A SIMPLIFIED APPROACH TO THE HYDRODYNAMICS OF BUTTERFLY VALVES

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Synopsis – In order to extend the prediction of butterfly valve characteristics to compressible flows and unusual geometries capable of giving a low self-closing torque, a simplified analysis of butterfly valve behaviour is necessary. In the paper an analysis based upon regarding the butterfly valve as two non-interacting jets is presented and compared with an orthodox "exact" solution for a plane butterfly in a parallel channel. It appears that the jet-flow analysis gives results closer to available experimental values, at least when the valve approaches closure, than does the "exact" analysis.

It is concluded that the jet-flow analysis will be useful for designing unusual butterflies. However, more careful experiments are needed, both with plane channels and pipes to give reliable data for comparison with theory.

LIST OF SYMBOLS

$C_{c1,2}$	Contraction coefficients at tip 1, 2 respectively
D	Circular butterfly diameter
F_n	Normal valve force
F^*, $(F^*)_3$	Two and three dimensional force coefficients $F_n/\varrho V_j^2 wL$; $F_n/\frac{\pi}{4}\varrho V_j^2 D^2$
h_1, h_2	Jet widths (see Fig. 1)
L	Valve length, channel width (see Fig. 1)
M	Moment on valve
M^*, $(M^*)_3$	Two and three dimensional moment coefficients $M/\varrho V_j^2 wL^2$; $M/\varrho V_j^2 D^3$
V	Fluid velocity parallel to butterfly
V_j	Jet velocity

V_0	Approach velocity
w	Valve length along pivot
x	Tip distance (see Fig. 1)
x_s	Value of x for minimum velocity
X	$\equiv x_s/L$
β	Valve angle (see Fig. 1)
Δ	Valve opening (see Fig. 1)
ΔP	Pressure at point in flow − downstream pressure
ζ	$\equiv V/V_j$
ϱ	Fluid density

1. INTRODUCTION

The spread of automatic control systems has resulted in many hydraulic elements being subjected to further study, particularly in respect of their input/output characteristics, the input being, for

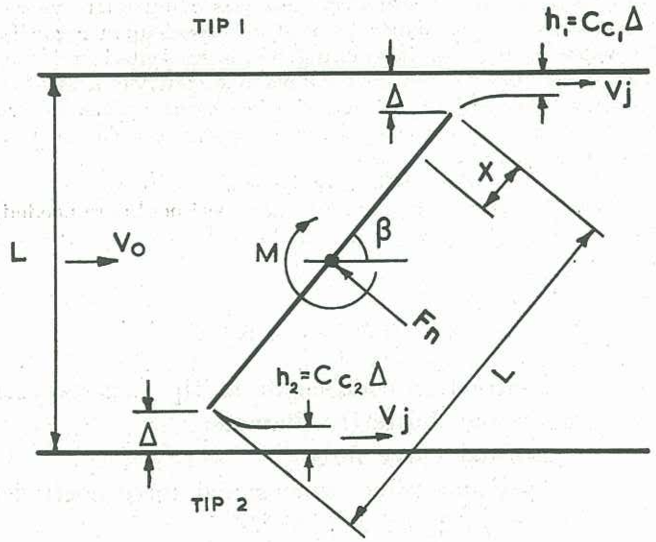

FIG. 1. Butterfly valve geometry.

example, measured as a mechanical displacement, while the output is measured as a rate of flow or a pressure. The butterfly valve has not been extensively studied in this respect, and although there are a number of papers (reviewed in Refs. (1) and (3)) dealing with

experimentally measured static characteristics of such valves, few attempts at analysis have been made, and little knowledge, if any, appears to exist on the dynamic characteristics of such valves.[3] The work of Sarpkaya[2] on static characteristics is a notable exception. He analyses the flow through the two-dimensional valve shown in Fig. 1, using an incompressible, inviscid flow model with Kirchhoff–Rayleigh[4]-type free streamline flow downstream of the valve. Because of the convergent geometry it is expected, by analogy

