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USE OF A FOUR-WIRE PROBE FOR THE MEASUREMENT OF TURBULENT VORTICITY FLUCTUATIONS

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ABSTRACT

The lateral vorticity components in the turbulent far-wake of a cylinder have been obtained with a compact four-wire arrangement that comprises an X-probe straddling two parallel hot wires. A useful check of measurement accuracy is provided by the reasonable agreement obtained between the vorticity-velocity cross products and the lateral derivative of the Reynolds shear stress. Measured root mean square values and spectra of the vorticity components are compared with measurements made in the same flow by Antonia, Browne and Shah (1988) using two parallel X-probes.

INTRODUCTION

Vorticity is an important defining property of turbulent flows, thus explaining the interest that its accurate measurement has attracted. Comprehensive reviews of measurement difficulties, the different measurement methodologies and of available results, have been compiled by Wallace (1986) and Foss and Wallace (1989). Ideally, it is desirable to obtain simultaneously all three components of the vorticity vector. Attempts by Wallace (see Balint et al., 1987) to provide such data with a nine wire probe of relatively small dimensions are encouraging. The present goal is more modest in that it consists in measuring (separately, not simultaneously) the lateral vorticity components.

Use is made of a fixed four-wire probe (see Figure 1) to measure either of the lateral vorticity

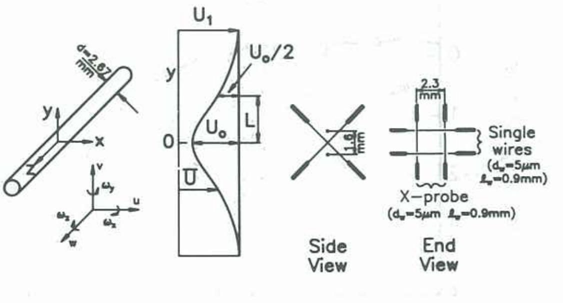

Figure 1 Experimental arrangement, showing the four-wire ω_z geometry.

components (ω_y or ω_z) in the far-wake of a circular cylinder. The main aim is to assess the performance of this probe by using different approaches. One of these consists in determining the correlations between these vorticity components and the lateral velocity components. Measurements of the vorticity components are also compared with those of Antonia et al. (1988a), hereafter referred to as I. Measurements in the latter paper were made in the same flow and experimental conditions using two (parallel) X-probes, which were displaced relative to each other in either y or z directions. Future aims are to analyse the statistics of ω_y and ω_z in the context of local isotropy and investigate the use of ω_z for detecting the turbulent/non-turbulent interface.

EXPERIMENTAL DETAILS

Measurements were made in the self-preserving wake of a circular cylinder, of diameter d = 2.67 mm, at a distance of 420d from the cylinder. The Reynolds number, based on the free stream velocity U_1 and d is 1170. At the measurement station, the mean velocity half-width L (see Figure 1) is 12.3 mm and the mean velocity defect U_0 is 0.36 m/s. An important advantage afforded by the experimental conditions is that the Kolmogorov microscale L_k is large enough (it increases from about 0.45 mm at $\eta = 0$ to about 0.7 mm at $\eta = 2$, where $\eta \equiv y/L$) to construct a probe with adequate spatial resolution. Another advantage is that the turbulence intensity level is quite small ($\overline{u^2}^{1/2}/\overline{U} \simeq 0.016$, $\overline{v^2}^{1/2}/\overline{U} \simeq 0.014$, $\overline{w^2}^{1/2}/\overline{U} \simeq 0.013$ at $\eta = 0$) so that the excursions of the velocity vector cone angle in the plane of the X-probe are too small to affect the probe performance. Also the effect of the w fluctuation should be negligible (the effect of w, as well as the influence of several other factors, on this type of probe was investigated by Foss et al., 1987).

