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SOLIDIFICATION AND DEFORMATION OF CRUST ON VISCOUS-GRAVITY FLOWS

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ABSTRACT

The effects of a solidifying crust on the dynamics and surface morphology of viscous gravity currents is investigated through laboratory experiments with wax beneath cold water and solution of the surface cooling problem. When cooling is not sufficient to cause solidification, the radius of a current spreading from a source increases in proportion to the square root of time. For more strongly cooled extrusions the presence of the solid crust greatly reduces the spreading rate. As the current spreads its solid crust also deforms. Progressively colder experiments reveal a sequence of surface morphologies resembling features observed on cooling lava flows. This sequence includes frontal levees, regularly spaced surface folds, multi-armed rift structures with transform faults and shear offsets, and bulbous lobate forms similar to pillow lavas seen under the ocean. This morphologic continuum with well-defined transitions involves many mechanisms of coupling between viscous fluids and a deformable solid. In volcanology and planetology it offers the prospect of more successful interpretation of natural lava flows.

INTRODUCTION

Spreading flows dominated by a balance between gravitational and viscous forces have been investigated both theoretically and experimentally because they are good analogs for numerous geophysical phenomena. Application of these purely viscous models to lava flows has been limited, however, because most lavas are covered by a solidified crust which thickens with time. Such a cooled surface can raise the bulk viscosity of the current, impart a yield strength, retard the radial velocity, and increase surface stresses. Deformation of the crust in response to stresses applied by the underlying viscous flow produces characteristic morphologic features. Surface deformations are used by geologists to interpret the rheology and em-placement history of lava flows. For example, regularly spaced surface ridges have been related to viscosity gradients at the surface in the presence of a compressive stress (Fink and Fletcher, 1978; Fink, 1980a). However, no comprehensive models that address the formation of more than an individual feature in response to local flow conditions have appeared.

Laboratory simulations of lava flows have used a number of materials. Among the simplest laboratory configurations were those of Huppert (1982), who used isothermal Newtonian silicone oils to model the radial spread of viscous gravity currents, and Blake (1989) who substituted kaolin slurry (a Bingham plastic) to examine the effect of yield strength on spreading rate. Hallworth et al. (1987) injected polyethylene glycol under water (to minimize surface tension effects and increase cooling rates to realistic values) and focussed on the the effects of slope and extrusion rate on the development of branching patterns which result from the temperature-dependent rheology.

We have used laboratory simulations and theoretical arguments to arrive at a model that explains surface deformations and the velocity of the flow front in terms of the growth rate of a mechanically distinct solid crust on the surface of very viscous gravity currents (Fink and Griffiths, 1989). An analysis predicts the heat transports away from the surface of a viscous fluid by turbulent convection compared to the rate of lateral heat advection by the viscous flow, and gives the distance from a vent at which surface solidification will begin. The morphology of a flow, resulting from compression, extension and shear of the solidifying surface, is found to be controlled by the rate of solidification of crust.

PARAMETERIZATION OF FLOWS WITH FREEZING CRUST

Consider a viscous liquid (such as lava or wax but which for convenience we will refer to as lava) extruding from a vent at a temperature T_ℓ and spreading over a rigid plane beneath water at an ambient temperature T_a. The variables are the angle β of the plane to the horizontal, the volume flow rate Q, the reduced gravity $g'=g\Delta\rho/\rho$ (where $\Delta\rho$ is the density difference leading to the motion and ρ is the density of the source fluid), the temperatures T_a and T_ℓ, as well as physical properties of both the lava and the ambient fluid (namely the solidification temperature T_s, the diffusion coefficients κ and κ_a, the viscosities ν and ν_a and the temperature-dependent rheology of the lava. Latent heat of solidification is negligible in lava crust, where quenching forms a glass, and we assume that the laboratory wax also solidifies without crystallization. The properties of the lava and crust directly determine the mechanics of the flow, while those of the ambient fluid enter only through their influence on the heat transport from the lava surface.

In the case of a constant volume flux issuing from a point source on to a planar surface with no temperature or viscosity gradients and no mechanically distinct crustal layer (and assuming a simple balance between buoyancy and viscous forces) there exists a similarity solution to the flow (Huppert, 1982). In this solution the depth H of the fluid and the axisymmetric radial velocity U_n of the leading edge of the spreading current scale as:

$$H \sim (Q\nu/g')^{1/4}, \quad U_n \sim (g'Q^3/\nu)^{1/8}\, t^{-1/2} \qquad (1)$$

The flow can also be described in terms of a global velocity scale $U \sim (g'Q/\nu)^{1/2}$.

