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HYDRAULIC ANALOGY STUDIES OF SHOCK ATTACHMENT TO WEDGES

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Synopsis – Transonic flow over two-dimensional wedge profiles may be investigated by the use of the hydraulic analogy. Keeping the water depth small the physical water flow around the wedge can be treated as a distorted dissimilar model of a corresponding prototype gas flow about the same profile and the transfer of data is effected by the application of transonic similarity laws.

Hydraulic analogy studies of bow shock wave detachment distances have indicated that, although there is a considerable disagreement with published gas dynamics data for shocks that are well detached, the conditions at the point of shock attachment can be predicted. This property of the analogy has been used to determine the Mach numbers at shock attachment for wedges moving with finite incidence to the free stream and it is shown that the results so obtained also satisfy transonic similarity laws. The influence of the afterbody on the attachment Mach number is also discussed.

LIST OF SYMBOLS

C_w	Velocity of surface wave propagation
F	Free stream Froude number
K	Transonic similarity parameter
l	Length of wedge upstream of sonic point (chord)
M	Free stream Mach number
r	Half thickness of wedge at sonic point
α	Angle of incidence
γ	Ratio of specific heats
δ	Displacement thickness of boundary layer
ν	Kinematic viscosity of water
θ	Total angle of wedge
τ	Thickness ratio

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Suffix 1 Analogue gas value ($\gamma = 2$)
2 Prototype gas value ($\gamma = 1{\cdot}4$ for air)
A Value for wedge A
B Value for wedge B

1. INTRODUCTION

The purpose of this paper is to describe the results of shock wave attachment studies, for wedges moving at finite incidence to the free stream, obtained by means of the hydraulic analogy.

Although the similarity of the equations of two-dimensional flow of a perfect gas and open channel potential liquid flow has been appreciated for over forty years, it is only in the last decade that a distinction has been made between the mathematical and physical analogies(1) and methods have been devised whereby experimental hydraulic data may be transferred to corresponding gas flow.

2. THE HYDRAULIC ANALOGY AS A DISTORTED DISSIMILAR MODEL

A mathematical analogy exists only between an ideal two-dimensional flow of water in a rectangular channel with a horizontal bottom and a two-dimensional isentropic flow of a perfect gas having the ratio of specific heats $\gamma = 2$. When employing the analogy to investigate transonic flow over wedge profiles the required information must be obtained from a real water flow, which has finite viscosity and surface tension, therefore *a priori* assumption must be made that the real water flow may also be regarded as equivalent to an inviscid analogue gas flow having $\gamma = 2$. It is now generally agreed that this is possible, provided that, (a) the depth of water does not exceed 0·25 in. (optimum depth is 0·200 in.), (b) the models have chord lengths from 1–2 ft, (c) the energy loss across the bow surge (shock) is small, and (d) the towed model technique is used. Bryant(2, 3) has shown that under these conditions the physical water flow around the wedge can be treated as a distorted dissimilar model of a corresponding inviscid prototype gas flow ($\gamma = 1{\cdot}4$) about the same profile.

The operating condition is obtained by applying transonic similarity laws to the analogy which must be operated in such a way that the transonic similarity parameter in a form due to Spreiter(4)

$$K = \frac{M^2-1}{[M^2(\gamma+1)\ (r/l)]^{2/3}}$$

has the same numerical value for affinely related model and prototype profiles.

Bryant has also pointed out that the development of boundary layer along the walls of the model profile has the effect of thickening the profile and that this introduces an "effective" slope distribution which is not that of the profile being tested. There is no corresponding boundary layer in the inviscid prototype gas flow and the boundary layer which forms in the real gas flow cannot be considered by the analogy since it can only give predictions for an inviscid gas flow. The effective profile is defined as having a semi-thickness

$$r_1 = r_2 + \delta \tag{1}$$

where δ is the displacement thickness of the laminar boundary layer for a distance l along a flat plate

$$\delta = 1{\cdot}73\sqrt{(\nu l/FC_w)} \tag{2}$$

Allowing for the boundary layer distortion the operating condition becomes

$$\frac{F^2-1}{\left[2\left(\gamma_1+1\right)\left(\frac{r_2+\delta}{l}\right)\right]^{2/3}} = \frac{M^2-1}{\left[M^2\left(\gamma_2+1\right)\left(\frac{r_2}{l}\right)\right]^{2/3}} \tag{3}$$

where

$$l_1 = l_2 = l \tag{4}$$

Inspection of Eq. (3) shows that the practicable method of transferring data is to test the prototype profile in water flow and to determine the Mach number of the corresponding prototype flow from this equation.

3. EXPERIMENTAL SHOCK ATTACHMENT RESULTS

Experimental work on bow shock wave detachment distances using the towed model hydraulic analogy tank at the University of New South Wales has been previously reported in Refs. 2 and 5. The results, as seen from Fig. 1, are considerably different from the gas dynamics data compiled by Warren(6) except in the range near shock attachment. It has been shown in Ref. 7 that a steady shock wave pattern can be established only after steady motion has been maintained for an infinite time; consequently the lack of agreement for shocks that are well detached is thought to be due to the fact that steady final flow conditions have not been reached in the relatively short (14 ft 7 in.) analogy tank. On the other hand the good agreement near the shock attachment is in accord with Rao[(8)], who has shown that the shock conditions at attachment are independent of acceleration.

