TENTH AUSTRALASIAN FLUID MECHANICS
CONFERENCE - UNIVERSITY OF MELBOURNE
11-15 DECEMBER 1989

A POSSIBLE RECIRCULATION LIMIT FOR EXPANDING SWIRLING FLOWS

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ABSTRACT
A simple, approximate criterion based on inlet conditions is proposed for the onset of recirculation in an expanding, swirling flow. It is compared with a criterion for vortex breakdown and one for recirculation in rapidly expanding swirling flows. Since recirculation is caused by the development of circumferential vorticity from the upstream axial vorticity, its occurrence is likely to be sensitive to the initial distributions of those vorticities.

1. INTRODUCTION

Spall *et al.* (1988) proposed a criterion for vortex breakdown in swirling flows, based on the Rossby number (Ro) just prior to breakdown. However, there are some swirling flows, such as in axisymmetric diffusers, where it would be more useful to have a criterion based on the inlet conditions only. Furthermore, their criterion may well be inappropriate for expanding flows. The reason is that "vortex breakdown", interpretting the term loosely to cover *any* occurrence of zero axial velocity, can be forced by the expansion only, so the physical processes involved may well differ from those in flows of approximately constant cross-section.

Only the simplest possible case of swirling inlet flow will be considered. The initial flow (before expansion) is assumed to have a constant axial velocity, U_i, and solid body rotation, so that W_i, the initial circumferential velocity, is given by $W_i = \omega r$, where r is the radius. From the breakdown criterion for high Reynolds number "wing-tip" vortices given by Spall et al. (1988), this flow will be non-recirculating provided

$$Ro_i \equiv U_i / \omega r_i > 0.65 \quad \text{approximately,} \tag{1}$$

where r_i is the outside radius of the rotating flow. (Here and throughout this paper, the subscript "i" denotes initial or inlet conditions.) If Ro_i is greater than 0.65 it is still possible for recirculation to occur in the expanding downstream flow. This is most easily demonstrated using the Bragg-Hawthorne equation for the streamfunction, Ψ, in an *inviscid* flow with a prescribed distribution of initial vorticity, Batchelor (1967). For the initial conditions described above, the equation is

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial r^2} - \frac{1}{r}\frac{\partial\psi}{\partial r} = \frac{2\omega^2 r^2}{U_i} - \frac{4\omega^2}{U_i^2}\psi \tag{2}$$

The physical explanation for the tendency towards recirculation comes from noting that the right hand side of equn (2) is $-r\,\Omega_\theta$, where Ω_θ is the circumferential vorticity. Along a streamline which has a radius r_0 in the initial flow, and a radius r in the expanding flow,

$$\Omega_\theta(\psi) = -2\omega^2 r\,(1 - r_0^2/r^2)/U_i\,. \tag{3}$$

Ω_θ is zero initially, but goes negative as the flow expands, and r increases above r_0. Thus $\partial U/\partial r$ is positive over most of the flow, and U_a, the streamwise velocity along the axis, decreases monotonically with axial distance. This conversion of axial into circumferential vorticity can be viewed as the inverse of the more common formation of secondary flow in an initially two-dimensional shear layer by a similar and also essentially inviscid process, eg Bradshaw (1987). The demonstration of its importance implies that the tendency towards recirculation in any flow will be strongly dependent on the initial distribution of U and W.

There are other considerations which suggest the inappropriateness of a Ro criterion for expanding flows. Firstly, the solution to equn (2) which is discussed below, indicates that Ro as defined by Spall et al. (1988) tends to infinity as U_a goes to zero. Secondly, there is the following dimensional argument. Continuity requires a typical U to vary as r^{-2} in the expanding flow, while conservation of angular momentum forces W to be proportional to r^{-1}. As r increases, the *local* Ro therefore decreases as Ro_i/r. The experimental results used below suggest this decrease is too slow to be used as a criterion for recirculation. The apparent contradiction between this and an infinite Ro is an indication of the difficulty in defining local parameters when there are large axial and radial gradients. This alone suggests the need for a criterion based on well-defined inlet conditions.

