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SOME OPEN QUESTIONS IN TURBULENCE MODELLING FROM VIEWPOINT OF ASYMPTOTIC THEORY

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ABSTRACT

Without using a turbulence model asymptotic theory for turbulent shear flows is able to predict the structure of the solution in the overlap layer between wall layer and defect layer. Since asymptotically wall–layer solutions are universal, turbulence models can be restricted to the defect layer, for which the overlap layer supplies boundary conditions.

INTRODUCTION

Asymptotic theory for turbulent shear flows considers the solution of the Reynolds–averaged Navier—Stokes equations for the limit of high Reynolds numbers. Without using any turbulence model this theory leads to certain conditions for the structure of the solutions, which turbulence models have to satisfy in order to be asymptotically correct.

According to asymptotic theory turbulent shear flows near walls have usually a two–layer structure: the wall layer, in which the viscosity is important and the flow shows universal features, and the defect layer, where viscosity is negligible and turbulent models become essential.

Matching the two solutions found for each layer separately is equivalent to a–priori statements about the structure of the solution in the overlap region between these two layers.

In this paper the overlap regions of turbulent shear flows will be considered from the viewpoint of asymptotic theory.

Examples will be Couette flows without and with pressure gradient as well as equilibrium boundary layers.

BASIC EQUATIONS

A turbulent Couette flow with constant physical properties (no viscous heating, impermeable walls) is considered according to Figure 1. There are given: H, ρ, c_p, ν, $Pr = \nu/a = O(1)$, τ_w, q_w. The distributions of velocity u(y) and temperature T(y) are to be found. Since the mean flow is independent of x, continuity equation leads to v = 0.

Basic equations are :

Momentum:

$$\frac{d}{dy}\left[\nu\frac{\partial u}{\partial y}+\frac{\tau_t}{\rho}\right]=0 \qquad (1)$$

Turbulent kinetic energy:

$$\frac{\tau_t}{\rho}\frac{du}{dy}+\nu\frac{d^2k}{dy^2}+\frac{dB}{dy}-\epsilon_u=0 \qquad (2)$$

production — viscous diffusion — turbulent diffusion — dissipation

Internal energy (enthalpy):

$$\frac{d}{dy}\left[-a\frac{dT}{dy}+\frac{q_t}{\rho\, c_p}\right]=0 \qquad (3)$$

Variance of temperature fluctuations:

$$\frac{q_t}{\rho\, c_p}\frac{dT}{dy}+a\frac{d^2k_\theta}{dy^2}+\frac{dB_\theta}{dy}-\epsilon_\theta=0 \qquad (4)$$

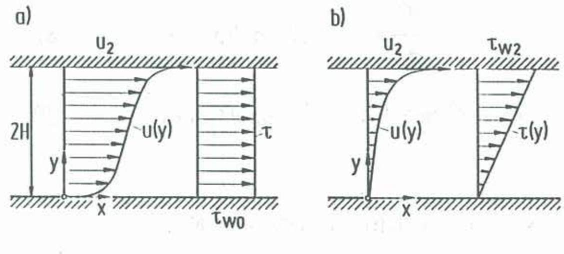

Figure 1 Couette–Poiseuille Flow
a) $\tau = \tau_w \neq 0$
b) $\tau_{w0} = 0$, $\tau \sim y$

TWO–LAYER STRUCTURE OF FLOW FIELD

Although the system of four equations is not closed without a turbulence model, formally the solutions can be determined for the limit $\nu \to 0$. Since the limiting solution $\nu = 0$ does not satisfy the boundary conditions at the wall, a singular perturbation problem has to be solved, which can be done by using the method of matched asymptotic expansions, see Van Dyke (1975). The flow has (in the upper and in the lower half of the flow field), a two–layer structure.

The characteristics of the two layers are:

Defect layer (core layer): ν negligible, thickness O(H), coordinate y, turbulent shear stress $\tau_t = O(\tau_w)$, turbulent heat flux $q_t = O(q_w)$, velocity u(y), temperature T(y).