In the present configuration (Figure 1), an X-probe straddles two parallel single hot wires. A similar configuration was previously used by Foss (1979) and Foss et al. (1987) with the difference that there was a lateral separation between the X-probe and the single hot wires. In the course of writing this paper, Professor Foss drew to our attention the work of Haw et al. (1988) who used a compact probe, of similar design to the present arrangement, to determine intermittency based on ω_z measurements. The arrangement shown in Figure 1 allows ω_z to be obtained. When the arrangement is rotated through 90° so that the X-probe lies in the (x,z) plane, ω_y is determined.

All hot wires were operated with constant temperature anemometers at an overheat of 0.5. Suitably amplified signals were digitised at a sampling frequency f_s of 2500 Hz/channel into a PDP 11/34 computer. Subsequent analysis was carried out on a VAX 780.

FORMATION OF VORTICITY COMPONENTS

Digital time series of ω_y and ω_z were determined using the approximations

$$\omega_y = \frac{\Delta u}{\Delta z} + \overline{U}^{-1}\frac{\Delta w}{\Delta t} \qquad (1)$$

$$\omega_z = -\overline{U}^{-1}\frac{\Delta v}{\Delta t} - \frac{\Delta u}{\Delta y} \qquad (2)$$

where Δt is the interval between successive digital samples; Δv (or Δw) is the difference between values of v (or w) which are separated by Δt; Δu is the difference between the longitudinal velocity fluctuations from the single hot wires which are separated in either the y or z directions. In (1) and (2), Taylor's hypothesis has been used to convert temporal derivatives into streamwise derivatives. This assumption should be satisfactory in view of the small turbulence level of the flow. The second assumption made in (1) and (2), i.e. the replacement of $\partial u/\partial z$ and $\partial u/\partial y$ with $\Delta u/\Delta z$ and $\Delta u/\Delta y$ respectively, is more critical since the correct choice of the separation (Δz or Δy) is not straightforward. Analyses for the effect of wire separation on spectra of $\partial u/\partial y$ or $\partial u/\partial z$ have been made (e.g. Wyngaard, 1969; Antonia et al., 1984) but they are based on a number of assumptions, e.g. local isotropy and a particular shape of the energy spectrum. While the latter assumption can be avoided (e.g. by following the procedure of Roberts, 1973), local isotropy applies only at high wavenumbers (e.g. Antonia et al., 1986). The more direct experimental approach of varying Δy or Δz has also been tried (Antonia et al., 1984 in a jet and Browne et al., 1987 in a wake) but the major difficulty relates to the extrapolation to zero separation (e.g. Browne et al., 1983). The magnitude of the separation ($\Delta y = \Delta z \simeq 1.6$ mm or $3.5L_k$ at $\eta = 0$), which is comparable to the streamwise separation ($\overline{U}\Delta t \simeq 1.3$ mm), was chosen on the basis of previous experimental results for the effect of Δy on the statistics of $\partial u/\partial y$ (Browne et al., 1987).

COMPARISON OF STATISTICS OF ω_y AND ω_z WITH PREVIOUS RESULTS

Several standard checks of the performance of hot wires were made before the vorticity components were calculated. Distributions of $\overline{u^2}^{\frac{1}{2}}$, $\overline{v^2}^{\frac{1}{2}}$, $\overline{w^2}^{\frac{1}{2}}$ were in good agreement with those presented in Browne et al. (1987), for the experimental uncertainty range reported in that paper. Distributions of $\overline{uv}$ were in close agreement with those reported in Browne and Antonia (1986) while the shear stresses $\overline{uw}$ and $\overline{vw}$ were sufficiently small, in conformity with the assumed two-dimensionality of the mean flow. It should also be noted that the possibility of cross-contamination between the X-probe and the single hot wires was checked by verifying that velocity statistics obtained with the X-probe were unaffected when the single wires were switched off and vice versa.

