TWO-PHASE BUBBLY-DROPLET FLOW THROUGH A CONTRACTION: EXPERIMENTS AND A UNIFIED MODEL

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ABSTRACT

We have derived consistent and general equations describing the motion of two phases through a contraction. One phase is continuous, the other dispersed and the range of density ratios is wide. The single-phase limit and the homogeneous-flow limit are contained within our equations. Comparisons between modeling and experimental data for two density ratios of 1000 and 1.26 showed very good agreement in predicting pressure from input flowrates in vertical flow.

INTRODUCTION

Flow systems involving a mixture of gas and liquid or liquid and liquid occur commonly in the petroleum industry. Development of a flowmeter capable of extracting the individual flowrate of each of two components in a two-phase mixture irrespective of their density ratio or the inclination of the pipe is of great importance. One of the fundamental requirements for designing such a device is to formulate correctly a mathematical model for a two-phase flow system with a wide range of density ratios.

The mathematical model described here is a unified model being able to treat air-water as well as oil-water mixtures. It is also based on the interstitial velocity, i.e. the velocity of the unperturbed liquid between the bubbles, rather than on the average velocities as is often done in the segregated or two-fluid models. General-purpose models used for engineering applications today are two-fluid models. But these models do not allow for the effects on the flowrate of the liquid caused by the presence of flowing bubbles or droplets, and in particular by the relative velocity between the bubbles and the liquid. The approach and derivation of the equations of motion reported here follows and extends that given in [1] who considered these effects.

DEFINITIONS AND ASSUMPTIONS

We consider statistically stationary flow in a vertical pipe of varying cross-sectional area $A(x)$, where the x-axis is streamwise vertically up the pipe. It is assumed that the bubbles are rigid spheres all of the same density ρ_b and volume V_b and with a small diameter compared to the pipe diameter. The word 'bubbles' is to be thought of here as the dispersed component of the flow and refer to air bubbles in water flow or oil drops in water flow. There is no explicit effect of liquid- and bubble-density variations, unsteady drag, liquid turbulence generated by mean shear and bubble wakes, the bubble-wall and liquid-wall friction forces. The velocity profiles are flat, akin to a turbulent single-phase pipe flow. It is further assumed that all bubbles at a given cross section move with the same velocity.

In this one-dimensional system, all flow variables are taken to be spatial averages over the pipe cross section and are dependent on x. We use the subscript b or B to refer to the dispersed phase (bubbles) while the subscript l or L refers to the continuous phase (liquid). If there are n bubbles per unit volume, the void fraction for example, normally defined as the ratio of the volume of the light phase over the total volume of both phases, is simply $\epsilon_b = nV_b$ and the liquid holdup is just $\epsilon_l = 1 - \epsilon_b$.

Bubbles are assumed to move with a velocity u_b. The bubble-phase superficial velocity u_B is given by $u_B \equiv Q_b/A$, where Q_b is the volume flowrate of the dispersed phase. The average bubble velocity is then

$$\langle u_b \rangle \equiv \frac{Q_b}{\epsilon_b A} = \frac{u_B}{\epsilon_b} \,. \qquad (1)$$

Note here that $\langle u_b \rangle = u_b$ since all bubbles are assumed to move with the same velocity at a given cross section of the flow.

For the liquid, the superficial velocity u_L is given by $u_L \equiv Q_l/A$, where Q_l is the liquid volume flowrate. The liquid average velocity is then

$$\langle u_l \rangle \equiv \frac{Q_l}{\epsilon_l A} = \frac{u_L}{\epsilon_l} \,. \qquad (2)$$

The concept of the interstitial velocity u_l serves to represent the liquid velocity in the space between the bubbles, similar to a background velocity field determining the motion of any bubble in a low void fraction mixture. When a body moves uniformly through an infinite volume of incompressible inviscid fluid at rest, it induces a drift in the fluid such that the drift-volume of fluid is equal to $C_m V$, where V is the volume of the body and C_m is the added-mass coefficient. For a rigid sphere C_m has the value 0.5 (see [2]). For the present formulation, bubbles are modelled as rigid spheres and so the virtual-mass concept must also be introduced. Then the liquid flowrate across a surface area A is

$$Q_l = u_L A = [(1-\epsilon_b)u_l + \epsilon_b C_m (u_b - u_l)]A \,, \qquad (3)$$

Combining Eq.(2) and (3), the liquid average velocity can now be rewritten as

$$\langle u_l \rangle = u_l + \frac{\epsilon_b}{\epsilon_l} C_m (u_b - u_l) \,. \qquad (4)$$