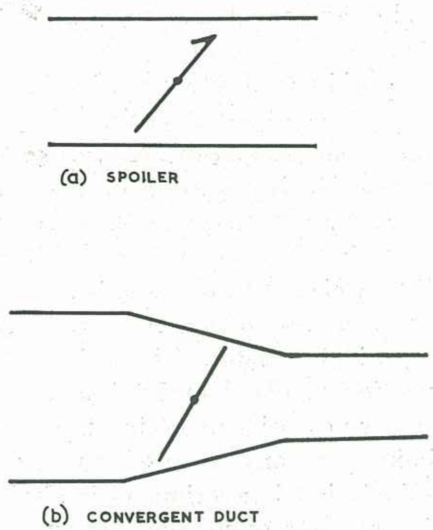

FIG. 2. (a) Spoiler (b) Convergent duct.
Suggested configurations for low torque valves.

with jet flows, that this flow model will be quite suitable except when the valve is nearly fully open, when turbulence will control the flow. Sarpkaya[2] employs the classical hodograph-conformal transformation method to determine the flow, but the results are sufficiently complicated to require considerable numerical integration in order to determine the moment and force coefficients of the valve.

Analysis and experiment reveal that there is a moment tending to close the valve, and from the control point of view it is desirable to eliminate, or at least reduce, this torque to a minimum. Other geometries, such as those sketched in Fig. 2, have therefore been suggested[3] as a means of reducing the peak closing torque on the valve. An analysis of the type made by Sarpkaya[2] seems too complex to yield useful guidance on the choice of configuration. Furthermore, many valves work with compressible fluids, and the author would hesitate to make any attempt to extend Sarpkaya's analysis in this direction. Hence a simplified approach to the prediction of forces, moments and discharges, which could be easily extended to the configurations of Fig. 2 or to compressible flow, is desirable.

The purpose of this paper is to explore the validity of considering the wings of the butterfly to produce, with the channel walls, jet flows from a funnel of angles 2β and $2(\pi-\beta)$ respectively (Fig. 1). This means that the second jet and channel wall are assumed to be far enough away to be ignored when considering the flow through the first. Clearly, as the fully shut position is approached, so the solution should tend to the "exact solution", produced by Sarpkaya[2]. As the fully open position is approached, the approximation will break down completely, but then the effect of the walls on the butterfly may be neglected as a first approximation and a free-stream Kirchhoff–Rayleigh[4] flow assumed.

Hence there is a good chance that the simple jet model (plus the free-stream model) will be useful. The object of this paper is to study the configuration of Fig. 1 in detail and compare the results with Sarpkaya's for a potential plane incompressible flow. This is a necessary preliminary to any more advanced work. Comparison with some three-dimensional experiments is also attempted.

A comprehensive account of jet flows is given by Birkhoff and Zarantonello[5], where the incompressible case (first studied by von Mises in 1913[6] for general jet angles) is discussed in some detail and a table of contraction coefficients is given. The results of the present paper are based on this flow. The compressible case has been worked out in principle by Jacob[7], but the results are not given in a directly useful form and further computation would be necessary to obtain contraction coefficients.

2. CONTRACTION COEFFICIENTS AND VALVE FORCES

For the valve in a parallel channel (Fig. 1) the effective contraction (or discharge) coefficient is $\frac{1}{2}(C_{c_1}+C_{c_2})$. Using the data of von Mises[6] for the jet case and that of Sarpkaya[1] for comparison, the result shown in Fig. 3 emerges. Except where the valve is less than

about 5–10 degrees from the fully-open position, the discharge coefficients compare within 3 per cent, and the jet flow prediction is generally too low. In view of the inviscid flow approximation, it seems hardly worth the effort to obtain further accuracy for the discharge.