When solidification occurs the parameter of primary importance in describing the flow is the degree of cooling necessary for solidification. This can be measured in terms of the overall temperature difference $\Delta T = T_\ell - T_a$, and we form the parameter

$$\Theta_s = (T_s - T_a)/(T_\ell - T_a). \qquad (2)$$

No solidification is possible if $\Theta_s < 0$, and Θ_s must be less than 1 for the lava to be liquid at the source. For $\Theta_s \gtrsim 0$ no solidification is expected because the surface is unlikely to reach its freezing point. For $\Theta_s \lesssim 1$ only a little cooling will give rise to a solid crust, which will tend to develop close to the vent.

A second factor influencing the formation of crust is the rate of radial advection of heat compared with the rate of cooling of the lava surface. Assuming that the heat transfer within the lava is by conduction alone a Peclet number Pe can be formed using (1):

$$Pe = UH/\kappa = (1/\kappa)(g'Q^3/\nu)^{1/4}. \qquad (3)$$

The greater the value of Pe, the smaller the heat loss from the surface and the thinner is the boundary layer at any given distance from the vent. Equivalently, the larger Pe the farther the lava surface moves before it reaches its solidification temperature. This study is concerned only with flows having Pe>>1.

The rate of cooling of the lava surface is not determined by Pe alone, however, but also depends critically on the magnitude of the convective surface flux compared with the conductive flux in the lava. Thus we define a modified Peclet number Π as the ratio of the rate of lateral advection to the rate of cooling of the lava surface. In order to find the appropriate form for Π it is necessary to consider the heat transfer by high-Rayleigh number convection and to solve for the surface (or contact) temperature T_c. We apply a one-dimensional cooling model to the spreading lava by placing ourselves in the frame of reference moving with the velocity of the lava surface. Conductive heat transfer in the lava is matched to the convective heat flux F(t) from its surface, with the initial condition $T(o) = T_\ell$ and a flux given by

$$F = \gamma \rho_a c_a [(g\alpha_a \kappa_a^2)/\nu_a]^{1/3}\, (T_c - T_a)^{4/3}, \qquad (4)$$

where ρ_a and c_a are the density and specific heat of the water and γ is a dimensionless constant whose empirical value is close to 0.1. The contact temperature T_c can be found from the general solution for conduction in the presence of the surface flux (Fink and Griffiths, 1989) and requires numerical solution of the equation

$$\theta_c = 1 - \int_0^\tau \{ [\theta_c(\zeta)]^{4/3} / (\tau - \zeta)^{1/2} \}\, d\zeta, \qquad (5)$$

where θ_c is the dimensionless contact temperature, $\theta_c = (T_c - T_a)/\Delta T$, τ is the dimensionless time, $\tau = t/\lambda$, and λ is the time scale over which T_c decreases from the vent temperature T_ℓ towards the water temperature T_a:

$$\lambda = \left(\frac{\rho c}{\rho_a c_a}\right)^2 \left(\frac{\pi}{\gamma^2}\right)\left(\frac{\nu_a}{g\alpha_a \Delta T}\right)^{2/3} \left(\frac{\kappa}{\kappa_a^{4/3}}\right). \qquad (6)$$

At $\tau=0$, $\theta_c=1$. At $\tau \to \infty$ the contact temperature approaches the water temperature and $\theta_c \to 0$. The dependence of the convective flux on the contact temperature implies that the flux decreases in time, causing θ_c to decrease more slowly than the relative simple behaviour $(1-\tau^{1/2})$ predicted for a constant surface flux.

A time scale t_A for lateral advection through a distance H is found from (1) and the velocity scale U: $t_A = [(\nu/g')^3 Q]^{1/4}$. From (3) and (6) we find the ratio of surface cooling time to advection time, which is our modified Peclet number:

$$\Pi = \left(\frac{\rho c}{\rho_a c_a}\right)^2 \left(\frac{\pi}{\gamma^2}\right)\left[\frac{\Delta\rho}{\rho\alpha_a \Delta T}\right]^{\frac{2}{3}} \left(\frac{\nu_a}{\nu}\right)^{\frac{2}{3}} \left(\frac{\kappa}{\kappa_a}\right)^{\frac{4}{3}} Pe^{1/3}. \qquad (7)$$

