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UNDERWATER EXPLOSION BUBBLE DYNAMICS IN THE NEIGHBOURHOOD OF BOUNDARIES

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ABSTRACT

Models of underwater explosion bubbles that assume a spherically symmetric bubble fail badly in the neighbourhood of boundaries. Perturbation methods are inadequate as the bubble experiences large deformations of shape with high speed jet penetration being observed.

The Boundary Integral Method (BIM) provides an efficient scheme for the computation of bubble motion in the neighbourhood of boundaries. In this study the BIM is used to model the growth and collapse of an explosion bubble in the neighbourhood of a rigid boundary. Variations in the parameters governing the motion reveal a number of interesting phenomena, in particular the possibility of bubble rebound and the growth of liquid jets upon rebound. Observed behaviours may be plausibly explained by considering the fluid momentum via the concept of the Kelvin impulse.

INTRODUCTION

When an explosion occurs underwater the gaseous remnants of the detonation are observed to be confined to a region which, in the initial stages of the motion, may be considered as spherical. Subsequently the bubble expands and collapses during which time great volumes of water are set in motion, a motion with the potential to cause great damage to marine craft.

The earliest models of explosion bubble phenomena considered a spherically oscillating bubble in an infinite fluid (Rayleigh (1917)). These simple models predict high speed fluid motion, especially during the collapse phase of the bubble. Experimental studies of small scale bubble phenomena have shown this high speed fluid motion but have also established that any asymmetry in the flow field, whether it be asymmetry due to gravity or motion induced by nearby boundaries, induces asymmetric bubble collapse with high speed jets often observed to form and penetrate the bubble (Gibson (1968), Benjamin and Ellis (1966)).

Recent theoretical studies have successfully predicted this phenomena in the restricted case of axisymmetric flow. It is clear that the pressure gradient due to gravity produces a jet directed in opposition to it whereas the flow field induced by oscillations in the neighbourhood of a rigid boundary tends to induce jet formation directed towards the boundary (Guerri et.al. (1981), Blake et.al (1986), Wilkerson (1989), Best and Kucera (1989)).

The model used for the cavitation bubbles of previous theoretical studies assumes a vapour of constant pressure within the bubble. In contrast to this the explosion bubble contains the compressible remnants of detonation. It is of fundamental interest as to the effect that the presence of compressible gas within the bubble will have upon jet formation. In particular the question arises as to whether the high pressures generated within the bubble during its contracted phase are sufficient to arrest the jet formation process. Also of fundamental interest is the manner in which the boundaries and buoyancy couple in the jet formation process. In the case where the bubble lies underneath the rigid boundary (a simplified model of an explosion beneath a ship hull) both buoyancy and the presence of the boundary will induce jet formation directed towards the boundary. In the case where the bubble is above the boundary (an explosion in the neighbourhood of the ocean floor) buoyancy and boundary effects are oppositely directed.

In this paper such questions are investigated theoretically via the Boundary Integral Method. Variations in the basic parameters which govern the motion result in a range of interesting behaviours, providing considerable insight into explosion bubble collapse phenomena.

THEORY

We will assume that the fluid is inviscid and incompressible and that the flow induced by the explosion bubble's oscillatory motion is irrotational ("Triple I" fluid). In this case we may introduce a velocity potential, ϕ, which is a solution of Laplace's equation. The remnants of the detonation are assumed to be ideal and to undergo adiabatic processes as the bubble oscillates. These gaseous products are thus characterised by an adiabatic coefficient γ which is typically chosen to be 1.25 (TNT). We may then write

$$p = p_0(v_0/v)^\gamma, \qquad (2.1)$$

where p and v are the pressure within and volume of the bubble respectively. The subscript "o" denotes initial quantities.

We will consider axisymmetric motion in the two geometries as shown in FIG.1. The motion may occur below a rigid boundary to simulate an explosion beneath a ship hull (FIG. 1a) or above a rigid boundary as in the case of an explosion in the neighbourhood of the ocean floor (FIG. 1b). In each case we impose the rigid boundary condition that there is no flow perpendicular to the rigid surface.