Wall layer: ν important, $\tilde{\varepsilon} := \delta_w/H = \nu/H\sqrt{\tau_t/\rho}$, thickness $\delta_w := \nu/\sqrt{\tau_t/\rho}$, coordinate $\tilde{y}^+ := y/\tilde{\varepsilon}$, turbulent shear stress $\tau_t = O(\tau_w)$, turbulent heat flux $q_t = O(q_w)$, velocity $u(\tilde{y}^+)$, temperature $T(\tilde{y}^+)$.

In each layer the flow equations are simplified for $\nu \to 0$ and solved separately.

OVERLAP LAYER

Matching of the two solutions rests on the existence of an *overlap layer*, where both the core solution and the wall–layer solution are valid in the limit $\nu \to 0$. The thickness of the overlap layer is small compared to H, but large compared to δ_w. However, the overlap layer has a finite thickness with respect to a coordinate $\tilde{y}$, which lies between y and $\tilde{y}^+$ and is called *intermediate coordinate:*

$$\tilde{y} := \frac{y}{\tilde{\varepsilon}^{\alpha}} \qquad 0 < \alpha < 1 \tag{5}$$

Matching by using the intermediate coordinate is called the *intermediate matching principle*, see Van Dyke (1975), p. 91, Schneider (1977), p. 205.

Since the solution in the overlap layer must be independent of ν, Pr, τ_w/ρ, and $q_w/\rho\, c_p$ (and also independent of wall roughness) as well as independent of H, the following relations must be valid for the solution in the overlap layer for $\nu \to 0$:

$$\frac{du}{d\tilde{y}} = f_u(\tilde{y}, \tau_t/\rho) \tag{6}$$

$$\tilde{\varepsilon}_u := \tilde{\varepsilon}^{\alpha}\, \varepsilon_u = f_\varepsilon(\tilde{y}, \tau_t/\rho, k) \tag{7}$$

$$\frac{dT}{dy} = f_\theta(\tilde{y}, \tau_t/\rho, q_t/(\rho\, c_p)) \tag{8}$$

$$\tilde{\varepsilon}_\theta := \tilde{\varepsilon}^{\alpha}\, \varepsilon_\theta = f_{\varepsilon\theta}(\tilde{y}, \tau_t/\rho, q_t/(\rho\, c_p), k_\theta) \ . \tag{9}$$

GENERAL MATCHING CONDITIONS

According to Π–theorem Eqs. (6) to (9) can be written in dimensionless form:

$$\mathrm{Lim}\, \frac{\tilde{y}}{\sqrt{\frac{\tau_t}{\rho}}} \frac{du}{d\tilde{y}} =: \mathrm{Lim}\, \frac{\tilde{y}}{\tilde{l}} = C_u = \mathrm{const.} \tag{10}$$

$$\mathrm{Lim}\, \frac{\tilde{y}\sqrt{\frac{\tau_t}{\rho}}}{\frac{q_t}{\rho\, c_p}} \frac{dT}{d\tilde{y}} =: \mathrm{Lim}\, \frac{\tilde{y}}{\tilde{l}_\theta} = C_\theta = \mathrm{const.} \tag{11}$$

$$\mathrm{Lim}\, \frac{\tilde{\varepsilon}_u\, \tilde{y}}{\left[\frac{\tau_t}{\rho}\right]^{3/2}} =: \mathrm{Lim}\, \frac{\tilde{y}}{\tilde{l}_\varepsilon} = C_\varepsilon \left[\mathrm{Lim}\, \frac{k}{\frac{\tau_t}{\rho}}\right] \tag{12}$$

$$\mathrm{Lim}\, \frac{\tilde{\varepsilon}_\theta \tilde{y}}{\left[\frac{\tau_t}{\rho}\right]^{3/2}} =: \mathrm{Lim}\, \frac{\tilde{y}}{\tilde{l}_{\varepsilon\theta}} = C_{\varepsilon\theta} \left[\mathrm{Lim}\, \frac{k_\theta \frac{\tau_t}{\rho}}{\left[\frac{q_t}{\rho\, c_p}\right]^2}\right] \ , \tag{13}$$

where

$$\mathrm{Lim}: \ = \lim_{\substack{\tilde{y}\ \mathrm{fixed} \\ \tilde{\varepsilon} \to 0}} \tag{14}$$

refers to a limiting process in the overlap layer ($\tilde{y}$ fixed).