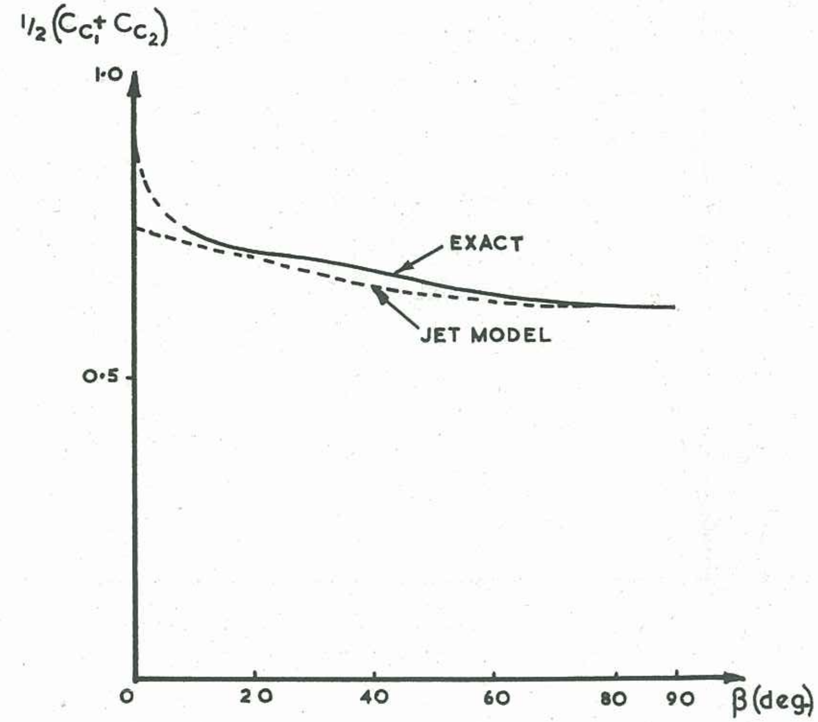

FIG. 3. Discharge coefficients.

By considering the momentum change in the direction of the channel, denoting the force on the valve by F_n, it is clear from Fig. 1 that

$$F^* \equiv \frac{F_n}{\varrho V_j^2 wL} = (1-V_0/V_j)^2/2\sin\beta \tag{1}$$

which may be computed using the relation between the geometry and the discharge coefficients. A comparison with Sarpkaya's values (Fig. 4) shows that the values of F^* found by this method are only a little too high. Hence the discharge and force on the valve plate may be found readily for many arrangements; unfortunately the prediction of the moment is more difficult and demands a closer examination of the flow.

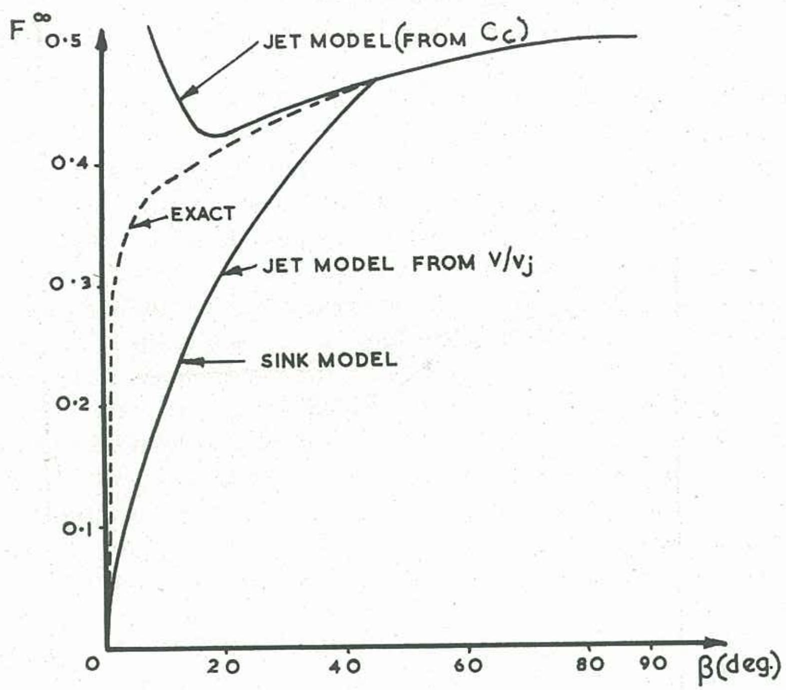

FIG. 4. Force coefficient $F^* = F_n/\varrho V_j^2 wL$.

3. DISTRIBUTION OF VELOCITY ALONG BUTTERFLY

The calculation of velocity (and pressure) distribution along the butterfly allows a direct determination of the moment coefficient. However, the introduction of the non-interacting jet model in place of the true flow leads to the difficulty that no true stagnation point occurs on the valve. When the valve approaches closure, this is relatively unimportant, as shown by Fig. 5, where it is clear that the pressure almost approaches the stagnation pressure at the point of intersection of the velocity profiles.

It was decided to explore the possibility of simply taking the jet velocity profiles for jet half angles β and $\pi-\beta$ and assuming that the

velocity profile along the valve plate is formed by the intersecting profiles after the manner shown in Fig. 5.