Finally, the solution to (5) predicts the time elapsed before the lava surface reaches the solidification temperature T_s: the lava surface will be solid if $\theta_c < \Theta_s$ as defined in (2). Equivalently, there is a dimensionless time τ_s such that the surface solidifies when $\tau > \tau_s$. Knowing Θ_s, we can find τ_s and, from (6), the actual time (t_s) to the beginning of solidification. The most direct means of describing the tendency for crustal growth is therefore in terms of a single dimensionless parameter Ψ, the ratio of the time t_s for the lava surface to reach its solidification temperature to a time scale t_A for horizontal advection through one current depth. From (6,7) this parameter is simply

$$\Psi = \Pi \tau_s(\Theta_s). \qquad (8)$$

When the above cooling problem is applied to a spreading gravity current, the solidification time corresponds to a distance travelled, r_s. There will be no crust at distances less than r_s from the vent, at least during the early stages of the axisymmetric flow. Solid crust will appear at larger radii. From (1) and (7) it can be shown that

$$r_s/H = (\Pi\tau_s)^{1/2} = \Psi^{1/2}. \qquad (9)$$

A further parameter required for a complete description of the flow and its crustal deformation is the ratio of the strength σ of the solid to the applied viscous stresses:

$$S = \sigma H/\rho \nu U = (\sigma/\rho)(g'^3 Q \nu)^{-1/4}. \qquad (10)$$

The effects of varying this strength parameter are of higher order than those due to changing Θ_s and Π. The strength of the solid is large for lavas (where $S \sim 10^2$) and for our laboratory wax flows (where $S \sim 10^4$). Under these conditions deformation and fracture of crust can occur only where it is extremely thin. Hence we assume that variations in S are of little significance when $S >> 1$.

EXPERIMENTS

Polyethylene glycol (PEG) is a water-soluble wax. We selected PEG 600 because it is liquid at room temperature and freezes at a temperature easy to maintain in the laboratory (17°-20°C depending on the batch). The wax was injected into a tank of cold water, rather than air, in order to reduce the influence of surface tension and to increase the cooling rate. We added sucrose to the water in order to reduce the density contrast. A constant flow rate was forced by a peristaltic pump with flow integrators. The effects of smooth and rough bases and a bottom slope were studied.

The first goal of our experiments was to quantify the influence of surface crust on radial spreading. We compared our data for flow front position versus time for constant effusion rate experiments with those obtained by Huppert (1982) for the spreading of isothermal viscous oils in air. Figure 1 is a representative plot of dimensionless radius versus time for representative experiments, including an example in which no crust formed. The theory provides a good description of the data when no crust forms excepting that the logarithmic decay is very slightly smaller (0.46 instead of 0.5), probably due to an increase in the wax viscosity as it cools with time. The effect of cooling on the spreading rate is clearly seen: the stronger the cooling, the smaller the radius for a given time. Solidification of crust retards spreading of the front, causing the wax to pile up to greater depths. In addition, at relatively large times the trends of the spreading rate plots show distinct kinks. We attribute the kinks to a transition from a Newtonian viscous-buoyancy balance (with an effective viscosity to which the crust contributes) to Bingham behaviour in which the buoyancy forces must overcome a yield strength for flow to continue (Blake, 1989).

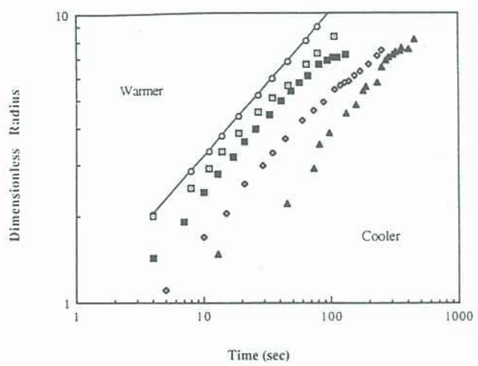

Fig.1. Dimensionless radius versus time for four experiments showing the decreased spreading rate and changes in morphology accompanying decreases in dimensionless temperature. The folding and rifting data sets exhibit distinct decreases in slope at large values of time. The large scatter in the pillow data reflects the irregular bud-like growth pattern of the flow. The straight line indicates the behaviour of isothermal, Newtonian flows. ⊡ - no crust; O - levees; ■ folds; ◊ - rifts; ▲ - pillows.)

SURFACE MORPHOLOGY

Our experiments generated a reproducible sequence of surface morphologies, and these resembled those found in lava flows. The structure depended on the relative rates of crust formation and radial spreading. For experiments with the slowest extrusion rates and lowest bath temperatures (smallest Π and highest Θ_s or lowest τ_s), cooling is most efficient and crust forms almost immediately as the wax emerges. This solid wax forms a connected shell (or tube if the flow is on a slope) which prevents the liquid interior of the current from advancing. This confinement raises the internal pressure in the dome or tube, stretching or bending the crust until its tensile strength is exceeded. At this point a break-out occurs, and the newly exposed interior wax again solidifies rapidly. This episodic process results in a collection of interconnected bulbous lobes, or pillows, whose lengths are inversely proportional to the temperature contrast ΔT. The extrusion builds up a dome which can be as high as it is wide. When dissected, the dome is found to consist of a large number of interconnected cavities of liquid wax.