These are the general matching conditions, see also Cebeci, Bradshaw (1984), p. 166, p. 336. It is worth mentioning that the different length scales, e.g. mixing length $\tilde{l}$, dissipation length $\tilde{l}_\varepsilon$ etc, have been defined such, that they are proportional to $\tilde{y}$ in the overlap layer. They are *not* turbulence models.

PHYSICS IN OVERLAP LAYER

It can be shown, that the following limits hold:

$$\mathrm{Lim}\, \frac{dB}{d\tilde{y}} = 0\ ; \quad \mathrm{Lim}\, \frac{dB_\theta}{d\tilde{y}} = 0 \ . \tag{15}$$

From these relations the following statements about the behaviour of the solution in the overlap layer for the limit $\tilde{\varepsilon} \to 0$ can be made:

1) In the overlap layer turbulence production is equal to turbulence dissipation. An equivalent statement for the variance of temperature fluctuations is valid. Therefore, this layer is called *equilibrium layer*.

   From Eqs. (2), (4) and (15) follows:

$$\mathrm{Lim}\, \frac{\tau_t}{\rho} \frac{du}{d\tilde{y}} = \mathrm{Lim}\, \tilde{\varepsilon}_u \tag{16}$$

$$\mathrm{Lim}\, \frac{q_t}{\rho\, c_p} \frac{dT}{d\tilde{y}} = \mathrm{Lim}\, \tilde{\varepsilon}_\theta \ . \tag{17}$$

2) In the overlap layer the turbulent kinetic energy is proportional to turbulent shear stress and the variance of temperature fluctuations is proportional to turbulent heat flux. Therefore, one says, that in the overlap layer there exists *structural equilibrium*, see Townsend (1961).

   From Eqs. (16), (17), (10) to (14) follows:

$$\mathrm{Lim}\, k = C_k\, \mathrm{Lim}\, \frac{\tau_t}{\rho} \tag{18}$$

$$\mathrm{Lim}\, k_\theta = C_{k\theta}\, \mathrm{Lim}\, \frac{\left[\frac{q_t}{\rho\, c_p}\right]^2}{\frac{\tau_t}{\rho}} \tag{19}$$

3) In the overlap layer the four length scales, defined by Eqs. (10) to (13), are by definition proportional to $\tilde{y}$. Mixing length $\tilde{l}$ and dissipation length $\tilde{l}_\varepsilon$ are identical. The turbulent Prandtl number is constant.

   From Eqs. (10) to (14), (16) and (17) follows

$$\mathrm{Lim}\, \tilde{l} = \frac{\tilde{y}}{C_u} = \frac{\tilde{y}}{C_\varepsilon} = \mathrm{Lim}\, \tilde{l}_\varepsilon \ , \tag{20}$$

$$\mathrm{Lim}\, \tilde{l}_\theta = \frac{\tilde{y}}{C_\theta} = \frac{\tilde{y}}{C_{\varepsilon\theta}} = \mathrm{Lim}\, \tilde{l}_{\varepsilon\theta} \ , \tag{21}$$

$$\mathrm{Lim}\ Pr_t = \frac{C_\theta}{C_u} = \frac{\kappa}{\kappa_\theta} = \mathrm{const} \quad . \tag{22}$$

The universal constants C_u, C_ϵ etc. are given in Table 1. In summary, the structure of the solution in the overlap region is known a priori.

	Normal	Blowing/ Suction Parameter v_w^+	Viscous Heating Parameter Ec_τ	Variable Properties Parameter β_q
$C_u = C_\epsilon$	$\frac{1}{\kappa}$	$\frac{1}{\kappa}(1+v_w^+ F_{v0})$	$\frac{1}{\kappa}$	$\frac{1}{\kappa}(1-\beta_q K_\rho F_{\rho 0})$
$C_\theta = C_{\epsilon\theta}$	$\frac{1}{\kappa_\theta}$	$\frac{1}{\kappa_\theta}(1+v_w^+ G_{v0})$	$\frac{1}{\kappa_\theta}$	$\frac{1}{\kappa_\theta}(1-\beta_q K_\rho G_{\rho 0})$

Table 1 Constants of matching and structural equilibrium.
($\kappa = 0.41$; $\kappa_\theta = 0.47$; $c_\mu = 0.09$; $c_{\mu\theta} \approx 1$;
$C_k = 1/\sqrt{c_\mu} \approx \kappa^{-4/3}$; $C_{k\theta} = 1/\sqrt{c_{\mu\theta}}$)

Although the derivations have been made for a Couette flow, the results are universal. The pressure gradient is in the limit $\bar{\epsilon} \to 0$ a higher–order effect as long as $\tau_w \neq 0$. The wall curvature is also negligible for curvature radii $R = O(H) >> \delta_w$. Therefore, the wall layers of all turbulent shear flows near walls are identical with the wall layer of the Couette flow in the limit $\nu \to 0$ for $\tau_w \neq 0$, see Mellor(1972).