EQUATIONS OF MOTION

Each component of the flow obeys its own mass-conservation law stating that the flowrate, once specified at the inlet, is constant throughout the pipe for steady flow and constant densities, i.e.

$$\frac{d}{dx}(Au_B) = 0 = \frac{d}{dx}(Au_L) \,. \qquad (5)$$

If there are n bubbles per unit volume, the momentum equation for the dispersed phase in an unsteady liquid flow can be written in terms of a generalised force equation as follows (see [3]):

$$\epsilon_b \rho_b \frac{D_b u_b}{Dt} = n F_b \,, \qquad (6)$$

where F_b is the resultant force acting on a rigid spherical bubble of volume V_b moving at a velocity u_b. The material derivative is defined as

$$\frac{D_b}{Dt} \equiv \frac{\partial}{\partial t} + u_b \frac{\partial}{\partial x} \,. \qquad (7)$$

If a bubble Reynolds number is defined in terms of the bubble diameter and slip velocity, we first consider the inviscid flow around a spherical bubble as in [3]. This is an appropriate model when the bubble Reynolds number is large (> 1000, equivalent to a diameter of $5mm$ and a rise velocity in water of 15 to $25cm/s$) and the water is pure. If the bubbles are also sufficiently small that there is local homogeneity in the flow velocity gradients, the bubble velocity will depend only on the interstitial liquid velocity and its first derivatives ([3]) and we then assume that drag can be added to the forces derived for inviscid flow and for a sufficiently low void fraction, that bubble-bubble interactions can also be neglected. F_b can thus be decomposed into four uncoupled contributions, $F_b = F_p + F_g + F_v + F_d$, which are now described.

The force F_p due to the pressure gradient in the liquid far from the bubble is given by

$$F_p = -V_b \frac{\partial p_l}{\partial x} = \rho_l V_b \left(\frac{D_l u_l}{Dt} + g \right) \,, \qquad (8)$$

where p_l is the undisturbed liquid pressure, g is the gravitational acceleration and

$$\frac{D_l}{Dt} \equiv \frac{\partial}{\partial t} + u_l \frac{\partial}{\partial x} \,. \qquad (9)$$

The gravitational force F_g exerted on the bubble in the absence of the liquid is simply $F_g = -\rho_b V_b g$.

The virtual-mass force F_v is the inertia force due to the local acceleration of the added mass of liquid travelling with the bubble (inviscid drag force), and is written as

$$F_v = -\rho_l V_b C_m \left(\frac{D_b u_b}{Dt} - \frac{D_l u_l}{Dt} \right) \,. \qquad (10)$$

The actual drag force F_d on the bubble is due to the viscous stresses changing the pressure distribution around the bubble; neglecting the non-uniformity and unsteadiness of the surrounding flow, F_d can be expressed in terms of V_t, the terminal rise (or fall) velocity of a bubble in an infinite stationary liquid (see [3]), as

$$F_d = -|\Delta\rho| V_b g \frac{(u_b - u_l)|u_b - u_l|}{V_t^2} \,, \qquad (11)$$

where $\Delta\rho = \rho_b - \rho_l$. Combining the above results and assuming steady flow, the momentum equation for the dispersed phase becomes

$$(\rho_b + \rho_l C_m) u_b \frac{du_b}{dx} - \rho_l C_m u_l \frac{du_l}{dx} =$$
$$-\frac{dp_l}{dx} - \rho_b g - g|\Delta\rho| \frac{(u_b - u_l)|u_b - u_l|}{V_t^2} \,. \qquad (12)$$

In the limit $\rho_b \ll \rho_l$ (i.e. for a gas-liquid system) flow, the one-dimensional momentum equation for a bubble reduces precisely to Eq.(28) in [1].

The liquid average pressure $\langle p \rangle$ can be written in terms of the interstitial pressure, p_l, i.e. the liquid pressure between the bubbles on a surface across the flow, as

$$\langle p \rangle = p_l - \frac{\rho_l C_m \epsilon_b (u_b - u_l)^2}{2\epsilon_l} \,. \qquad (13)$$

Eq.(13) together with Eq.(4) clearly illustrate the difference in going from an average-variable formulation to Eq.(12) by introducing extra terms of order ϵ_b.

It becomes apparent that Eq.(12) is identical to an average-variable equation in the cases when ϵ_b tends to zero (single-phase limit) or when the slip velocity $u_b - u_l$ is zero (homogeneous-flow limit).