For slightly lower cooling rates crust first appears at some distance from the centre. The liquid over the vent moves away at roughly the same rate that cust forms. The result is a rift-like structure with 2 or 3 wedge-shaped solid plates maintained at a constant distance from each other. The liquid wax continues to fill the radial gaps between the solid plates, and divergent flow within these zones forms striations and transform faults.

In runs with still lower cooling rates, solidification only occurs in narrow zones forming radial strips of thin crust. Because of the outward decrease in radial velocity the crust experiences radial compression. If this compression is strong enough, it results in surface folding. Eventually the folded strips may merge to form a continuous set of circular folds surrounding the vent (Fig.2).

For experiments with the lowest cooling rates crust appears only near the flow front. As a result, it does not undergo sufficient shortening to allow folding. The solid crust may be able to form a complete ring confining the current until the flow spills over or breaks through the levee.

There was usually only one dominant structure which culd be ascribed to any given set of emplacement conditions. This morphology is related to the flow rate and

Fig.2. Photograph of an experiment ($Pe=7.92x10^3$, $\Theta_s=0.84$, t=162 s) showing formation of folds in the case of a horizontal base. Wavelength of folds is approximately 0.3 mm.

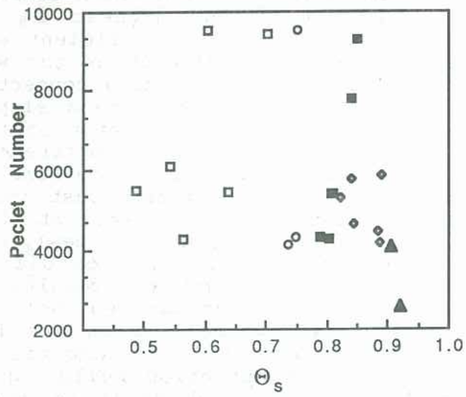

Fig.3. The morphology of the wax surface as a function of Pe and Θ_s for the case of a constant influx and a horizontal base. ⊡ - no crust; O - levees; ■ folds; ◊ - rifts; ▲ - pillows.)

temperature conditions represented by Pe and Θ_s (Fig.3). According to (7,8), however, use of the parameters $[\Pi,\tau_s]$ or Ψ allows more reliable extrapolation of our experimental results to other systems. Figure 4 shows the same data as Figure 3. Transitions between morphologic types are related to specific values of Ψ.

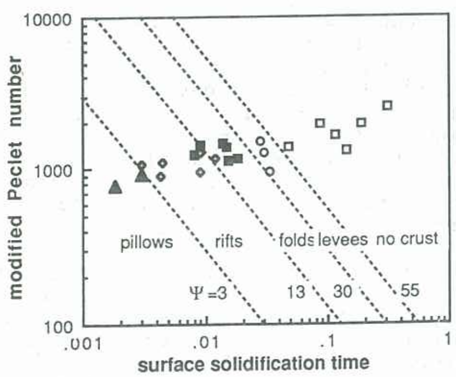

Fig.4. The same data as on Fig.3 but expressed as a function of the modified Peclet number (Π) and the dimensionless time to solidification (τ_s). Lines representing constant Ψ are a measure of the distance from the vent to the solidification front, and are predicted to separate the different morphologic fields. Symbols as in Fig.3.

SUMMARY

Laboratory experiments show that the rate of spreading of viscous-gravity currents is, in an initial phase, influenced by a solidifying crust in a manner which closely matches the spreading of a Newtonian viscous current having a greater effective viscosity. The effective viscosity increases with increasing solidification rate. At a later time a transition from the viscous-buoyancy balance to Bingham behaviour occurs. Buoyancy forces must thereafter balance the strength of the crust, and the speed of the nose decreases more rapidly.

In return, the viscous fluid applies stresses to the solid crust, leading to characteristic solidification patterns and deformation of the solid. The experiments indicate that this surface morphology is primarily controlled by flowrate and cooling rate. For a given flowrate (or Pe), progressively increasing the temperature contrast (Θ_s) leads to production of marginal levees, surface folds, rift-like fractures, or bulbous pillows. Analysis of the cooling of the viscous flow by high-Rayleigh number convection predicts that Pe and Θ_s can be combined into a single dimensionless parameter, Ψ, which describes the time taken to solidify the flow surface. The surface morphology appears to depend on this single parameter, even though the processes by which surface structures form are complex and involve strain rates and crust strength as well.

Application of these results in order to constrain emplacement conditions for natural lava flows on earth and elsewhere in the solar system will require calibration through detailed field observations and measurement of material properties. Initial comparison with the formation of submarine pillow basalts shows reasonable agreement. Radiative cooling of subairial flows may be treated in a similar fashion.

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