Since the behaviour of the wall–layer flows is sufficiently known or can be determined universally, turbulence modelling and numerical calculations can be restricted to the core layer (defect layer). The general matching conditions will serve as boundary conditions for the solutions in the core layer. A thousandfold repeated computation of the universally known wall layer over and over again is unnecessary and senseless.

In the following the effects so far neglected (blowing/suction, viscous heating, variable properties) will be investigated with respect to their influence on the universal wall layer and particularly on the overlap layer. The parameters characterizing the various effects are assumed to be small. The results are summarized in Table 2.

The dimensionless variables have been introduced by using the following reference values:

length: H; velocity: friction velocity $u_\tau := \sqrt{\tau_{w0}/\rho_0}$; shear stress and pressure: τ_{w0} ; temperature: friction temperature $T_\tau := -[q_w/\rho\, c_p\, u_\tau]_0$; heat flux: q_{w0} .

Dimensionless parameters are:

$$v_w^+ := \frac{v_w}{u_\tau} \ , \quad Ec_\tau := \frac{u_\tau^2}{c_p\, T_\tau} \ , \quad \beta_q := \frac{T_\tau}{T_0} \ ,$$

$$y^+ := \frac{y}{\epsilon} \ , \quad \epsilon := \frac{\nu}{H\, u_\tau}$$

The matching condition (10) reads in dimensionless form

$$\lim_{y^+ \to \infty} \frac{y^+}{\sqrt{\frac{\tau_t}{\rho}}} \frac{du^+}{dy^+} = \lim_{\substack{\bar{y}\ \mathrm{fixed} \\ \bar{\epsilon} \to 0}} \frac{\bar{y}}{\sqrt{\frac{\tau_t}{\rho}}} \frac{du^+}{d\bar{y}} = \lim_{y \to 0} \frac{y}{\sqrt{\frac{\tau_t}{\rho}}} \frac{du^+}{dy} \tag{23}$$

and Eqs. (11) to (13) accordingly.

BLOWING AND SUCTION

In order to keep the Couette flow independent of x, at the lower wall fluid is injected and the same amount of fluid is sucted off at the upper wall or vice versa. It is worth mentioning, that in the overlap layer the distributions of velocity and temperature are $\sim \ln^2 y$, see also Rotta (1970), τ_t, k, q_t and k_θ are $\sim \ln y$. The constants in the matching conditions (Table 1) also depend on v_w^+ (due to convection), whereas the coefficients F_{v0} and G_{v0} follow from a turbulence model for the core layer (k–ϵ–model: $F_{v0} = G_{v0} = -2.7$; Rotta–method, see Rotta (1986): $F_{v0} = -4.6$). It is interesting to note that for this problem in the textbook by Tennekes, Lumley (1972), p. 55, four different approaches are offered. Unfortunately, all four are asymptotically not correct.

VISCOUS HEATING

By taking viscous heating into account the heat flux is not constant, but $q_t \sim \ln y$, whereas the temperature is $\theta^+ \sim \ln^2 y$, see also Rotta (1959).

VARIABLE PROPERTIES

It is assumed that density ρ, viscosity η, and heat conductivity λ are functions of temperature. Since only small heat transfer parameters β_q are considered, it is sufficient to use the property laws:

$$\alpha = 1 + K_\alpha\, \beta_q\, \Theta^+ \ ; \ K_\alpha := \left[\frac{d\alpha}{dT}\frac{T}{\alpha}\right]_0 \ ; \ \alpha = \rho, \eta, \lambda \ .$$

Again, the matching constants C_u, C_ϵ etc. depend also on $\beta_q K_\rho$ (due to convection), see also Walker et al. (1987). Like in the case of blowing/suction u^+ and θ^+ are $\sim \ln^2 y$, whereas τ_t, q_t are $\sim \ln y$, see also Rotta (1959). The main effects are due to density variations. Variations of viscosity and heat conductivity lead only to additive constants in the distributions of u^+ and θ^+.

