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HYDRAULIC JUMPS ON STEEP SLOPES AND AT SMALL FROUDE NUMBERS

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ABSTRACT

Laboratory experiments have shown that the transition between super-critical flow down a steep slope and a deep tail water can take place through three different types of flow. A normal hydraulic jump forms when the Froude number of the approach flow is large enough. At smaller Froude numbers the high velocity flow plunges as a jet beneath the tail water; Froude number at transition between the normal jump and the plunging jet varies with Reynolds number and with bed slope. In the range of Froude numbers in which the plunging jet occurs, another type of flow, the surface jet, can exist as a stable alternative. However, the maximum value of Froude number at which the surface jet can persist decreases as the Reynolds number increases and it is not clear whether the surface jet can occur in flows at very large Reynolds numbers. The properties of normal jumps on slopes from 0.167 to 0.40 have been measured.

INTRODUCTION

During hydraulic model studies of a flood control weir, to be built as a road embankment across a tidal channel, it was noted that the flow over the downstream slope of the weir was bi-stable for a wide range of conditions (Cardno & Davies Australia Pty Ltd (1981)). Either a plunging jet or a surface jet could occur for the same imposed conditions of flow and downstream water level, as illustrated in Fig.1. This had significant implications for the hydraulic design of the structure. The surface jet caused large velocities at the sides of the channel whereas the plunging jet caused large velocities at the bed of the channel. It was not known whether the two states of flow could occur in the full size structure or whether the duality was caused by scale effect in the model and it was considered necessary to provide protection against scour in both regions of high velocity. Subsequently, the series of laboratory studies described here was undertaken to provide better understanding of the phenomena.

HYDRAULIC JUMPS ON STEEP SLOPES: PREVIOUS STUDIES

The hydraulic jump is a natural phenomenon in which a rapid, shallow, free surface flow is transformed into a slower, deeper flow with a large loss of flow energy. The Froude number of the rapid flow is greater than unity while that of the slow flow is less than unity. The Froude number of free surface flow in a rectangular cross section, **F**, is defined as $V/(gd)^{\frac{1}{2}}$, where V is the cross-section mean velocity, d is the depth of flow and g is the acceleration due to gravity. The excess kinetic energy in the flow passing over the spillways of dams is often dissipated in a hydraulic jump, located in a stilling basin, and the characteristics of hydraulic jumps on horizontal and sloping beds have been studied extensively. The phenomena illustrated in Fig.1 are related to the characteristics of hydraulic jumps on steep slopes and several studies of such jumps have been published.

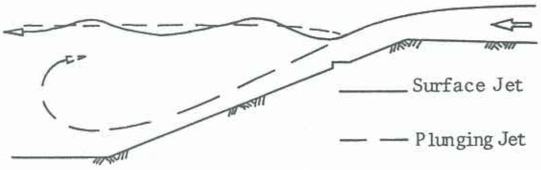

FIG.1 Bi-stable conditions observed in model study of flow over embankment

Bakhmeteff and Matzke (1938) report experiments with hydraulic jumps on six bed slopes ranging from 0.0 to 0.07. The characteristics of the jumps on the slopes were found to be similar in general to those on the horizontal bed. It is noted that, for steeper slopes, the jet "plunges into the tail water and follows the slope downward with comparatively slow expansion and obviously relatively small losses" and that such phenomena should be "differentiated from the jump proper, as observed in mildly sloped channels". No measurement of "plunging jets" is reported.

The characteristics of hydraulic jumps on a slope of 0.167 were measured by Kindsvater (1944), who describes only normal jumps. However, several discussers of the paper comment on the possibility of a plunging jet forming.

Bradley and Peterka (1957) present the results of an extensive set of measurements of normal hydraulic jumps on thirteen slopes from 0.052 to 0.280. The purpose of the study was to provide data for design of hydraulic jump energy dissipators with sloping beds. No mention is made of the possibility of a plunging jet occurring.

The characteristics of hydraulic jumps on slopes are summarised in various texts such as Woodward and Posey (1941), Chow (1959), Elevatorski (1959) and Henderson (1966). In only two of these is it noted that a plunging jet may form instead of a normal jump in some circumstances and that, when it does, high velocity water shoots out under the pool with little localised impact. Woodward and Posey (1941) suggest that the plunging jet may occur if the slope is greater than 0.167 to 0.20 and Elevatorski (1959) suggests that slopes greater than 0.25 be avoided.