The upper limit for expanding flows is provided by sudden expansions for which Hallett (1988) gives a criterion in terms of (in effect) Ro_i and the expansion ratio, the ratio of the downstream to inlet radius. His criterion is more complicated than the one derived here, but is adequate for the experiments considered. However, his analysis implies that there is a value of Ro (about 1.4) above which the downstream flow *cannot* recirculate. This seems unlikely and may be traceable to his use of assumed downstream profiles that do *not* conserve circulation along a streamline. For combustor applications where there is considerable turbulence, this non-conservation may be justified, but it cannot be for the flows considered here. It appears, therefore, that different flows will need different recirculation or breakdown critera. At present, the choice of an appropriate criterion is not clear, and will remain so until more experimental results are available.

The next Section describes the simple analysis based on equn (2). This is followed by a description of the available experimental results. Section 4 describes the combining of the analysis and measurements, while the last Section gives the major conclusions.

2. ANALYSIS

It seems that the only appropriate analytic solution to equn (2) is for the case where $\partial^2\Psi/\partial x^2$ can be neglected. From equn (7.5.24) of Batchelor (1967), U_a is given by

$$U_a / U_i = 1 + (1 / r^2 - 1) (2r / Ro_i) J_1 (2r / Ro_i) \ . \qquad (4)$$

Here J_1 is the Bessel function of the first kind of order one and, for the rest of this paper, r denotes the (local) outside radius of the expanding flow normalised by the initial radius. Equn (4) allows U_a to go through zero, and a recirculation zone to form. The theory is not valid within that zone, because the right hand side of equn (2) applies only to streamlines originating in the initial flow. However, determining the *onset* of recirculation in terms of the critical radius, r_c, for which $U_a = 0$, does not violate the constraint on the origin of the streamlines. With r_c also normalised by r_i, we have

$$r_c^2 = u / (u-1), \quad \text{where } u \equiv r_c / [Ro_i J_1(2r_c/Ro_i)] \qquad (5)$$

so that r_c is the critical expansion ratio. Equn (5) was derived also by Bossel (1968), but is an immediate consequence of Batchelor's prior analysis. It has two limits that must be considered. Firstly, as $2r_c / Ro_i \rightarrow j_{1,s}$, the first zero of J_1, $u \rightarrow \infty$, $r_c \rightarrow 1$, and $Ro \rightarrow 2 / j_{1,s} = 0.522$. Contrary to the explanation following equn (3), this suggests a criterion for vortex breakdown for flows of constant cross-section. It will be shown later to be inappropriate. Secondly, as $r_c/Ro_i \rightarrow 0$, it follows from the series expansion of J_1, eg equn (9.1.10) of Abramowitz and Stegun (1955), that

$$Ro_i / r_c^2 \rightarrow 2^{-1/2} \qquad (6)$$

Strictly, the second limit should be obtained from equn (4), because of the possibility that U_a may not go to zero for insufficiently small swirl, but it is easy to show that this would not effect equn (6). The two limits suggest that a value of Ro_i/r_c^2 of between 0.5 to $2^{-1/2}$ may give a general, approximate criterion for the onset of recirculation. Fig. 1 shows equns (1) and (6), together with the general solution for Ro_i/r_c^2 from equn (4).