If we let $\mathbf{p}$ denote the position vector of some point in the domain, Ω, of the flow, then with the help of Green's Theorem we may write the solution to Laplace's equation as

$$c(\mathbf{p})\phi(\mathbf{p}) = \int_S \left(\frac{\partial \phi}{\partial n}G - \phi\frac{\partial G}{\partial n}\right) dS, \qquad (2.2)$$

with

$$c(\mathbf{p}) = \begin{cases} 2\pi & \text{if } \mathbf{p} \in S \\ 4\pi & \text{if } \mathbf{p} \in \Omega. \end{cases} \qquad (2.3)$$

The Green's function G is

$$G = \frac{1}{|\mathbf{p}-\mathbf{q}|} + \frac{1}{|\mathbf{p}-\mathbf{q}'|} \qquad (2.4)$$

where $\mathbf{q}$ is the position vector of a point on the surface S, $\mathbf{q}'$ is the position vector of the image of $\mathbf{q}$ reflected about the rigid boundary and $\frac{\partial\phi}{\partial n}$ is the normal derivative of the potential on S. In applications to bubble dynamics we proceed to develop an algorithm for the solution of (2.2) by choosing S as the surface of the bubble. We select node points (r_i, z_i) (cylindrical co-ordinates) on the surface and represent the bubble shape using cubic splines. We choose cubic and linear representations for ϕ and $\frac{\partial\phi}{\partial n}$ respectively. Collocation at the node points yields a system of linear equations from which we can determine $\frac{\partial\phi}{\partial n}$ at the bubble surface, given the surface shape and distribution of potential on it.

If we denote by $\mathbf{X}(t)$ the bubble surface then knowledge of ϕ and $\frac{\partial\phi}{\partial n}$ here allows calculation of the fluid velocity $\mathbf{u}(\mathbf{X},t)$ at the bubble surface. The differential equation,

$$\frac{d\mathbf{X}}{dt} = \mathbf{u} \qquad (2.5)$$

describing the evolution of the bubble surface is then solved using the 4th order Runge-Kutta method. In this manner we follow the motion of the node points in time. Similarly we solve the differential equation

$$\frac{D\phi}{Dt} = \frac{\partial\phi}{\partial t} + |\nabla\phi|^2 \qquad (2.6)$$

describing the evolution of the potential on the bubble surface, where we eliminate $\frac{\partial\phi}{\partial t}$ by making use of the Bernoulli equation evaluated at the surface of the bubble,

$$\frac{\partial\phi}{\partial t} + \frac{1}{2}|\nabla\phi|^2 + \alpha\,(v_0/v)^\gamma + \delta^2\,(z-\xi) - 1 = 0. \qquad (2.7)$$

In this equation we have introduced a length scale $\bar{R}$ (typically of the order of the bubble radius) and scaled time with respect to $\bar{R}\sqrt{\rho/p_\infty}$, where p_∞ is the far field pressure at the height ξ at which the motion is initiated. The vertical co-ordinate is denoted by z. The dimensionless parameters α and δ are defined as

$$\alpha = p_0/p_\infty, \quad \delta^2 = \rho g \bar{R}/p_\infty. \qquad (2.8)$$

The parameter α is a measure of the strength of the explosion and δ determines the significance of buoyancy forces.

A variable time increment Δt is used as the solution proceeds in time with Δt given as

$$\Delta t = \frac{\Delta\phi}{\max\limits_S\left[1 + \alpha\,(v_0/v)^\gamma + \delta^2\,|z-\xi| + \frac{1}{2}|\mathbf{u}|^2\right]}, \qquad (2.9)$$

with $\Delta\phi$ constant. Having chosen Δt in this manner the change in potential at any point on the bubble surface for the given time iteration is bounded above by $\Delta\phi$. Furthermore, the above choice of Δt provides a most efficient use of computational resources.

Finally we impose initial conditions upon our system. We suppose that initially the bubble is spherical with an initial potential that vanishes everywhere on the bubble surface, corresponding to a stationary surface. The motion is thus generated impulsively from rest by the very high initial pressure p_0.