In the tests reported in this paper the property of the analogy that conditions at the point of shock attachment can be predicted using the hydraulic analogy has been utilized to determine the Mach numbers at shock attachment for wedges moving with finite incidence to the free stream. These tests were designed not only to obtain the attachment Mach numbers but to serve also as a further check on the range over which distorted dissimilar modelling technique and transonic flow theory will collapse and predict experimental data.

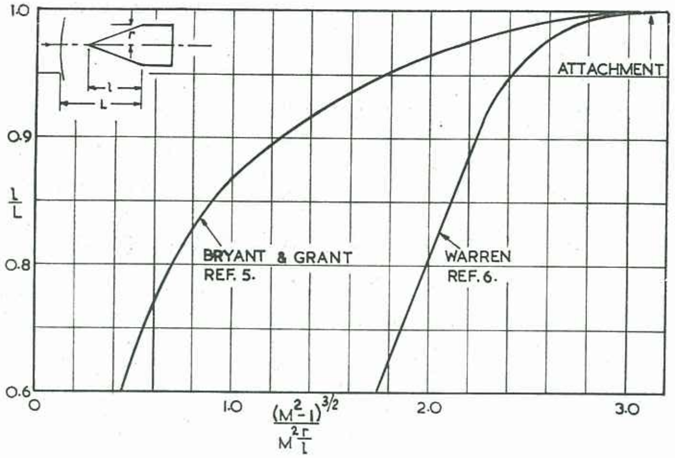

Fig. 1. Two-dimensional bow shock wave detachment distances as function of Spreiter's parameter.

Transonic flow theory states the flow pattern around two wedges A and B will be similar if the similarity parameter for the two wedges has the same numerical value, i.e.

$$\frac{M_A^2-1}{[M_A^2(\gamma+1)\tau_A]^{2/3}}=\frac{M_B^2-1}{[M_B^2(\gamma+1)\tau_B]^{2/3}} \tag{5}$$

where the thickness ratio τ, in general, represents simultaneous change in the thickness ratio, camber and angle of incidence. Equation (5) is satisfied if the angles of incidence are set in proportion to the thickness ratios,

$$\frac{\alpha_A}{\alpha_B}=\frac{\tau_A}{\tau_B} \tag{6}$$

Figure 2 shows the shock attachment Mach and Froude numbers plotted against angles of incidence for 20, 15 and 9 degree wedges, having chord lengths of 12 in., 15 in. and 15 in. respectively. Figure 3 shows the same results plotted with the Spreiter similarity parameter (in a modified form) as the ordinate. In each case the full lines are drawn through the experimental points and the dashed lines represent the attachment conditions predicted by Eq. (6) using the

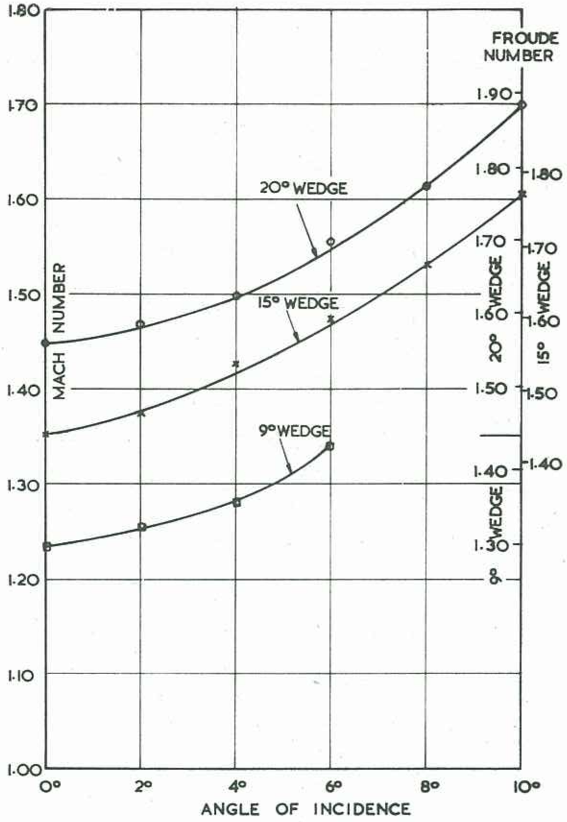

Fig. 2. Attachment Mach and Froude numbers as function of angle of incidence.

experimental values for the 20 degree wedge as the independent variable.

Figure 3 shows that the predicted and experimental values for the 15 degree wedge are in good agreement up to 6 degrees of incidence, whilst on the other hand there is a considerable disagreement for the 9 degree wedge beyond 1 degree of incidence.