3. EXPERIMENTAL RESULTS

Also included in Fig. 1 are the experimental results of Hagiwara *et al.* (1986) and Clausen (1987), the only available results suitable for testing the proposed criterion. No correction in either case was made to r_c for the effects of the wall boundary layer, so the resulting Ro_i/r_c^2 are slightly low. In both experiments, the inlet flow closely approximated the ideal assumed here, and the expansion was acheived by a conical diffuser with an included angle of 20°. Hagiwara et al. (1986) varied the Reynolds number by a factor of five without significantly affecting the onset of recirculation, while Clausen's (1987) results were taken at an even higher Reynolds number. Thus the inviscid analysis of the previous Section should be of some use. Hagiwara et al. (1986) also tested a variety of other diffuser geometries, some of which will be referred to below. Clausen's (1987) measurements were obtained on the centre-line at the diffuser outlet, where recirculation should first occur. A PELA Flow pulsed-wire anemometer was used to measure the axial and circumferential velocities. The probe of this instrument contains a central wire that is pulsed to provide a heat tracer. There are two sensing "cold-wires", located upstream and downstream of the pulsed wire which are used to determine the time of flight of the heat tracer and the direction in which it travels. Apart from the possible problem of flow interference, the pulsed wire anemometer has two main disadvantages. It is restricted to velocities less than about 12 m/s and has a comparatively large measuring volume, but neither of these is critical for the present application. The results in Fig. 1 were obtained by increasing Ro_i until recirculation was detected by the pulsed wire anemometer. The complete exit profiles corresponding to the conditions in Fig. 1 can be found in Clausen & Wood (1987). The unfilled symbols in Fig. 1 denote non-recirculating flow. Fig. 2, taken from Clausen (1988), shows the experimental and theoretical variation of U_a with Ro_i/r^2 and the percentage time that the measured instantaneous axial velocity, u_a, lay within half a

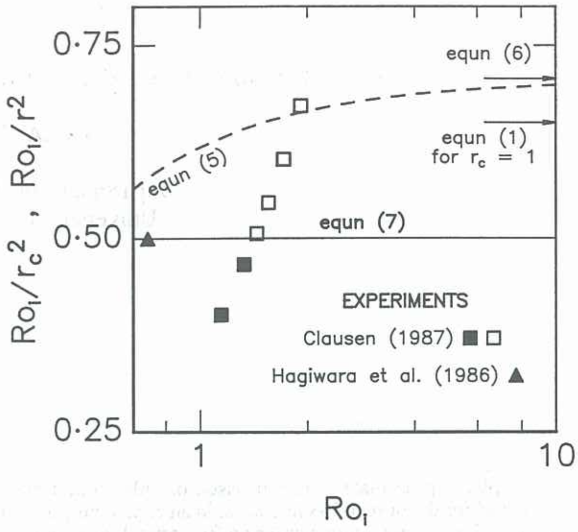

Figure 1..Ro_i/r_c^2 and Ro_i/r^2 plotted against Ro_i. Filled symbols denote recirculating flow, unfilled symbols non-recirculating flow. Note that the minimum Ro_i on the horizontal axis is 0.65.

digitising window of zero. The large increase in that percentage as Ro_i/r^2 falls below about 0.5 indicates the rapid onset of instantaneous flow reversals; these were measured, but their percentage occurrence was not recorded. Because of this rapid rise, and the more gradual reduction in U_a as it approaches zero, the appearance of instantaneous flow reversals has been chosen as the indicator of recirculation, even though the theoretical development given above relates only to mean flows. The first value of Ro_i for which recirculation occurred was 1.45. Since r_c was 1.69, $Ro_i/r_c^2 = 0.47$ or $Ro_i/r_c = 0.86$. The measurements of Hagiwara et al. (1986) were made with a laser-Doppler anemometer, with the aim of studying the recirculation zone for combustion applications. The result included in Fig. 1 was read from their Fig. 4.11 which gave the mean streamlines for a recirculating flow, so that the result lies on the boundary of the recirculation zone. Ro_i and r_c were 0.7 and 1.2 respectively, giving $Ro_i/r_c^2 = 0.49$ and $Ro_i/r_c = 0.58$.

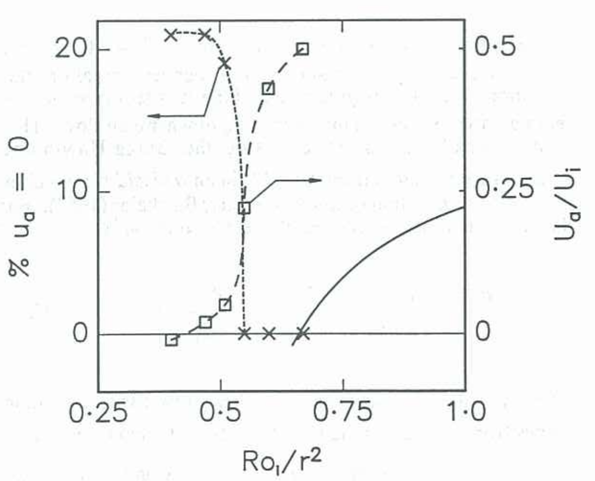

Figure 2..Measured percentage of zero u_a, X ; and measured variation of U_a/U_i, , from Clausen (1988). Solid line shows equn (4). Dashed curves for visual aid only.