The momentum-balance equation for the liquid component can be derived mathematically by integrating the Navier-Stokes equations over a fixed control volume of fluid containing and possibly intersecting bubbles (see [1]). The final result is an equation similar to that for the dispersed phase, where the rate of change of liquid momentum is equal to the sum of various forces acting on the control volume:

$$\rho_l (2u_l - \epsilon_b u_b) \frac{du_l}{dx} + \rho_l u_l^2 \frac{1}{A} \frac{dA}{dx} - \epsilon_b \rho_l C_m \left(u_b \frac{du_b}{dx} - u_l \frac{du_l}{dx} \right)$$
$$+ H = -\epsilon_l \frac{dp_l}{dx} - \epsilon_l \rho_l g + \epsilon_b g |\Delta\rho| \frac{(u_b - u_l)|u_b - u_l|}{V_t^2}$$

with

$$H = \frac{\rho_l C_m}{10} \left\{ 4 (u_b - u_l)^2 \frac{\epsilon_b}{\epsilon_l} \frac{1}{A} \frac{dA}{dx} - \frac{d}{dx} \left[(u_b - u_l)^2 \frac{\epsilon_b}{\epsilon_l} \right] \right\}$$

Here also, it is easy to see this equation reduces to Bernoulli's equation when the slip velocity vanishes (homogeneous-flow limit) as well as for the single-phase flow limit ($\epsilon_b = 0$).

To arrive at a two-component pressure equation, the two momentum equations above are combined by eliminating the drag term between the two equations. This leads to:

$$\frac{dp_l}{dx} + (\epsilon_l \rho_l + \epsilon_b \rho_b) g =$$
$$-\rho_l (2u_l - \epsilon_b u_b) \frac{du_l}{dx} - \rho_l u_l^2 \frac{1}{A} \frac{dA}{dx} - \epsilon_b \rho_b u_b \frac{du_b}{dx} - H \quad (14)$$

Similarly, a velocity equation can be derived by eliminating the pressure gradient between the two momemtum equations:

$$\rho_l (2u_l - \epsilon_b u_b) \frac{du_l}{dx} + \rho_l u_l^2 \frac{1}{A} \frac{dA}{dx} - \rho_l C_m \left(u_b \frac{du_b}{dx} - u_l \frac{du_l}{dx} \right) -$$
$$\epsilon_l \rho_b u_b \frac{du_b}{dx} + H = g \Delta\rho \left\{ \epsilon_l + \frac{|\Delta\rho|}{\Delta\rho} \frac{(u_b - u_l)|u_b - u_l|}{V_t^2} \right\} \quad (15)$$

It is worth noting that Eq.(14) reduces to the single-phase Bernoulli equation for ϵ_b equal to zero and collapses to a "two-phase Bernoulli equation" when $u_b = u_l$, with ρ_l being replaced by the mixture density $\rho_m \equiv \epsilon_b \rho_b + \epsilon_l \rho_l$.

EXPERIMENTS AND COMPARISONS

The experiments described in this paper were performed in a multiphase flow loop in Schlumberger Cambridge Research. The working section is made up of $76.2mm$ nominal bore clear acrylic plastic pipe. The generated air bubbles or oil droplets were about $5mm$ in diameter which gives a ratio of pipe size to bubble size roughly around 15.

In order to fully characterise the pressure behaviour in the straight and contracted sections, the pressure variation along the length of the pipe was measured using eight tappings (Fig.1), of which three provided a pressure drop measurement in the straight section and across the contraction. The tappings themselves could be connected selectively to a single differential pressure transducer by means of a fluid switch. Pressure profiles along the pipe were measured for an air/water or an oil/water mixture and for a range of superficial velocities u_L and u_B. The flow conditions were chosen to cover a representative range of void fractions, and to allow parametric comparisons (for example constant liquid flowrate with varying gas rate). The conditions tested are listed in Table 1.

Pressure profiles along the pipe as predicted from the model are shown in Fig.2 and Fig.3. On each graph the associated experimental points are also shown. In general it can be seen that the agreement is good, both for air/water (Fig.2) and oil/water (Fig.3) cases. Some detail effects are present in the experiments which are not represented in the model. For example the 'overshoot' in the pressure at $x = 1.0$ in each case is the sign of a small flow separation which occurs at the end of the conical contraction due to the sharp corner leading into the subsequent straight section of pipe. This

Table 1: **Conditions for pressure measurements**

Water superficial velocity (m/s)	Air superficial velocity (m/s)	Oil superficial velocity (m/s)
0.97	0.65	
0.97	0.4	
0.97	0.15	
0.48	0.47	
0.09	0.08	
1.35	0.37	
0.583		0.391
0.291		0.383
0.293		0.205
0.770		0.096
0.088		0.089
0.215		0.494

would obviously not be seen in the one-dimensional model since the tappings are not actually modelled numerically but the pressure is obtained as a field throughout the pipe.