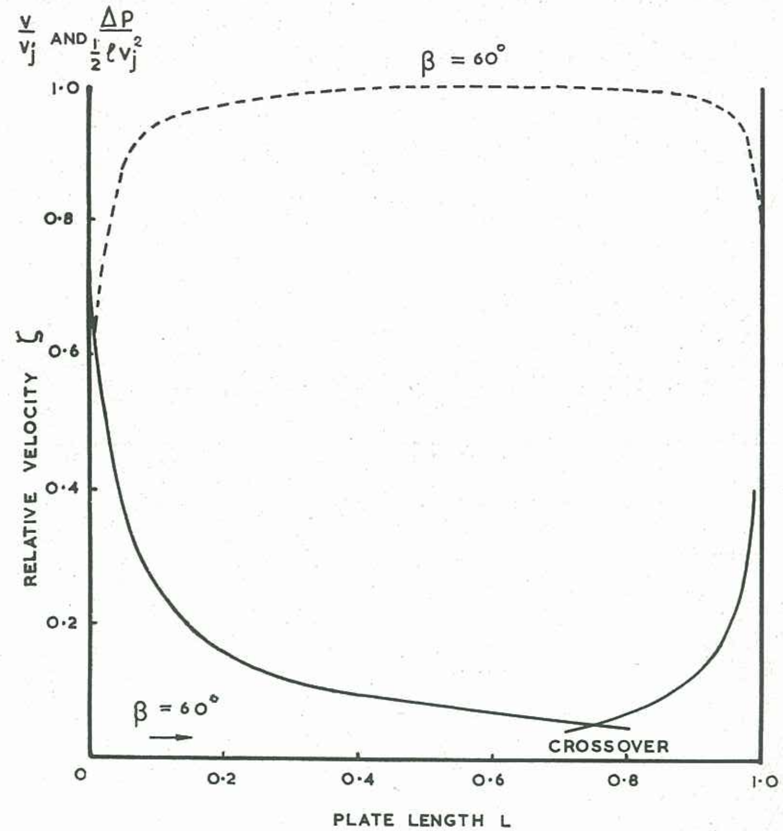

FIG. 5. Typical approximate velocity and pressure profiles.

To do this it was necessary to compute the velocity on the walls of the jet for various values of β. The expression for the velocity V/V_j is given in terms of the opening Δ and β as[5]

$$\frac{x}{\Delta} = \frac{C_c}{\beta}\left[1 - \frac{1}{\zeta} + 2\int_{\zeta}^{1} f(y)\,dy\right] \tag{2}$$

where $\zeta = V/V_j$, the velocity ratio and $f \equiv \dfrac{y^{\left(\frac{\pi}{\beta}-2\right)}}{1+y^{(\pi/\beta)}}$. The results are shown in Figs. 6, 7 and were obtained using the SILLIAC computer to about 1 in 1000 accuracy. Clearly the profiles will bear little quantitative resemblance to the truth for small angles, but since the moment depends only on the *difference* between the pressure ordinates on each side of the pivot, it is found that the

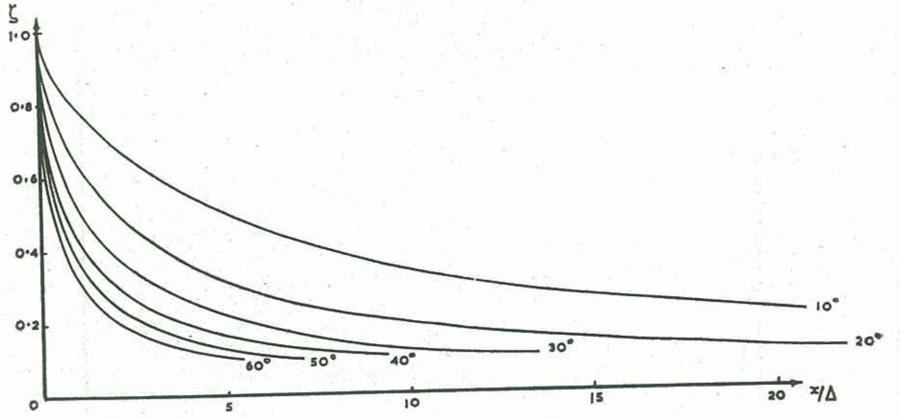

FIG. 6. ζ as a function of x/Δ for $\beta = 10° - 60°$.