Probability density functions (pdf) and corresponding moments of ω_y and ω_z have been obtained as well as spectra of these quantities. The pdfs are shown in Figure 2 for $\eta \simeq 1$. The pdf of ω_y is approximately symmetrical, the negligible value of the skewness of ω_y reflecting this symmetry. Note

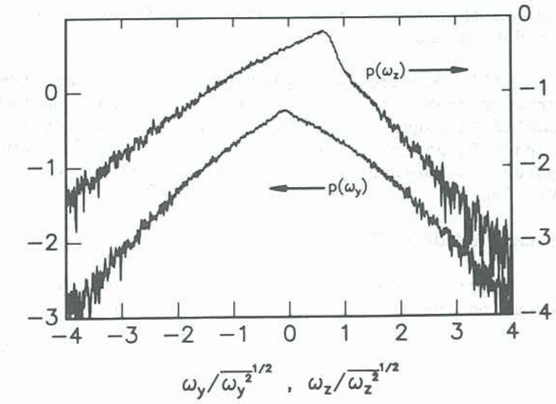

Figure 2 Probability density functions of ω_y and ω_z at $\eta \simeq 1$.

that symmetry of the pdf of ω_y follows from the requirement of symmetry with respect to the z direction. The pdf of ω_z is negatively skewed, as was also reported in I. The rms distributions in Figure 3 indicate that $\overline{\omega_y^2}^{\frac{1}{2}}$ has a local maximum at $\eta \simeq 1$ whereas $\overline{\omega_z^2}^{\frac{1}{2}}$ decreases away from the centreline. The maximum in $\overline{\omega_y^2}^{\frac{1}{2}}$ may be associated with the stretching of vortex loops in the central part of the wake whereas, speculatively, the peak in $\overline{\omega_z^2}^{\frac{1}{2}}$ near $\eta = 0$ may be due to the presence of spanwise vortices with either positive or negative ω_z in the vicinity of the centreline. The present behaviour of $\overline{\omega_y^2}^{\frac{1}{2}}$ and $\overline{\omega_z^2}^{\frac{1}{2}}$ is qualitatively similar to that reported in I but the magnitudes of $\overline{\omega_y^2}^{\frac{1}{2}}$ and $\overline{\omega_z^2}^{\frac{1}{2}}$ reported in I are smaller than the present values by about 30%. We have not been able to explain this difference. The value of Δy (or Δz) used here is identical to that in I. In view of the magnitude of $\Delta y/L_k$, one would expect both the present values of $\overline{\omega_y^2}^{\frac{1}{2}}$ (or $\overline{\omega_z^2}^{\frac{1}{2}}$) and those of I to be slightly attenuated (the maximum attenuation is unlikely to exceed 10%).

Notwithstanding the difference in the magnitudes of $\overline{\omega_y^2}^{\frac{1}{2}}$ and $\overline{\omega_z^2}^{\frac{1}{2}}$, the present normalised pdf, normalised moments and normalised spectra of ω_y and ω_z are in reasonable agreement with those obtained in I. The spectra of ω_y and ω_z (Figure 4) are

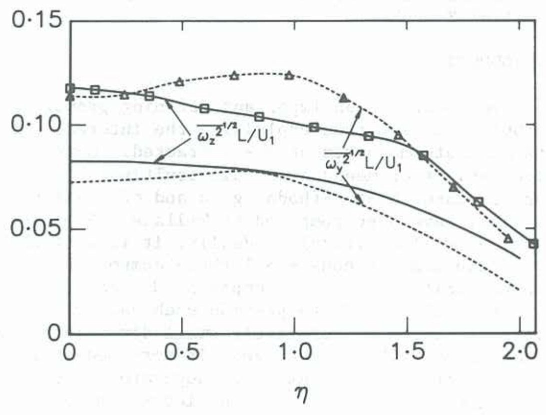

Figure 3 Distributions of rms values of ω_y and ω_z.