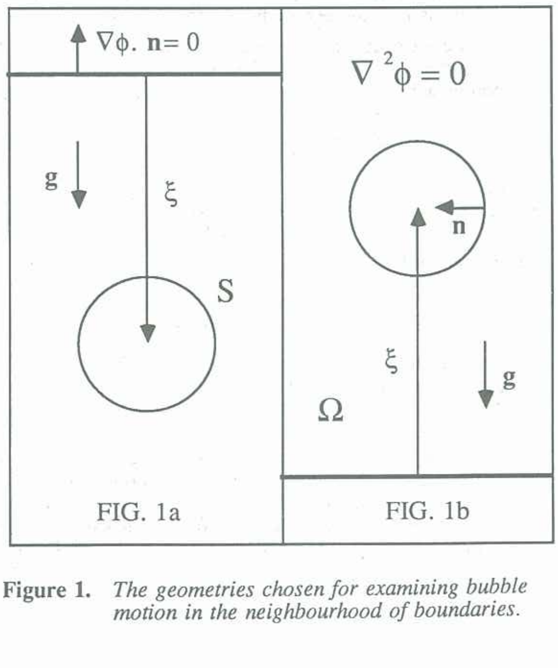

Figure 1. *The geometries chosen for examining bubble motion in the neighbourhood of boundaries.*

COMPUTATIONAL RESULTS

The motion of explosion bubbles characterised by $\alpha = 50.0$ and in the neighbourhood of rigid boundaries has been determined via the Boundary Integral Method of the previous section. The profiles of the bubble shapes at successive times are shown in FIG's 2(a),...,(f). It should be noted that in all the cases presented the expansion phase of the motion is not shown, as it is during this phase of the motion that the bubble remains spherical to a good degree of approximation. The outermost profiles in FIG's 2(a),...,(e) show approximately the maximum size of the bubbles, which have grown from an initial radius, R_0, of 0.1. In all the cases presented the detonation occurred at $\xi = \pm 1.0$ ($+1.0$ for detonation above the rigid boundary, -1.0 for detonation below the rigid boundary).

Figure 2(a) shows the collapse phase in the case of no buoyancy, that is $\delta = 0.0$. As the bubble collapses the greater mobility of the flow away from the boundary leads to a faster flow at the pole of the bubble furthermost from the boundary. The resultant perturbation in the surface shape grows rapidly, forming a high speed liquid jet that rapidly penetrates the bubble. In fact the jet formation and penetration of the bubble occurs in about 3% of the bubble lifetime (the time from denotation to the instant of complete penetration).

It is conjectured (Holt (1977), Benjamin & Ellis (1966)) that the bubble then evolves into a toroidal bubble with an attached vortex ring. It is clear that the flow domain becomes multiconnected, however the vorticity generated by the impact will be localised (in the initial phases of the motion) in the neighbourhood of the impact site. Thus, to model this as a vortex ring is most likely an oversimplification. The velocity of the jet as a function time throughout the collapse phase is also of interest. The plot of jet velocity vs. time can be found in Fig 3(a). As the collapse occurs we have a rapid acceleration of the jet tip which, just prior to the impact, is reduced almost to zero corresponding to the jet moving with approximately constant speed.

Figures 2(b) and 2(c) show the collapse phase for bubbles in the same geometry, but characterised by $\delta = 0.5735$ and $\delta = 0.2769$ respectively, in order to investigate the coupling of buoyancy and the generalised Bjerknes effect (the influence of the flow field induced by the presence of the boundary which causes the acceleration of the bubble towards it and ultimately the formation of the jet directed towards the boundary) in the collapse phenomena. In this case both have the mutual effect of inducing jet formation directed against the rigid boundary.