DYNAMIC SIMILITUDE

The phenomena occurring in the flows under consideration are affected by gravity, viscosity and surface tension. Dimensional analysis leads to the conclusion that the characteristics of the flow are functions of three non-dimensional parameters, viz:- Froude number ($\mathbf{F} = V/(gd)^{\frac{1}{2}}$), Reynolds number ($\mathbf{R} = 4Vd/\nu$) and Weber number ($\mathbf{W} = V/(\sigma/(\rho d))^{\frac{1}{2}}$), where ρ, ν and σ are, respectively, the density, kinematic viscosity and surface tension of the liquid. For full size flows in real structures the effects of viscosity and surface tension are negligibly small and the properties of normal hydraulic jumps are functions only of **F**. However, in laboratory studies, viscosity and surface tension may have measurable effect, particularly in the cases of plunging and surface jets.

EXPERIMENTS

Since the published literature provided no detailed information concerning the occurrence of the submerged jet and no mention of bi-stable phenomena involving the surface jet, laboratory experiemnts were carried out at The University of Queensland to provide better understanding of these phenomena.

The experiments were conducted in a flume 250mm wide. The bottom slope could be adjusted over a range from 0.167 to 0.40. The flow approached the top of the slope in a horizontal channel and could be controlled by a vertical sluice gate at the top of the slope or could enter the slope uncontrolled. The flow was supplied from a constant head tank through a pipeline in which a concentric orifice provided the means for measurement of discharge. At the downstream end of the slope the tail water level was controlled with a sluice gate. Water surface levels were measured with manually operated point gauges which had a resolution of 0.2mm. Panels of perspex in the side of the flume permitted observation of the flow on the slope. Kindsvater (1944) classified the hydraulic jump on a slope into four types, depending on whether the jump is complete on the slope or whether part of it is over a horizontal bed at the downstream end of the slope. All of these experiments were arranged so that the jump (or alternate flow type) was complete on the slope, as illustrated in Fig.2, corresponding to Case 4 of Kindsvater (1944) and Case D of Bradley and Peterka (1957).

The experiments were carried out under the direction of the author as a series of three undergraduate thesis projects. In the first, Baker and Steele (1982) measured the characteristics of hydraulic jumps on five slopes from 0.167 to 0.40 and established the conditions under which the normal jump changed to a plunging jet. Sewell and Vandeleur (1983) obtained extra data on normal jumps and plunging jets on slopes 0.167 and 0.286, refined the description of the plunging jet, observed the bi-stable phenomena on the slope 0.286 and studied the effects of scale on normal jumps and plunging and surface jets. Pekol and Romano (1984) obtained extra data on normal jumps and plunging jets on slopes 0.286 and 0.40, established the conditions under which surface jets form over these slopes and investigated further the effects of scale on the three different types of flow.

THE THREE FLOW TYPES

The general characteristics of the three types of flow which have been observed are illustrated schematically in Fig.2.

Normal Hydraulic Jump. The normal jump on a slope, Fig.2a, is similar in general to the jump on a horizontal bed but is longer. It is characterised by the large surface roller, which displays substantial unsteadiness and intense, large scale turbulence. Air is entrained extensively throughout the jump. There is a distinct height of jump, h, as defined in Fig.2a. In these experiments, the length of the jump, L_b, was measured at that location where the surface flow reverses; this is unambiguous and easily found by dye injection.

Plunging Jet. The super-critical approach flow dives under the downstream tail water when the plunging jet occurs, Fig.2b. The high velocity flow extends down the slope much further than is the case for a normal jump. There is less air entrainment and the tail water is much less disturbed than for a normal jump and there is a relatively small depression where the plunging jet dives under the surface. The "length of the jump" is not easily defined for the plunging jet. In these experiments L_b was measured at that location where the greatest upwelling to the surface of entrained air bubbles occurred. This was a reasonably definite location as illustrated in Fig.2b.

Surface Jet. When the surface jet occurs, the super-critical approach flow "rides" over the top of the tail water instead of plunging beneath it. The surface profile is distinctly undular as illustrated in Fig.2c and has some similarity to the profile of the "undular jump" which forms on a horizontal bed when the Froude number of the approach flow is less than 1.7. A reverse roller forms beneath the surface jet where it leaves the slope and little air entrainment occurs.