ZERO WALL SHEAR STRESS

Couette–Poiseuille Flow

By superposition of a proper pressure gradient to the Couette flow a special *Couette–Poiseuille* flow can be produced such that the wall shear stress at the lower wall is zero. In this case the general matching conditions, the statements about equilibrium layer, and structural equilibrium are still valid. It follows for the overlap layer, i.e. for the limit $y \to 0$:

$$\tau_t \sim y,\ u^+ \sim y^{1/2},\ \nu_t \sim y^{1/2},\ l \sim y,\ k \sim y,\ \epsilon_u \sim y^{1/2} \tag{24}$$

Instead of friction velocity the characteristic velocity is now

$$u_s := \left[\frac{\nu}{\rho}\frac{dp}{dx}\right]^{1/3} \ . \tag{25}$$

For details see Gersten (1987). Most functions undergo a drastic change in their structure in the transition from $\tau_w \neq 0$ to $\tau_w = 0$. There are exceptions: the length scales l, l_ϵ as well as the structural constant C_k and matching constants C_u , C_ϵ .

All existing models are not able to describe the drastic changes when approaching the point $\tau_w = 0$ (separation), because the model constants have to be different in the cases $\tau_w \neq 0$ and $\tau_w = 0$. A turbulent model based on equations for l and C_k is therefore proposed.

EQUILIBRIUM BOUNDARY LAYERS

The asymptotic theory for turbulent boundary layers has been given by Mellor (1972). The wall layer is locally identical with the wall layer of the equivalent Couette flow with the same wall shear stress. All results about the overlap layer are also valid. A particular family of turbulent boundary layers is characterized by self–similar solutions of the defect layer. They are called *equilibrium boundary layers* and are characterized by two dimensionless parameters, the *Clauser parameter* β and Reynolds number Re_1 (or shape parameter H_{12} or pressure gradient parameter m). Figure 2 shows the shape parameter as function of m and Re_1. There are two different curves for the limit $Re_1 \to \infty$. The line OB refers to cases $\tau_w \neq 0$, where after Mellor (1972) the boundary layer has a two–layer structure with a logarithmic velocity law in the overlap region. The curve BA refers to cases $\tau_w = 0$. After Klauer (1989) these boundary layers have a three–layer structure, see also Melnik(1986). The layer closest to the wall is again locally identical with the wall layer of the particular Couette–Poiseuille flow with $\tau_w = 0$, where

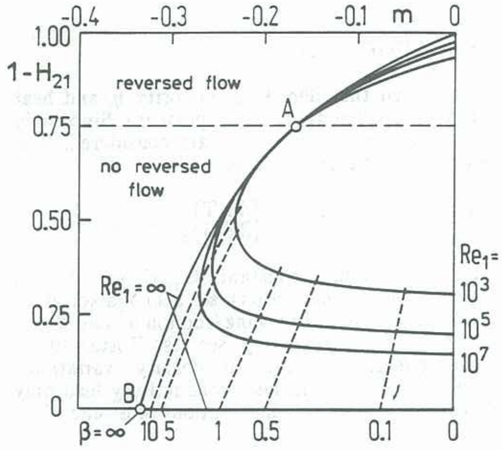

Figure 2.
Equilibrium Boundary Layers,
$H_{21} = \delta_2/\delta_1$; $m = (\delta_1 U)/(\delta_1 U')$; $\beta = \delta_1(dp/dx)/\tau_w$
OB: Logarithmic velocity law, two layers.
AB: Square–root velocity law, three layers.

the overlap layer has a square–root velocity law. Again, no existing turbulence model is able to describe the transition form $\tau_w \neq 0$ to $\tau_w = 0$ without changing the model constants. Here, again, a 1–C_k–model could be the solution.

