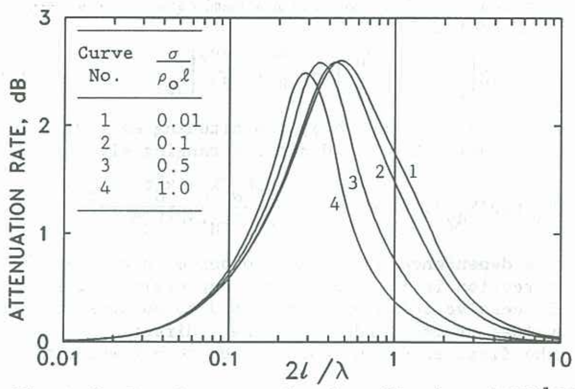

Figure 7 Sound attenuation in a lined rectangular duct. $M=0$, $R_{1z} = \frac{1}{2}R_{1y}$, $R_{1y}d/\rho_oc_o = 2.7$.

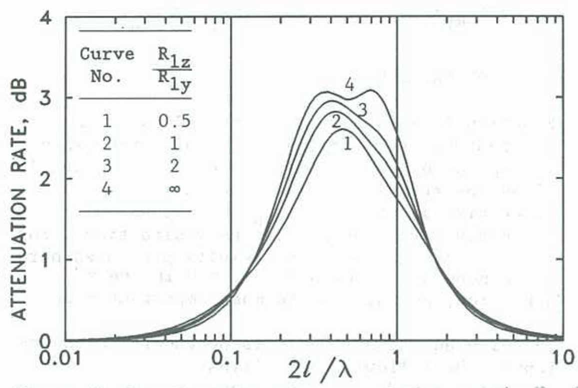

Figure 8 Sound attenuation in a lined rectangular duct. $M=0$, $R_{1y}d/\rho_oc_o = 2.7$, $\sigma/\rho_o\ell = 0.01$.

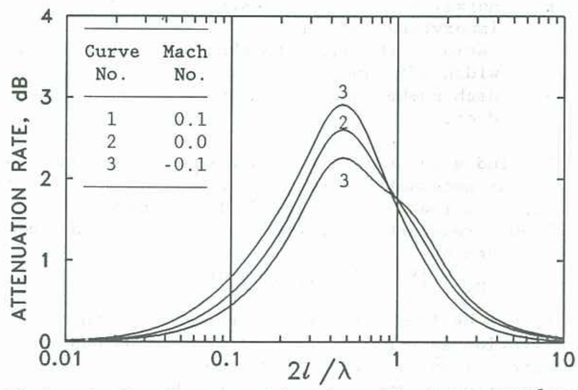

Figure 9 Sound attenuation in a lined rectangular duct. $R_{1z} = \frac{1}{2}R_{1y}$, $R_{1y}d/\rho_oc_o = 2.7$, $\sigma/\rho_o\ell = 0.01$.