Fig.4 shows the percentage difference for the model predictions relative to the experimental pressure drops across the contraction versus bubble void fraction; the left graph is for the air/water cases and the right one for the oil/water cases. It is clear that the model is accurate to better than 10% up to 30% bubble void fraction. In the dilute region (<10%) the agreement in both cases is excellent. Since the model assumes no bubble-bubble interactions other than those caused by the change in interstitial velocity, it is not surprising that above 20% bubble void fraction there is an increasing discrepancy between model and experiment. The variation seems to be quadratic, consistent with the limitations of the model. Note also that the accuracy within 5% for the single-phase cases quantifies essentially the variability in the experimental data relative to Bernoulli's equation for loss-free flows. It is also apparent that the experimentally derived single-phase discharge coefficient of 0.984 (i.e. an underestimate of 1.6%) is consistent with the zero-void fraction limit of the two-phase results.

CONCLUSION

We have derived a consistent and general set of equations to describe the motion of two phases in flow through a contraction. One phase is continuous, the other dispersed and the range of density ratios is wide. Given the density and flowrate of each component of the flow, the pressure, velocities and void fractions can be computed at any location along the pipe. The single-phase limit as well as the homogeneous-flow limit are both contained within our set of equations. The full mathematical model requires only the terminal rise velocity of a single bubble, V_t. This is in contrast to simpler models where extensive empirical calibrations are required. The use of the interstitial velocity and virtual-mass concept removes the empiricism between the relative velocity, the void fraction and the terminal rise velocity.

Comparisons with experiments demonstrate that, although the model is one-dimensional and neglects local bubble-bubble interactions, it is nevertheless robust in dealing with vertical flow and a wide range of density ratios. V_t was obtained from standard correlations for the fluids used and was not artificially adjusted for these specific experiments. Both when $\rho = 1000$ or $\rho = 1.26$, the agreement between the mathematical predictions and the experiments was good in the vertical flow cases. The implication of this conclusion is that the model should perform well at any intermediate values of density ratio.

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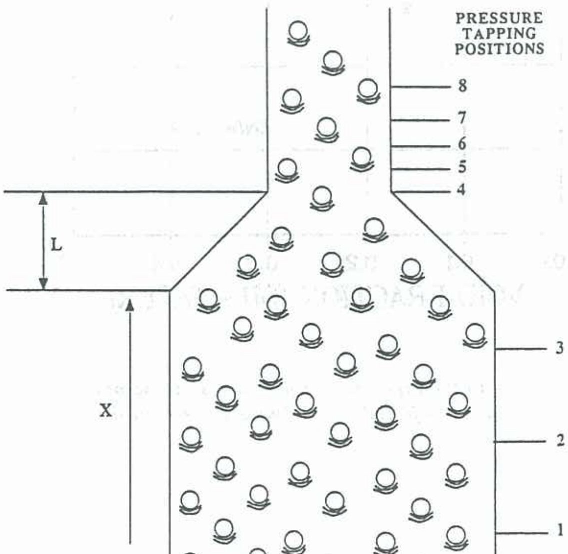

Figure 1 *Contraction geometry and pressure tappings.*

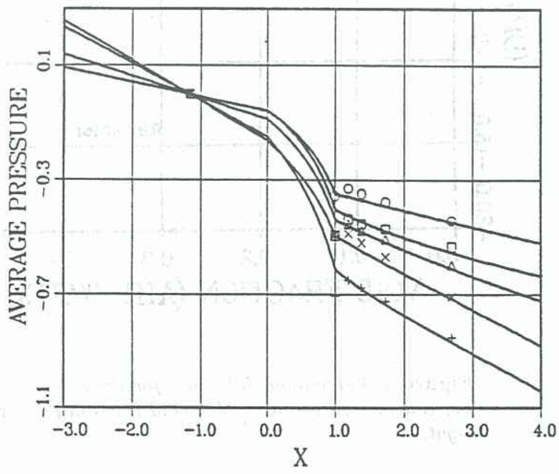

Figure 2 *Liquid pressure versus streamwise coordinate x for air/water cases. □ water u_L in the upstream section 1.35m/s, air u_B in the upstream section 0.37m/s; ○ water 0.97m/s, air 0.65m/s; △ water 0.97m/s, air 0.4m/s; + water 0.97m/s air 0.15m/s; × water 0.48m/s, air 0.47m/s.*