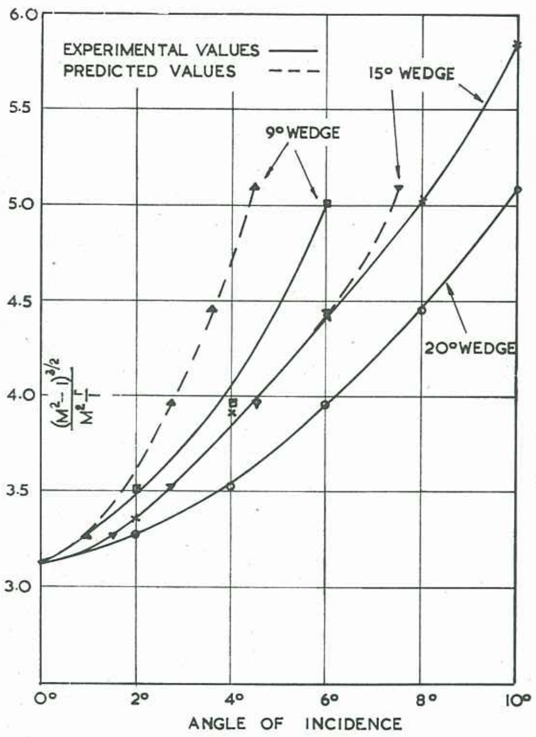

FIG. 3. Spreiter's parameter at attachment as function of angle of incidence.

4. DISCUSSION OF RESULTS

Examination of experimental data on hydraulic jumps shows that there are three distinct types of hydraulic jumps for free stream Froude numbers between $F = 1$ and $F = 2$:

(a) smoother undular
(b) broken undular, and
(c) plain, fully turbulent.

The change from type (a) to type (b) occurs at a free stream Froude number $F \approx 1{\cdot}26$, whilst the change from type (b) to type (c) is much harder to define. It has been suggested by Binnie and Orkney(9) that this change starts to take place at $F = 1{\cdot}55$, which is somewhat lower, than the figure of $F = 1{\cdot}73$ given by Bakhmeteff and Matzke.(10)

If the lower figure of $F = 1{\cdot}55$ is accepted for the change from a broken undular jump to a fully turbulent jump then the experimental results for the 9 degree wedge fall in the former and those for 20 and 15 degree wedges in the latter régime. Different modes of energy dissipation are associated with these two régimes. In the fully turbulent hydraulic jump the energy is dissipated in turbulence, whilst in the broken undular type jump the energy is mostly radiated in a wave form (capillary waves) and only a small amount is lost in turbulence. Also higher energy losses are encountered in the fully turbulent régime and, therefore, one cannot expect to transfer data from one régime to the other successfully. It is also to be remembered that validity of the analogy requires that the losses across the hydraulic jump be small. From the viewpoint of the analogy the formation of a fully turbulent jump would seem desirable, since this type of jump closely resembles a shock wave in the gasdynamics case. Nevertheless, the transonic flow theory has been shown to hold for up to $F \approx 1{\cdot}75$, provided that one works within one hydraulic jump régime.

To determine if afterbodies have any effect on shock attachment conditions when wedges are moving with finite incidence to the free stream, tests were carried out with 15 and 20 degree wedges without afterbodies and with afterbodies equal in length to half the distance from sonic point to the leading edge. Over the range tested (i.e. up to 10 degrees of incidence) no significant difference in attachment Mach numbers was recorded. This suggests that there was no appreciable movement of sonic points away from the shoulders with increasing incidence and Mach number.

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TRANSDUCERS FOR DYNAMIC FLUID PRESSURES

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Synopsis – "Dynamic" pressures are defined as those which are varying too fast for the human senses to follow without the aid of memory or recording devices. Accurate measurement demands that the in-service behaviour of the measuring instrument be known in sufficient detail so that the results obtained from it can be soundly interpreted. This paper is concerned with transducers for the conversion of dynamic pressures to electrical signals. The object is to simplify the task of the potential user who has to choose transducers for specific applications. The basic principles of these devices are reviewed and the performance characteristics described.

The relationships between size, accuracy, stability, sensitivity, frequency response and immunity from environmental changes are discussed, and supported by data from various sources. A survey is made of some types of mechano-electric effects that have practical application to pressure transducers. Like many other problems, that of selecting or designing a transducer is solved most successfully if a suitable compromise between many conflicting factors can be reached. This is illustrated by examples of several types of pressure transducers, applied in diverse fields, including references to the latest available data on the newest designs.

1. INTRODUCTION

Experimental investigations in the field of Hydraulics and Fluid Mechanics almost always involve measurement of the pressure of the working fluid, at one or more points in the system and at different instants in time. Under some conditions, these pressures may remain steady enough for readings to be taken by observation of gauges, dials, manometers or other, more sophisticated, devices which can be understood directly by the human senses. On many occasions, however, when the dynamic behaviour of a system is being studied, pressures are varying too fast for human senses to follow so that