RESULTS

Normal Hydraulic Jump

The lengths of normal jumps measured on the various slopes are plotted in as L_b/TW versus the Froude number of the approach flow in Fig.3. The downstream depth, TW, is defined in Fig.2. The results of all three thesis projects are in close agreement and show that L_b/TW is essentially constant for a given slope over the range of $\mathbf{F}_1$ tested. The values of L_b/TW measured for plunging jets are not shown in Fig.3 but it is interesting that the definition of L_b adopted for this case leads to similar values to those obtained for normal jumps.

Fig.3 includes results from Bradley and Peterka (1957) which show L_b/TW varying with $\mathbf{F}_1$. Reference to the original data shows that the curves are the results of interpolation and averaging among data points which have much larger scatter than those from the experiments reported here. Bradley and Peterka (1957) adopted as the end of the jump "the point where the high velocity jet began to lift from the floor, or a point on the level tail water surface immediately downstream from the surface roller, whichever occurred farthest downstream".

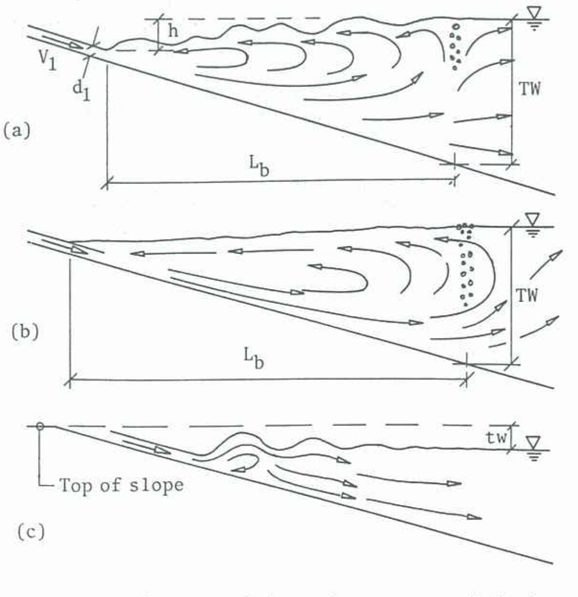

FIG.2 Possible transitions from super-critical to sub-critical flow on steep slopes: (a) Normal jump; (b) Plunging jet; (c) Surface jet.

This could result in different values for L_b from those measured in these experiments but, nevertheless, our constant value for slope 0.25 agrees with the horizontal part of the curve for that slope. The curves from Bradley and Peterka (1957) show L_b/TW as substantially constant in the range $4.5 < \mathbf{F}_1 < 13$ and Henderson (1966) obtained empirical expressions from their curves for slopes less than 0.30 which lead to,

$$L_b/TW = (6.1 + 4\,So)/(1 + 11.2\,So^{3/2}) \qquad (1)$$

where So is the bed slope. This relationship is shown on Fig.4 with the results from our experiments. The agreement is generally good.

Plunging Jet

In experiments on all slopes ranging from 0.167 to 0.40, it was found that a plunging jet formed at small values of $\mathbf{F}_1$ and that transition to a normal jump occurred as $\mathbf{F}_1$ increased. The experimental procedure followed was to maintain a constant flow rate and constant tail water level downstream while the value of $\mathbf{F}_1$ was increased progressively by lowering the upstream sluice gate and, so, increasing the velocity of the flow at the top of the slope. As $\mathbf{F}_1$ was increased, the plunging effect became less pronounced until a transitional range of $\mathbf{F}_1$ was reached; for values of $\mathbf{F}_1$ greater than this range, normal jumps formed.

The data in Fig.3 show some overlap in the ranges of $\mathbf{F}_1$ for the occurrence of normal jumps and plunging jets. The overlaps correspond to measurements made at different flow rates and, hence, different values of $\mathbf{R}$ for the same value of $\mathbf{F}_1$. This is an indication of the effect of viscosity on the occurrence of the plunging jet and it appears from limited data that the value of $\mathbf{F}_1$ at which the jet plunges decreases as $\mathbf{R}$ increases. The lines on Fig.3 which are designated as the transition between the normal jump and the plunging jet correspond to the measurements at the largest value of $\mathbf{R}$, which was 4×10^5 except for slope 0.167, for which the largest value of $\mathbf{R}$ was 2.5×10^5. The value of $\mathbf{F}_1$ for transition from plunging jet to normal jump at $\mathbf{R} = 4 \times 10^5$ is plotted against bed slope in Fig.4, which shows that the transition occurs at smaller values of $\mathbf{F}_1$ as the bed slope is decreased. This suggests that the plunging jet will not occur for slopes smaller than some limit value, as yet not determined accurately.