In the case $\delta = 0.5735$ we note that the collapse and jet formation is initiated prematurely (compared with the case $\delta = 0.0$). In association with this observation we note that the jet is of much greater breadth and inspection of the plot of jet velocity vs. time (FIG 3(b)) reveals a peak jet velocity much less than in the case $\delta = 0.0$. The results of the computation for $\delta = 0.2769$ (FIG 2(c)) reflect similar behaviour with jet formation initiated at an intermediate time, with a jet of intermediate breadth and final velocity. Note that in all the cases presented thus far the jet achieves a "terminal velocity" prior to impact against the upper surface of the bubble. Furthermore, we observe that in each case the liquid jet completely penetrates the bubble on collapse and then presumably evolves into a vortex system. The presence of the compressible gas within the bubble has shown no discernible effect in these examples. (Such as the rebound phenomena reported experimentally by Benjamin and Ellis (1966) and others, and shown theoretically by Best and Kucera (1989)).

If we change the geometry such that the motion occurs above a rigid boundary (FIG 1(b)) then the generalised Bjerknes effect and buoyancy forces are in opposition and the possibility exists that these effects may couple such that the bubble survives in connected form beyond the first collapse.

The bubble collapse as shown in FIG 2(d) is characterised by $\delta = 0.5735$. The detonation has occurred at $\xi = 1.0$. In this case the effect of buoyancy is predominant with a broad upwards directed jet forming and completely penetrating the bubble. If we observe the jet velocity as a

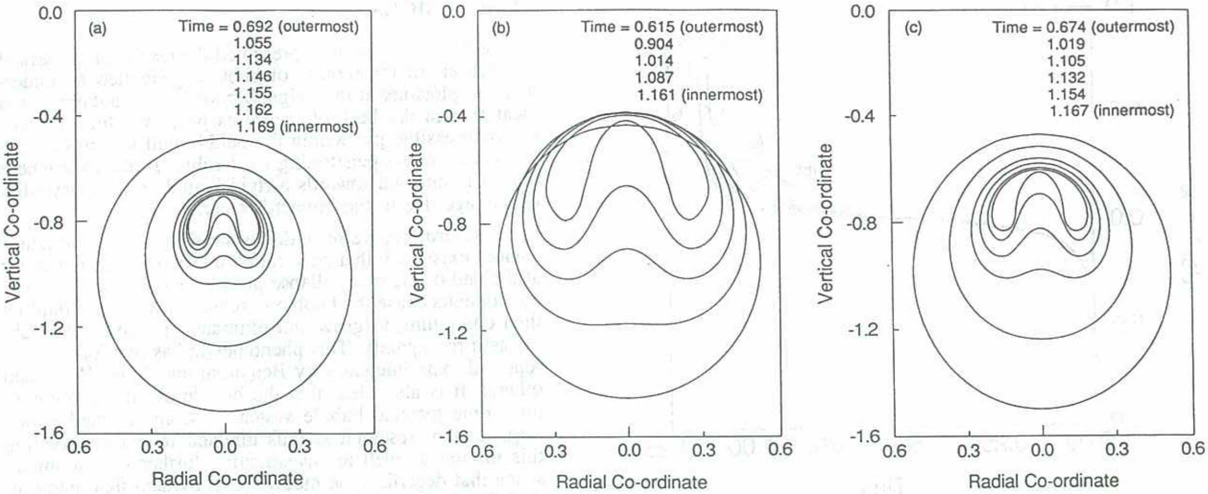

Figure 2. *Motion below a rigid boundary. Times shown are for successive profiles.*
(a) $\delta = 0.0$, Collapse phase. (b) $\delta = 0.5735$, Collapse phase.
(c) $\delta = 0.2769$, Collapse phase.