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	Blowing/Suction Parameter v_w^+	Viscous Heating Parameter Ec_τ	Variable Properties Parameter β_q
$\lim_{y\to 0} \tau_t - 1$	$v_w^+ U_{00}$	0	$-\beta_q K_\rho \theta_{00}$
$\lim_{y\to 0} \frac{du^+}{dy} - \frac{1}{\kappa y}$	$v_w^+ \frac{1}{\kappa y}\left[\frac{U_{00}}{2} + F_{vo}\right]$	0	$-\beta_q K_\rho \frac{1}{\kappa y}\left[\frac{\theta_{00}}{2} + F_{\rho o}\right]$
$\lim_{y\to 0} \nu_t - \kappa y$	$v_w^+ \kappa y \left[\frac{U_{00}}{2} - F_{vo}\right]$	0	$-\beta_q K_\rho \kappa y \left[\frac{\theta_{00}}{2} - F_{\rho o}\right]$
$\lim_{y\to 0} l - \kappa y$	$-v_w^+ \kappa y F_{vo}$	0	$+\beta_q K_\rho \kappa y F_{\rho o}$
$\lim_{y\to 0} u^+ - U_{00}$	$v_w^+\left[\frac{1}{4} U_{00}^2 + F_{vo}(U_{00} - C) + B_v\right]$	0	$-\beta_q \left\{K_\rho\left[\frac{1}{4}\theta_{00} U_{00} + U_{00}(F_{\rho o} + A_\rho(Pr)) + B_\rho(Pr)\right] + K_\eta C_\eta(Pr)\right\}$
$\lim_{y\to 0} q_t - 1$	$v_w^+ \Theta_{00}$	$-Ec_\tau U_{00}$	$-\beta_q K_\rho \Theta_{00}$
$\lim_{y\to 0} \frac{d\theta^+}{dy} - \frac{1}{\kappa_\theta y}$	$v_w^+ \frac{1}{\kappa_\theta y}\left[\theta_{00} - \frac{U_{00}}{2} + G_{vo}\right]$	$-Ec_\tau \frac{U_{00}}{\kappa_\theta y}$	$-\beta_q K_\rho \frac{1}{\kappa_\theta y}\left[\frac{\theta_{00}}{2} + G_{\rho o}\right]$
$\lim_{y\to 0} a_t - \kappa_\theta y$	$v_w^+ \kappa_\theta y \left[\frac{U_{00}}{2} - G_{vo}\right]$	0	$-\beta_q K_\rho \kappa_\theta y \left[\frac{\theta_{00}}{2} - G_{\rho o}\right]$
$\lim_{y\to 0} l_\theta - \sqrt{\kappa\,\kappa_\theta}\, y$	$-v_w^+ \sqrt{\kappa\,\kappa_\theta}\, y \frac{F_{vo} + G_{vo}}{2}$	0	$+\beta_q K_\rho \sqrt{\kappa\,\kappa_\theta}\, y\, G_{\rho o}$
$\lim_{y\to 0} \theta^+ - \theta_{00}$	$v_w^+\left[\frac{1}{2}\theta_{00}^2 + G_{vo}(\theta_{00} - C_\theta) + B_{\theta v}\right]$	$-Ec_\tau\left[\frac{1}{2}\theta_{00} U_{00} + A_{\theta E} U_{00} + B_{\theta E}\right]$	$-\beta_q \left\{K_\rho\left[\frac{1}{4}\theta_{00}^2 + \theta_{00}(G_{\rho o} + A_{\theta\rho}(Pr)) + B_{\theta\rho}(Pr)\right] + K_\lambda C_\lambda(Pr)\right\}$
$\lim_{y\to 0} Pr_t - \frac{\kappa}{\kappa_\theta}$	$v_w^+ \frac{\kappa}{\kappa_\theta}(G_{vo} - F_{vo})$	0	$-\beta_q K_\rho \frac{\kappa}{\kappa_\theta}\left[G_{\rho o} - F_{\rho o}\right]$

Table 2 Solutions in overlap layer (boundary condition for core layer)

$$U_{00} = \lim_{y^+\to\infty} \left[\frac{1}{\kappa}\ln y^+ + C\right] \quad ; \quad \theta_{00} = \lim_{y^+\to\infty} \left[\frac{1}{\kappa_\theta}\ln y^+ + C_\theta(Pr)\right]$$

F_{vo}, G_{vo}, $F_{\rho o}$, $G_{\rho o}$ depend on turbulence model, all other functions are universal.