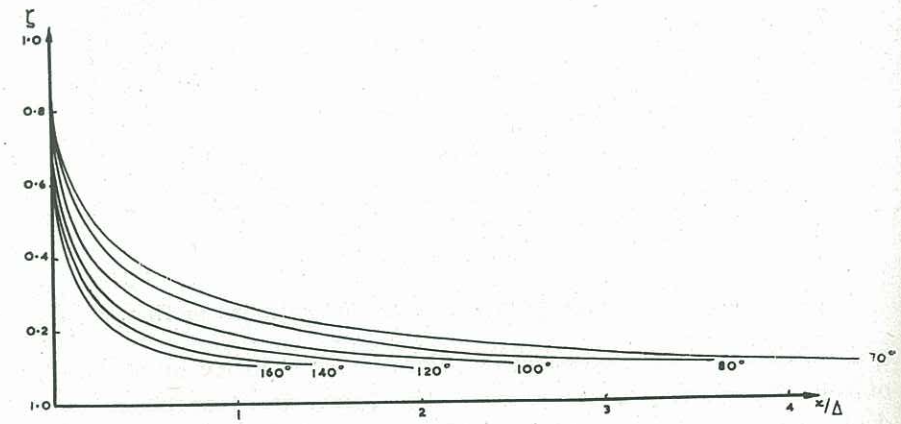

FIG. 7. ζ as a function of x/Δ for $\beta = 70° - 160°$.

error in the centre of pressure is much less than might be supposed, at least when $\beta > 20$ degrees.

On the other hand, values of the valve force F_n calculated from the jet model by using the velocity profiles of Figs. 6, 7 give too small a value for F_n, due to not reaching the stagnation point. This is shown in Fig. 4.

4. MOMENT COEFFICIENTS – JET MODEL

From the pressure distributions the moment coefficients $M^* \equiv M/\varrho V_j^2 w L^2$ were found by integration. The results are shown in Fig. 8.

Good agreement with the moment coefficient for 45 degrees determined by Sarpkaya[1] using relaxation is observed. (The value

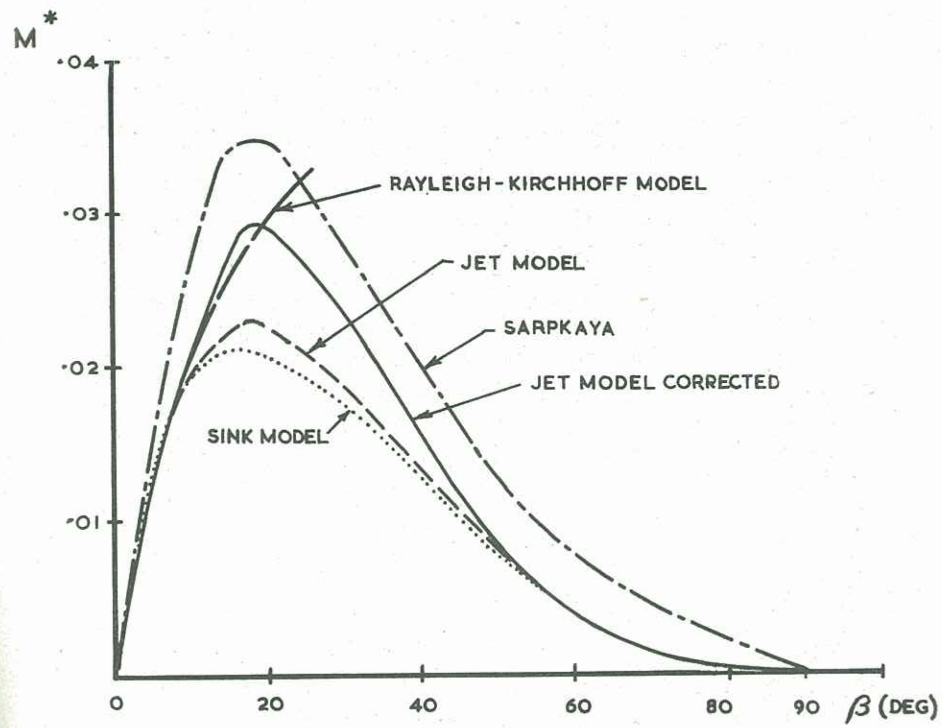

FIG. 8. Moment coefficient M^*.

of about 0·010 for M^* (45 degrees) was determined from diagrams in Ref. (1); a graph in the paper gives M^* about 0·0165.)