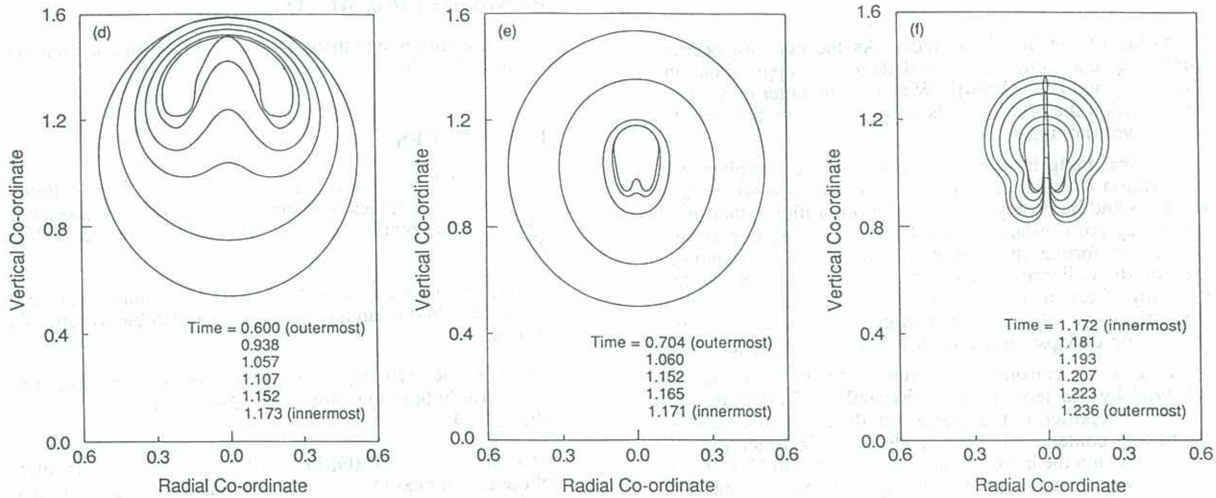

Figure 2. *Motion above a rigid boundary. Times shown are for successive profiles.*
(d) $\delta = 0.5735$, Collapse phase. (e) $\delta = 0.2769$, Collapse phase.
(f) $\delta = 0.2769$, Rebound phase.

function of time (FIG 3 (d)) we see that the jet reaches a terminal velocity but one greater than in the case where the buoyancy and Bjerknes effects are similarly directed (FIG 3(b)).

Similarly the case $\delta = 0.2769$ and $\xi = 1.0$ has been investigated and we have the intriguing result shown in FIG 2(e), (f). In this case we observe that the jet formation is incomplete as the bubble collapses to a minimum volume, however the compression of the gas inside the bubble causes it to rebound, the jet continuing to grow as the bubble re-expands until the very thin jet ultimately penetrates the upper surface of the bubble. If we consider the jet velocity as a function of time we see that jet formation is delayed in comparison with the other examples. Furthermore the jet velocity peaks and beyond this time the jet decelerates. (The non smooth nature of the velocity curve beyond this peak is due to slight numerical instability). We note that the peak of jet velocity is achieved in the small jet which exists as the bubble rebounds. It is the momentum concentrated in this jetting motion that continues to drive the thin jet into the bubble as it re-expands. We note also the elements of collapse in from the sides of the bubble. This perturbation appears as the bubble rebounds but does not result in collapse and the formation of a ring jet as has been reported elsewhere (Blake et.al. (1986)).

DISCUSSION

A number of the important features of the bubble collapse presented above may be plausibly explained by considering momentum conservation for the fluid motion, within the formalism of the Kelvin impulse.

The Kelvin impulse, **I** is defined as

$$\mathbf{I} = \oint_S \phi \mathbf{n} dS \tag{4.1}$$

and corresponds to the impulse that would have to be applied over the surface of the bubble to generate the given motion at any instant from rest. (**n** is the unit normal interior to the bubble surface.)

Alternatively, an equal and opposite impulse applied over the surface of the bubble will bring the observed motion to rest. Discussions of the concept of impulse in the context of bubble dynamics may be found in Benjamin and Ellis (1966) and Blake (1988). The impulse corresponds to the apparent inertia of the bubble and as such is analagous to the particle momentum of classical mechanics.