4. DISCUSSION

The criterion of equn (6) appears conservative, in other words, it under-estimates r_c for a given Ro_i. Fig. 1 suggests recirculation will not occur provided

$$Ro_i / r_c^2 > 0.5 \quad \text{approximately.} \qquad (7)$$

This correlation appears preferable to one based on a local Ro as Ro_i/r_c was 0.86 for Clausen's (1988) results but only 0.58 for the other case. Equn (7) must be considered preliminary, because of the small amount of experimental data on which it is based and the difficulty in defining precisely the onset of recirculation. It must also be considered approximate, given the possible differences in the shape of the expanding flow, the steadiness of, and the amount of turbulence in, the initial flow. Probably, the only major simplification is that, in practice, Ro_i is should not be much different from unity. Equn (7) is likely to be in error as $r_c \rightarrow 1$, as it does not reduce to equn (1). We note that equn (2) has been used to investigate vortex breakdown, without relying on equn (3), but these attempts have been severely criticised by Liebovich (1984) because (2) does not allow for non-axisymmetric modes.

Hagiwara et al. (1986) showed that the onset of recirculation was approximately independent of the shape of the diffuser wall *downstream* of r_c. This result gives some confidence that elliptic effects, which are included in equn (2) but not in the reduced form considered subsequently, are not of first order. The effect of the upstream wall shape was not studied in detail, but their measurements for a 35° diffuser suggest a slightly lower value of Ro_i/r_c^2 for the same Ro_i (0.71) as in Fig. 1. Clausen's (1988) finite-difference solutions to equn (2) for a fixed diffuser shape showed that the x-dependent term did not have much effect at $Ro_i \sim 1$ but reduced Ro_i/r_c^2 by around 25% at $Ro_i \sim 4$. The full equation, therefore, is likely to give a distribution of Ro_i/r_c^2 which is more constant and lower than from equn (6). However, the distribution must depend to some extent on the actual geometry of the expansion, which is not accounted for in equn (5). Generally, the importance of the x-dependent term is likely to increase as r_c increases, but only if the expansion is rapid (just as the adequacy of the boundary layer approximation depends on the rate of growth of the boundary layer thickness rather than the thickness alone). It was also mentioned that Hallett's (1988) analysis of the rapid case, is strictly not applicable to inviscid flow. Furthermore, the use of the reduced form of the normal momentum equation, $\partial P/\partial r = \rho W^2/r$, is unlikely to be adequate in rapidly expanding flows. Therefore it is not possible to assess either the lower limit of applicability of equn (7) (as $r_c \rightarrow 1$), or the upper limit in terms of r_c or Ro_i/r_c^2.

5. CONCLUSIONS

We have considered the onset of recirculation in the expanding flow whose initial conditions comprise a uniform axial velocity and solid-body swirl. It was suggested that recirculation occurs whenever Ro_i/r_c^2 falls below 0.5. This criterion combines the features of an inviscid theoretical analysis with some recent experimental results, and is based on the inlet conditions only. The latter feature is an advantage over the local criterion of Spall et al. (1988) for vortex breakdown, which may be limited to flows of approximately constant cross-section. The paucity of experimental results precluded an assessment of the bounds of applicability of the two critera. Hallett's (1988) analysis of rapidly expanding flows was shown not to be applicable to those considered here, so that the upper limit on the present criterion is not known. It was emphasised that the conversion of axial into circumferential vorticity which forces recirculation should be sensitive to the initial distribution of these quantities. Thus, the present criterion may turn out to be appropriate only to those initial flows which have a nearly-constant axial velocity and solid body rotation.

ACKNOWLEDGEMENTS
This work was supported by the Australian Research Council. The experimental work was done while P.D.C. held a Commonwealth Postgraduate Scholarship.

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