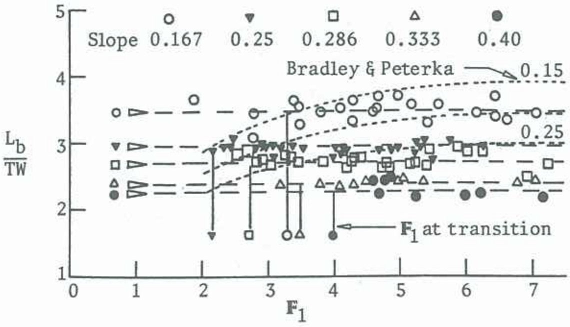

FIG.3 Lengths of normal jumps on slopes

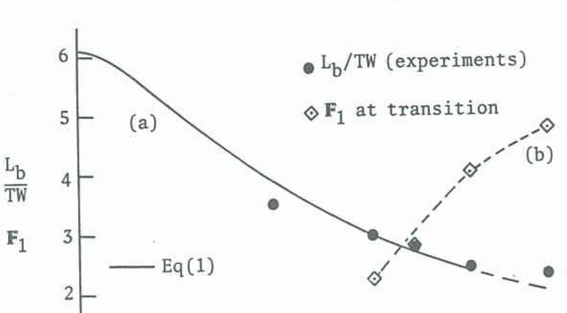

FIG.4 (a) Length of normal jump as function of bed slope; (b) Froude number at transition from plunging jet to normal jump. ($\mathbf{R} = 4 \times 10^5$,

Surface Jet and Bi-Stable Phenomena

A surface jet did not form without some external intervention in these experiments but Sewell and Vandeleur (1983) and Pekol and Romano (1984) found that, at small values of $\mathbf{F}_1$ for which a plunging jet normally formed, a surface jet could be generated by some temporary disturbance of the flow and that, once formed, it persisted indefinitely as an alternative flow type.

Sewell and Vandeleur (1983) produced stable surface jets on a slope 0.286 at conditions of flow rates and tail water levels otherwise dynamically similar to some of those for which the bi-stable phenomena had first been observed on the slope 0.40 in the hydraulic model studies referred to above. Further, they found that once the surface jet was established it would persist while the tail water level was lowered progressively. At a flow rate corresponding to $\mathbf{R} = 5.2 \times 10^4$, the surface jet persisted as the vertical distance from the top of the slope to the tail water level, "tw" in Fig.2, was increased from 2 to 30mm and at a flow rate corresponding to $\mathbf{R} = 1.2 \times 10^4$ it persisted as tw was increased from 4 to 51mm. However, the surface jet would not form at the lower tail water levels.

Pekol and Romano (1984) converted plunging jets into surface jets by dragging a horizontal flat plate downstream across the free surface at the plunge point in such a way as to produce a local reduction in water level upstream from the plate and a local increase downstream. At a constant tail water level, a stable surface jet could be established at a small flow rate and then the flow rate was increased progressively until the surface jet became unstable and reverted to a plunging jet. The value of $\mathbf{F}_1$ at which the surface jet became unstable was determined for a range of tail water levels over slopes 0.286 and 0.40. Pekol and Romano (1984) noted that a stable surface jet could not be formed at a given tail water level until a certain flow rate had been exceeded. Once formed, the surface jet would persist indefinitely until the flow rate was increased beyond an upper limit, at which the surface jet reverted to the plunging jet. The conditions measured at this upper limit of stability for the slopes 0.286 and 0.40 are shown in Fig.5 as the value of $\mathbf{F}_1$ at the limit of stability versus tw/d_1. Stable surface jets were formed at small negative values of tw, i.e. with the tail water level above the top of the slope, as well as for positive values.