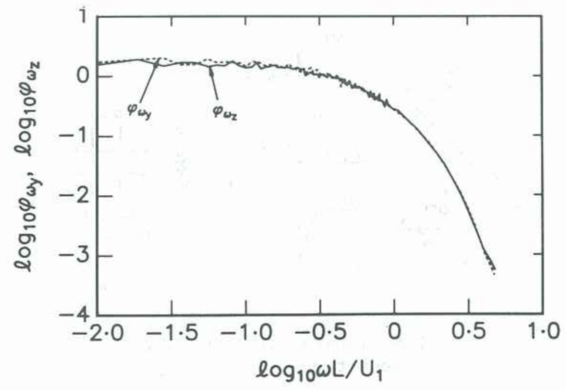

Figure 4 Spectra of ω_y and ω_z at $\eta \simeq 1$.

virtually identical at all wavenumbers, in agreement with local isotropy (the isotropic forms of these spectra were given in I). It should be emphasised however that local isotropy is only validated for relatively high wavenumbers. The spectral results presented in Antonia et al. (1988b) indicated that local isotropy is satisfied only for length scales smaller than about $3L_k$.

ACCURACY TEST FOR ω_y AND ω_z

A possible way of checking the accuracy of ω_y and ω_z is provided by the identity

$$-\frac{\partial}{\partial y}\,(\overline{uv}) \;=\; (\overline{v\omega_z}-\overline{w\omega_y}) + \tfrac{1}{2}\,\frac{\partial}{\partial x}\,(\overline{u^2}-\overline{v^2}-\overline{w^2}) \quad . \qquad (3)$$

The identity simplifies the approximate form (Tennekes and Lumley, 1972, p.79), viz.

$$-\frac{\partial}{\partial y}\,(\overline{uv}) \;\simeq\; \overline{v\omega_z} - \overline{w\omega_y} \qquad (4)$$

if the streamwise gradients are assumed to be small compared with the lateral gradients.

Figure 5 indicates that the right side of (4) is dominated by the contribution from $\overline{v\omega_z}$. Both $\overline{v\omega_z}$ and $\overline{w\omega_y}$ change sign near $\eta \simeq 0.7$, which corresponds approximately with the location at which $|\overline{uv}|$ is maximum. Figure 6 shows that eq. (4) is well

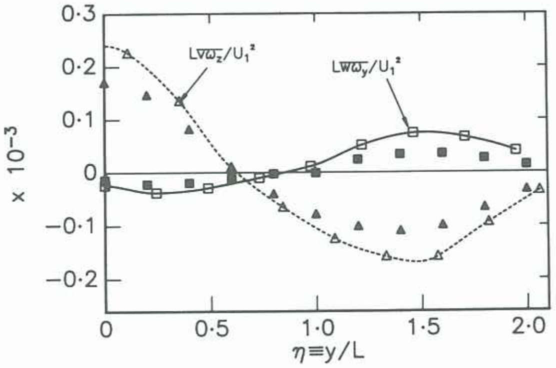

Figure 5 Distributions of $\overline{v\omega_z}$ and $\overline{w\omega_y}$. The present results are given by the open symbols. The filled in symbols are calculated using the present values of $\overline{v\omega_z}/\overline{v^2}^{\frac{1}{2}}\,\overline{\omega_z^2}^{\frac{1}{2}}$ and $\overline{w\omega_y}/\overline{w^2}^{\frac{1}{2}}\,\overline{\omega_y^2}^{\frac{1}{2}}$ with the values of $\overline{\omega_z^2}^{\frac{1}{2}}$ and $\overline{\omega_y^2}^{\frac{1}{2}}$ given in Antonia et al. (1988a).