The Rayleigh–Kirchhoff[4] value of the moment coefficient M^* and the predicted value of M^* using the jet model but corrected for the low value of F^*, are also given. The corrected curve is obtained effectively by using the jet model to determine the centre of pressure using the approximate velocity distribution while F^* is determined from the coefficients of contraction. The disagreement with Sarpkaya's results is marked, being especially serious as the valve approaches the closed position. This was a very unsatisfactory result and it was felt to be necessary to try and estimate how the moment coefficient behaves as the butterfly becomes almost closed. To do this the jet model was replaced by the sink model, described in the next section.

5. MOMENT COEFFICIENT–SINK MODEL

It may be noted that when the valve is sufficiently closed, and reasonable distances (in terms of valve opening) from the tips of the butterfly are considered, one can replace the jet flow by sinks of suitable strength, located where the valve plate produced cuts the channel walls. The exactness or otherwise of this approach can be found from considering the magnitude of the integral term in Eq. (2) in comparison with $1/\zeta$, which represents a sink.

Ignoring all terms except the first gives an analysis which asymptotically approaches the exact solution in the limit $\beta \to 90$ degrees and which steadily becomes worse as β diminishes. However, the relaxation solution at 45 degrees furnishes a guide by which the errors can be estimated. Denoting the valve opening by Δ (Fig. 1) and the jet width by $h_{1,2}$ it is clear from continuity that

$$h_{1,2}\, V_j = C_{c1,2} \Delta V_j \tag{3}$$

and since in the present case

$$2\Delta = L(1-\sin\beta) \tag{4}$$

it is clear that, for an equivalent sink strength

$$\frac{V}{V_j} \equiv \zeta = \frac{\Delta C_c}{\alpha(x+\Delta \operatorname{cosec}\alpha)} \equiv \frac{C_c(\alpha)(1-\sin\alpha)}{2\alpha\left[\dfrac{x}{L}+\dfrac{1-\sin\alpha}{2\sin\alpha}\right]}$$

where $\alpha=\beta$ or $\pi-\beta$ and x is the distance along the valve measured from the butterfly tip.

Denoting the "crossover" or "stagnation" point by $x_s/L = X$ measured from tip 1 of the butterfly, it is easily shown that the

moment coefficient M^* is given by

$$M^* = \frac{1}{2}\int_0^X \frac{\Delta p_1}{{}^1\!/_2\,\varrho V_j^2}\left(\frac{1}{2}-\frac{x}{L}\right) d\left(\frac{x}{L}\right) - \frac{1}{2}\int_0^{1-X} \frac{\Delta p_2}{{}^1\!/_2\,\varrho V_j^2}\left(\frac{1}{2}-\frac{x}{L}\right) d\left(\frac{x}{L}\right) \tag{6}$$

where

$$\frac{\Delta p}{{}^1\!/_2\,\varrho V_j^2} = 1 - \frac{C_c^2(1-\sin\beta)^2}{4\alpha^2\left[\dfrac{x}{L}+\dfrac{1-\sin\beta}{2\sin\beta}\right]^2}$$

X is found by equating velocities V, i.e.

$$\frac{C_c(\beta)(1-\sin\beta)}{2\beta\left[X+\dfrac{1-\sin\beta}{2\sin\beta}\right]} = \frac{C_c(\pi-\beta)(1-\sin\beta)}{2(\pi-\beta)\left[(1-X)+\dfrac{1-\sin\beta}{2\sin\beta}\right]} \tag{7}$$

For β just less than 43 degrees, the solution X of (7) is greater than unity, hence the acute angled sink profile extends right across the plate. For $\beta > 45$ degrees the result for M^* is

$$M^* = K(\beta)\left[\frac{1}{\gamma}+2\ln\left(\frac{\gamma}{\gamma+X}\right)+2-\frac{1+2\gamma}{\gamma+X}\right] - K(\pi-\beta)\left[\frac{1}{\gamma}+2+2\ln\left(\frac{\gamma}{\gamma+1-X}\right)-\frac{1+2\gamma}{\gamma+1-X}\right]$$

where

$$\gamma = \frac{1-\sin\beta}{2\sin\beta},\quad 2K(\beta) = \left[\frac{C_c(\beta)(1-\sin\beta)}{2\beta}\right]^2 \tag{8}$$