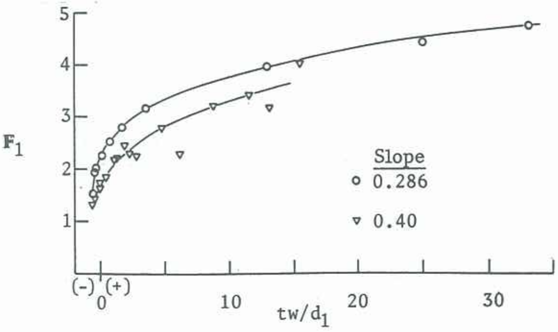

FIG.5 The surface jet; conditions at upper limit of stability before transition to plunging jet

Scale Effects

It was noted above that the value of $\mathbf{F}_1$ at which the plunging jet converts to a normal jump decreases as $\mathbf{R}$ increases. Systematic experiments were carried out to determine whether the results of measurements on normal jumps and plunging jets were affected by the scale. The discharge was set at different constant values, the imposed conditions adjusted to satisfy Froude law scaling and the other quantities were measured. In the absence of "scale effect", all measured quantities should scale according to the Froude law. The largest ratio of discharges achieved was 3:1 and in this limited range the results for plunging jumps indicated only small scale effects which were reduced further when the surface tension was reduced by 7 per cent with detergent. The results for normal jumps indicated significant scale effects which were reduced but not removed by reduction of the surface tension by 7 per cent. It appears from these results that measurements at much larger flow rates would be required to establish conditions in which the effects of viscosity and surface tension are negligibly small.

In the case of the surface jet, the value of $\mathbf{F}_1$ at the upper limit of stability is likely to be a function of $\mathbf{R}$ and perhaps of $\mathbf{W}$. The results for $\mathbf{F}_1$ in Fig.5 are plotted against $\mathbf{R}$ in Fig.6 and it appears that $\mathbf{F}_1$ does vary with $\mathbf{R}$, decreasing in magnitude as $\mathbf{R}$ increases. One might infer from Fig.6 that a surface jet would not form under any conditions at very large values of $\mathbf{R}$. These results suggest that the surface jet was not likely to be observed in full size flows and that it is essentially a laboratory phenomenon. However, a type of flow which appeared to be very similar to the surface jet was observed when flood flows over-topped the raised embankment floodways on the approach to the Fitzroy River crossing at Willare in Western Australia in 1986 (Van Kleef and Goh (1988)). The geometric conditions were similar to those which applied to the hydraulic model tests in which the surface jet was first observed but the value of $\mathbf{R}$ for the flood flow over the embankment was of order 10^7!

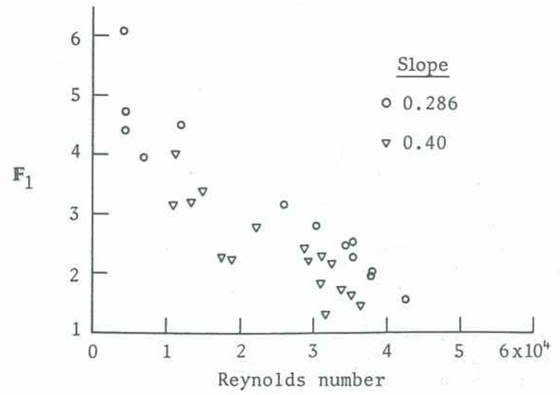

FIG.6 The surface jet; $\mathbf{F}_1$ at transition from surface jet to plunging jet.

CONCLUSION

The experiments reported here have demonstrated that three different types of flow can occur when supercritical flow down a steep slope meets a deep tail water.

When the Froude number of the approach flow is large enough, a normal hydraulic jump forms. The characteristics of such normal jumps on slopes from 0.167 to 0.40 were measured and generally good agreement was obtained with available published data.

When the Froude number of the approach flow is smaller than that necessary for the production of a normal jump, the high velocity flow plunges as a jet beneath the tail water. The value of the Froude number at transition decreases as the Reynolds number of the flow increases and for flows at the same Reynolds number it decreases as the bed slope decreases. It appears that the plunging jet will not occur for slopes smaller than some limit value less than 0.167 but not yet determined accurately.

In the range of Froude numbers of the approach flow for which the plunging jet forms, another type of flow, the surface jet, can exist as a stable alternative to the plunging jet. The surface jet reverts to the plunging jet as the flow rate is increased, all else remaining unchanged. The value of Froude number at which this transition occurs decreases as the Reynolds number of flow increases and the trend in the results of the laboratory experiments suggests that the surface jet would not occur at the very large Reynolds numbers which apply in "full size" flows. Nevertheless, phenomena which appear to be similar to the surface jet were observed recently in large flood flows passing over a road embankment.

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