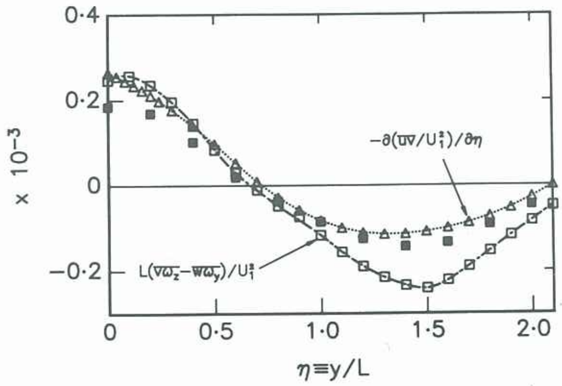

Figure 6 Distribution of the terms representing the left and right sides of eq. (4). The present results are given by the open symbols. The filled in symbols are calculated using the present values of $\overline{v\omega_z}/\overline{v^2}^{\frac{1}{2}}\,\overline{\omega_z^2}^{\frac{1}{2}}$ and $\overline{w\omega_y}/\overline{w^2}^{\frac{1}{2}}\,\overline{\omega_y^2}^{\frac{1}{2}}$ with the values of $\overline{\omega_z^2}^{\frac{1}{2}}$ and $\overline{\omega_y^2}^{\frac{1}{2}}$ given in Antonia et al. (1988a).

supported by the data in the range $0 < \eta < 1$, implying that the probe performs adequately in this fully turbulent part of the flow. For $\eta > 1$, $|\overline{v\omega_z} - \overline{w\omega_y}|$ becomes larger than the shear stress gradient, the maximum deviation occurring near $\eta \simeq 1.5$. This departure cannot be attributed to the neglect of the streamwise derivatives in eq. (3), the neglect being easily supported either by an order of magnitude analysis or by measurements. It is also unlikely to be associated with the presence of a turbulent/non-turbulent interface, the departure tending to decrease near the edge of the wake. A possible reason may be the slight increase in length scale in this region which may adversely affect the measurements of $\partial u/\partial y$ and $\partial u/\partial z$. The correlation $\overline{w\omega_y}$ may be interpreted as a vortex-stretching force and Figure 5 shows that the magnitude of $\overline{w\omega_y}$ is indeed largest in this flow region.

In view of the discrepancy (Figure 3) between the present rms vorticity values and those in I, the distributions of $\overline{v\omega_z}$ and $\overline{w\omega_y}$ were recalculated using : (i) the present values of $\overline{v\omega_z}/\overline{v^2}^{\frac{1}{2}}\,\overline{\omega_z^2}^{\frac{1}{2}}$ and $\overline{w\omega_y}/\overline{w^2}^{\frac{1}{2}}\,\overline{\omega_y^2}^{\frac{1}{2}}$; (ii) the best fit curves to the relatively large body of data for $\overline{v^2}^{\frac{1}{2}}/U_1$ and $\overline{w^2}^{\frac{1}{2}}/U_1$, presented in Browne et al. (1987); and (iii) the rms values of ω_z and ω_y obtained in I. The assumption here is that normalised quantities such as the correlation coefficient are not significantly affected by the probe performance, unlike the actual correlations and rms values. Step (ii) is not critical since, as noted earlier, the present values of $\overline{v^2}^{\frac{1}{2}}/U_1$ and $\overline{w^2}^{\frac{1}{2}}/U_1$ are in close agreement with those of Browne et al. (1987). The recalculated values of $\overline{v\omega_z}$ and $\overline{w\omega_y}$ are shown in Figure 5 while the difference is plotted in Figure 6. The latter figure shows that the recalculation yields slightly poorer agreement with eq. (4) near $\eta = 0$ but improved agreement in the outer part of the wake. This tends to suggest that the shapes of the $\overline{\omega_z^2}^{\frac{1}{2}}$ and $\overline{\omega_y^2}^{\frac{1}{2}}$ distributions presented in I may be more appropriate than the present distributions in the outer wake.

CONCLUSIONS

The four-wire arrangement of Figure 1 appears to yield reasonable statistics of ω_y and ω_z in a self-preserving wake. The difference between the values of $\overline{v\omega_z}$ and $\overline{w\omega_y}$ obtained with this arrangement is in reasonable, although not perfect, agreement with the lateral gradient of the Reynolds shear stress. The discrepancy seems to reflect the difference between the present rms vorticity values and those reported in Antonia et al. (1988a).

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