For $\beta=40$ degrees $X=1$, and the second term disappears. These results are plotted on Fig. 8. They compare quite closely with the jet and relaxation approximations, at least for $\beta>45$ degrees. It may be noted that $dM^*/d\beta=0$ when $\beta=90$ degrees. Since $M^*(\pi/2-\delta) = -M^*(\pi/2+\delta)$, the curve is antisymmetric about $\pi/2$ and hence $M^*(\pi/2-\delta)=0(\delta^3)$. In the same way we find

$$F^* = \frac{1}{2} - K(\beta)\left[\frac{2}{\gamma}-\frac{2}{\gamma+X}\right] - K(\pi-\beta)\left[\frac{2}{\gamma}-\frac{2}{\gamma+1-X}\right] \tag{9}$$

where $K(\beta)$, γ are defined in Eq. (8).

This is compared to other evaluations in Fig. 4. Agreement with the jet model is close.

6. TWO TO THREE-DIMENSIONAL TRANSITION

The calculations have so far dealt with two-dimensional valves whereas practical interest is mainly in three-dimensional valves. There is clearly no hope of a simple approach to the three-dimensional case, for the flow pattern is obviously most complex. In the face of such difficulties, the simplest course is to make the sweeping assumption which appears to work in the case of truly axisymmetrical flows[8, 9, 10] and assume that the characteristics of the three-dimensional flow associated with a given pipe area/orifice area ratio are the same as those for a two-dimensional flow of the same area ratio. Specifically, we assume, for a given area ratio and given β, that C_c and F^* are the same in the two and three-dimensional cases. For the butterfly valve the channel area/orifice area is $1/(1 - \sin\beta)$ for both two and three-dimensional cases, so they do preserve area ratios for a given β.

Consider in turn the C_c, F^* and M^* coefficients. We take thin strips (width dw) perpendicular to the valve axis and suppose they behave in the same manner as regards C_c and F^* in both three and two dimensions. Then $(C_c)_{3D}=(C_c)_{2D}$ and no correction is needed to predict the three-dimensional value. This has been confirmed experimentally.[1] F^* is defined to be $F_n/\varrho V_j^2 Lw$ in the two-dimensional case.

Comparison with the circular pipe case shows that if

$$(F^*)_3 = F_n/\varrho V_j^2 D^2 \tag{10}$$

then, at least near closure, the values of $(F^*)_3$ will be only $\pi/4$ of F^*. Hence it appears that a factor $\pi/4$ should be included in (10) so that

$$(F^*)_3 = F_n/\frac{\pi}{4}\varrho V_j^2 D^2 = F^* \tag{11}$$

and no further correction is needed in comparing F^* and $(F^*)_3$. If the moment coefficient $(M^*)_3$ is defined to be

$$(M^*)_3 = \frac{M}{\varrho V_j^2 D^3} \tag{12}$$

then a multiplying factor must be inserted to make a useful comparison. Unfortunately, this factor is more difficult to find for M^* than for F^*. Guided by the calculation for F^*, it appears that a circular butterfly of diameter D is then equivalent to a square butterfly of side $L = D\sqrt{(\pi/4)}$.

Hence it would appear reasonable to define $(M^*)_3 = (4/\pi)^{1\cdot5} M/\varrho V_j D^3$, except that the centre of pressure will lie closer to the pivot for the circular valve than for the square valve. Assuming that each strip behaves as if it were in a two-dimensional flow, one has, for the moment dM of a strip of the circular valve,

$$dM = \varrho V^2 M^* L^2 dw \tag{13}$$

where $L^2 = D^2 - 4w^2$, w being the distance of the strip (width dw) from the centre-line of the pipe.

Integration gives

$$M = \tfrac{2}{3} M^* \varrho V_j^2 D^3 \tag{14}$$

which implies that the moment is decreased by $\frac{2}{3}$ of the square plate case due to the shape of the plate. Combining with the result found above, it is suggested that a fair comparison of $(M^*)_3$ and M^* is obtained with

$$(M^*)_3 = \frac{2}{3}\left(\frac{4}{\pi}\right)^{1\cdot5} M^* \approx 0{\cdot}957 M^* \tag{15}$$

This relationship has been used in the rest of the paper. It is clear that only careful experiment could decide whether or not Eq. (15) is reasonable; for the present it is taken as a working hypothesis.