Recall the observation that the broader jets observed have associated with them a smaller velocity than the narrower jets, the breadth of the jet being somewhat proportional to the buoyancy parameter in the case where buoyancy and the Bjerknes effect are similarly directed. In these cases the coupling of effects gives rise to a much larger de-

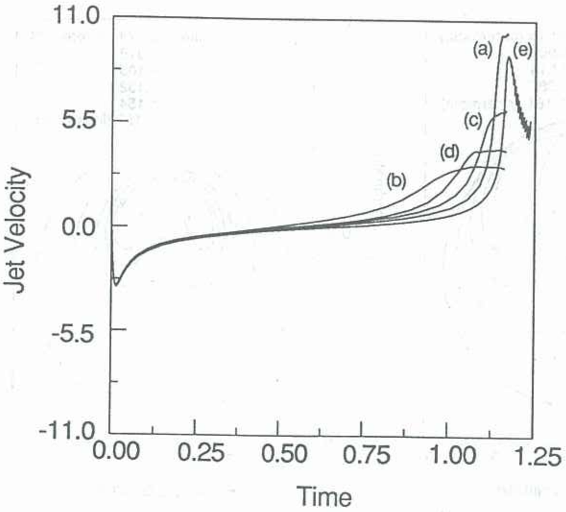

Figure 3. *Jet velocity vs time for the bubble motions illustrated in Fig. 2. The label on each graph corresponds to the bubble as illustrated in Fig. 2.*

formation of the bubble surface. As the collapse occurs impulse is conserved to a good degree of approximation (Benjamin and Ellis (1966)). With a much larger mass contained within the broad jet this impulse conservation results in a slower moving jet.

By reasoning in a similar manner we may explain the observation of a faster moving jet in the case where the Bjerknes and buoyancy effects are in opposition rather than when they are similarly directed. The opposing effects delay the jet formation process so that for a given bubble lifetime the collapse occurs over a shorter time. Since the opposing effects result in a smaller perturbation of the bubble surface we observe a narrower jet. Conservation of impulse on collapse thus requires a faster moving jet.

Finally, we consider the intriguing "terminal velocity" achieved by the jets in all the examples. Similar results have been reported in the case where the compressibility of the bubble contents has been neglected. (Constant vapour pressure within the bubble, Blake and Prosperetti (1989).) It is clear, then, that the compressibility of the bubble contents is not a major factor in causing this phenomenon. We can gain some insight into this phenomenon by considering the momentum of the fluid motion.

As discussed by Benjamin and Ellis (1966) the Kelvin impulse of the motion has two contributions, one consisting of an equivalent rigid body motion, and one associated with deformation of the bubble shape with respect to the centroid. As reported by Best and Kučera (1989) the upwards rise of deforming bubbles is significantly less than that predicted for bubbles that remain spherical. (If the bubble remains spherical, the only contribution to the impulse is due to equivalent rigid body motion (to the lowest order of approximation).) This reflects the fact that on collapse the impulse (which is more or less conserved on collapse) becomes shared between motion of the centroid and deformation with respect to the centroid. Equivalently we may consider that on collapse fluid momentum is manifesting itself in the jet rather than motion of the centroid. Thus, as the collapse proceeds the momentum transfer to the jet results in a rapid acceleration of the jet. As the bulk of the momentum is transferred to the jet it reaches a "terminal velocity". In any case, continued acceleration of the jet would eventually violate the conservation of impulse on collapse.

The above considerations, although not completely free from objection, display the value of momentum considerations in attempting to understand the features of bubble collapse phenomena.

CONCLUSIONS

In this paper we have presented the results of numerical simulations of the motion of bubbles generated by underwater explosions in the neighbourhood of boundaries. It is clear that in the bulk of the cases presented the presence of compressible gas within the bubble will not prevent the formation of jets penetrating the bubble. In the case where the jet is directed towards a rigid boundary the possibility of damage due to the impact is clear.

Furthermore, we have demonstrated that when circumstances exist such that penetration by the jet does not occur at the end of the first collapse phase, the compressible bubble contents cause the bubble to rebound, with the liquid jet then continuing to grow and ultimately penetrate the bubble as it re-expands. This phenomenon has previously been reported experimentally by Benjamin and Ellis (1966) and others. It is also clear that the bubble eventually evolves into some toroidal bubble system with an attached vortex ring. Future research is thus directed towards modelling this motion as well as investigating further the parameter space that describes the motion. It is evident that many interesting collapse and rebound modes exist. Experimental work is also essential in order that theoretical predictions may be validated.

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