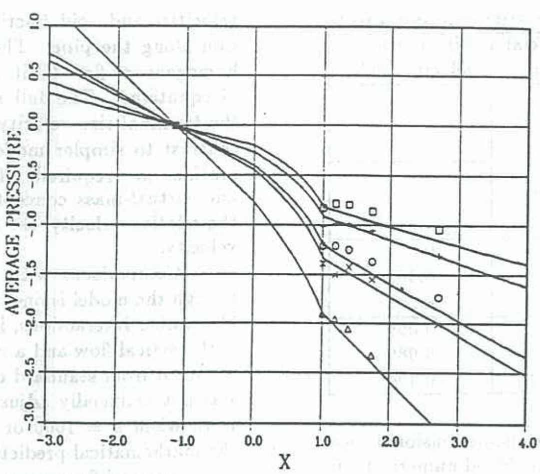

Figure 3 *Liquid pressure versus streamwise coordinate x for oil/water cases. □ water u_L in the upstream section 0.58m/s, oil u_B in the upstream section 0.39m/s; ○ water 0.29m/s, oil 0.38m/s; △ water 0.29m/s, oil 0.21m/s; + water 0.77m/s, oil 0.096m/s; × water 0.22m/s, oil 0.49m/s.*

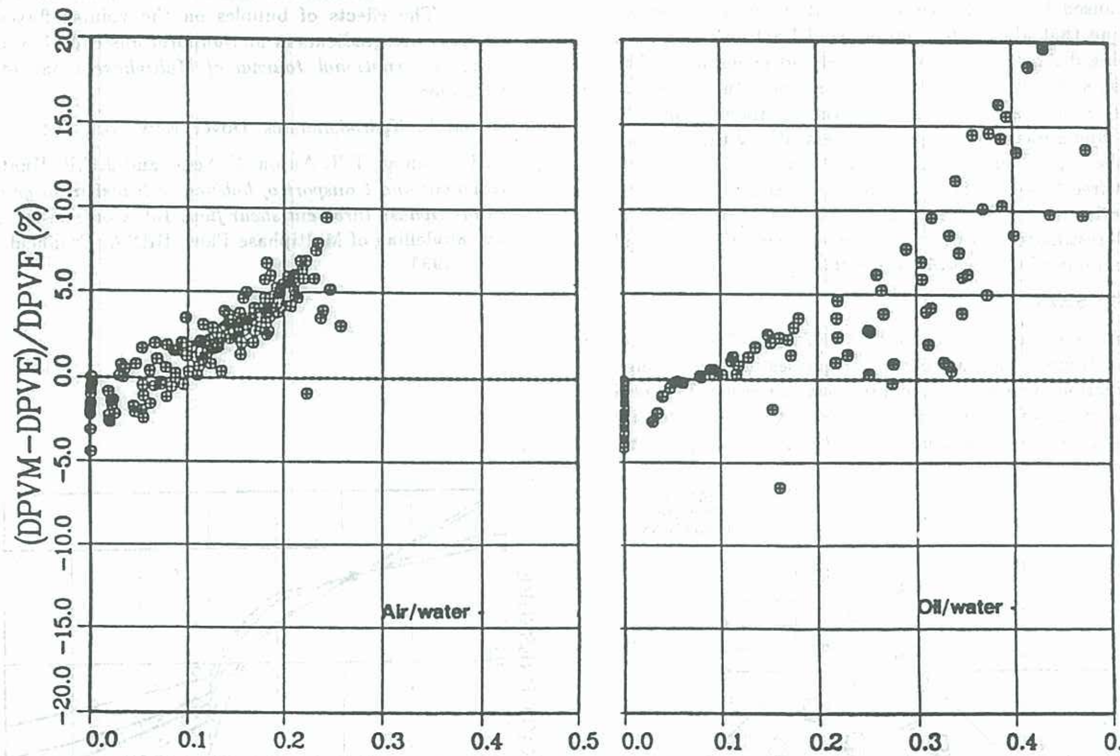

Figure 4 *Percentage difference for the model predictions relative to the experimental pressure drops across the contraction versus bubble void fraction for air/water cases on the left and for oil/water cases on the right.*