7. DISCUSSION

It is clear that the main differences between Sarpkaya's work and the present paper occur as $\beta \to 90$ degrees. His graph shows that $dM^*/d\beta \to$ constant as $\beta \to 90$ degrees, while the present treatment gives $dM^*/d\beta = 0$ at that point (Fig. 8); since the jet and sink models become almost exact in this region, it is concluded that there is an error in his torque curve. This conclusion is strengthened by the disagreement of the torque coefficient value M^* given on his graph for $\beta = 45$ degrees and the value determined directly from the relaxation curve quoted elsewhere in the paper.

The final results obtained (which must be regarded as provisional) are shown in Fig. 9, comparison being made with experimental data quoted in Ref. (1). Although there is a discrepancy between the experimental data (whose accuracy is unknown) and the values predicted from the sink model it is clear that the present prediction is much better than Sarpkaya's, particularly as $\beta \to 90$ degrees. For small angles ($\beta \sim < 19$ degrees) it is clear that the Kirchhoff–Rayleigh[4] treatment is adequate, and in view of the physical limitations of the inviscid model, no further refinement is warranted. The chief unknowns are the uncertain experimental accuracy and the 2–3-dimensional "factor". It is hoped that this can be determined properly in the future. Certainly, it appears that the jet model will

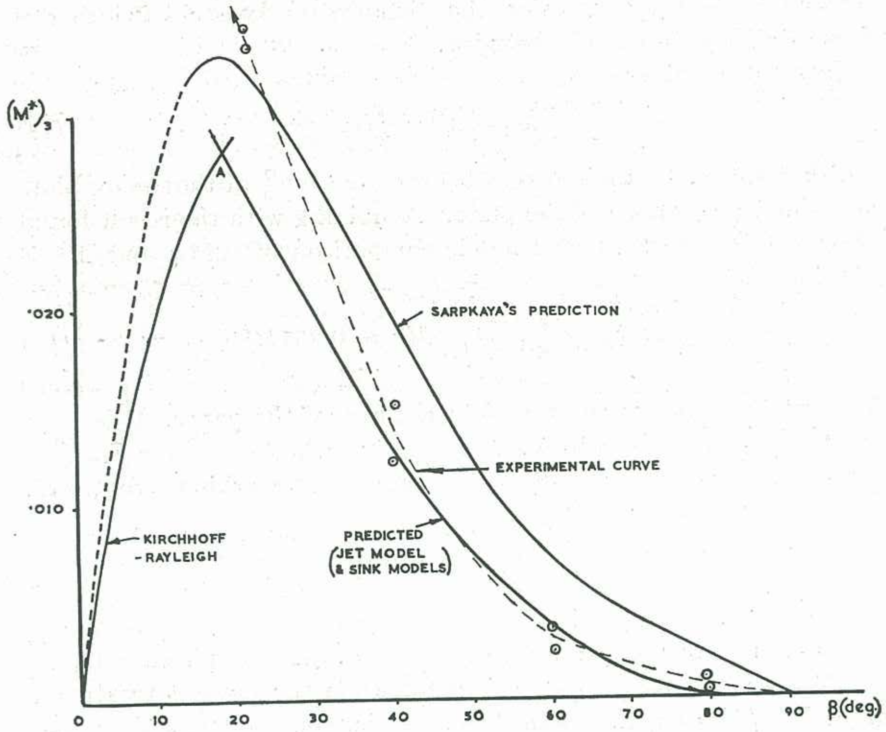

FIG. 9. Tentative results for circular butterfly.

suffice for guiding the choice of any proposed combination of wall angles, and it is certainly feasible to extend the analysis to the compressible case. It is therefore concluded that the present analysis is successful in predicting discharges, forces, and moments for surprisingly large openings; and near $\beta = 0$ the Kirchhoff–Rayleigh model (no wall effect) will suffice. As a corollary to the last remark, it appears to be useless to expect a convergent (Fig. 2b) (or divergent) channel to significantly alter the torque coefficient near the open position. Hence only for systems in which V_j increases

considerably as the valve approaches closure (so that the torque maximum occurs for a higher value of β than does the torque coefficient maximum) is the convergent–divergent wall scheme expected to be of practical interest in reducing peak torque on the valve.

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