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ON THE HORIZONTAL DIFFUSION OVER THE SEA SURFACE

EIICHI INOUE

Division of Meteorology, National Institute of
Agricultural Sciences, Nishigahara, Tokyo, Japan

Synopsis — Taking the recent developments in the study of atmospheric diffusion into account, several oceanic phenomena of horizontal diffusion are dealt with. The visible diameter of dye patches floating on sea surfaces and the visible outline of dye plumes injected continuously from fixed sources are theoretically calculated with the similarity theories of turbulence. The diameter of floating dye patches increases with $t^{3/2}$ for the small amount of time t after its initiation and the time-mean width of continuous dye plumes increases with x for the small amount of distance x from the source. Comparing these theoretical results with experimental values the turbulent characteristics of oceanic surface flows are estimated, and the possible applications of these estimations to practical problems of water-pollution and of model experiments are discussed.

LIST OF SYMBOLS

x, y, z	Distances measured from the centre of dye patch floating over the sea surface, x and y lying horizontally and z directing vertically downward
X, Y, Z	Displacements of dye particles along three directions from the centre of patch
Q	Source intensity of dye released at the centre at time $t=0$
χ	Concentration of dye
χ_h	Vertically integrated dye concentration
χ_*	Visible lower limit of χ_h
ε	The mass rate of turbulent energy dissipation at the sea surface
r_0	Radius of visible domain of dye patch
T	Time after the release of dye when the dye patch disappears

$R_L(\xi)$	Lagrangian correlation between velocities of a same dye particle at time ξ_0 and $\xi_0+\xi$
U	Mean flow velocity
v	Turbulent flow velocity
Δ	Scale of turbulent eddy
τ	Life time of turbulent eddy

1. INTRODUCTION

Even though the theories of turbulent diffusion are still under development, many practical diffusion problems in the atmosphere such as air-pollution have been fairly successfully dealt with by means of recent theories. In particular, in the atmosphere the diffusing smoke puffs and plumes have sometimes been considered as some sort of meteorological tool for measuring several characteristics of the atmospheric turbulence.(1, 2, 3) Similarly, in the ocean the diffusing dye patches and plumes can be good oceanographic tools for measuring the oceanic turbulence. In this paper, some possible methods of estimating oceanic turbulence, with data presented by several oceanographers, will be discussed. Furthermore, some possible applications of the knowledge on oceanic turbulence to future research in physical oceanography will be proposed.

2. TWO TYPES OF TURBULENT DIFFUSION

Present theories of turbulent diffusion are almost always concerned with the statistical nature of diffusion phenomena. In other words, the very irregular form of single smoke puffs or plumes cannot be precisely represented by the theoretical calculations, but the statistical configurations of such puffs or plumes can be, provided that a number of them are conveniently superposed after sufficient trials have been carried out.

For example, theoretically a visible outline of a dispersing smoke puff in the atmosphere is represented by a circle. In practice, however, such an outline is rarely circular, and mostly very complicated. Recently, some trials have been proposed to express such a complicated shape in terms of turbulence in wind observed at the same place and during the same period.(5) To the present author, however, the practical value of such a treatment seems rather obscure.

In this paper, for the sake of simplicity, turbulent diffusion phenomena are divided into two types. One is the diffusion of smoke puffs generated instantaneously by such a burst of anti-aircraft shell or fireworks. This is called here the instantaneous floating-source-type diffusion, since the diffusion is usually represented with regard to its floating center of gravity. The other is the average diffusion of smoke emitted continuously from a fixed source, such as an exit from a stack, for a sufficiently long time, and is called here the continuous fixed-source-type diffusion. For this latter case, the diffusion is usually described with regard to the position of fixed source.

For the diffusion of dye over sea surfaces, the same divisions are considered necessary.

3. VISIBLE OUTLINE OF FLOATING DYE PATCHES

Recently several experiments of dispersing dye over sea surfaces have been carried out to investigate the diluting activity of sea-water. These experiments are related to water pollution(6–8) problems due to sewage and atomic waste products.

It may be plausible to assume that the concentration distribution of dye within the dyed portion is statistically of the Gaussian form. With respect to the floating center, $x=y=z=0$, the concentration distribution may be represented by

$$\chi = \frac{Q}{2^{1/2}\pi^{3/2} < X^2 >^{1/2} < Y^2 >^{1/2} < Z^2 >^{1/2}} \cdot \exp\left\{-\frac{1}{2}\left(\frac{x^2}{< X^2 >}+\frac{y^2}{< Y^2 >}+\frac{z^2}{< Z^2 >}\right)\right\}$$

where x and y extend horizontally, z vertically and $< X^2 >^{1/2}$ and others denote respectively the standard deviations of concentration distribution. The source intensity Q is normalized by the formulation of

$$Q = \int_0^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \chi \, dx\, dy\, dz$$

The intensity of colour viewed from an aircraft high above the sea surface may be represented by

$$\chi_h = \int_0^{\infty} \chi \, dz$$
$$= \frac{Q}{2\pi < X^2 >^{1/2} < Y^2 >^{1/2}} \exp\left\{-\frac{1}{2}\left(\frac{x^2}{< X^2 >}+\frac{y^2}{< Y^2 >}\right)\right\}$$

which is of quite the same form as that of floating smoke puffs in the atmosphere being viewed from the ground surface.(9, 10)

When the horizontal scales of both $<X^2>^{1/2}$ and $<Y^2>^{1/2}$ are sufficiently smaller than that of the horizontally largest eddy over the sea surface, but sufficiently larger than that of the smallest eddy, the similarity theory(2, 11) of turbulence gives rise to

$$A^2 = <X^2> = <Y^2> = \frac{2}{3}\varepsilon t^3$$

The scale of the largest eddy depends upon many factors, such as the depth of sea, the distance from the coast and the temperature distribution of sea-water. Unfortunately, however, little has been known of the general nature, except Nan'niti and Okubo's fairly crude estimations.(6) The scale of the smallest eddy is roughly estimated by $\nu^{3/4}\varepsilon^{-1/4}$, where ν is the kinematic viscosity of sea-water.(12) Assuming that $\nu \approx 10^{-2}$ cm^2 sec^{-1} and $\varepsilon \approx 10^{-4}$ cm^2 sec^{-3}, the scale of the smallest eddy is roughly given by 0·3 cm.

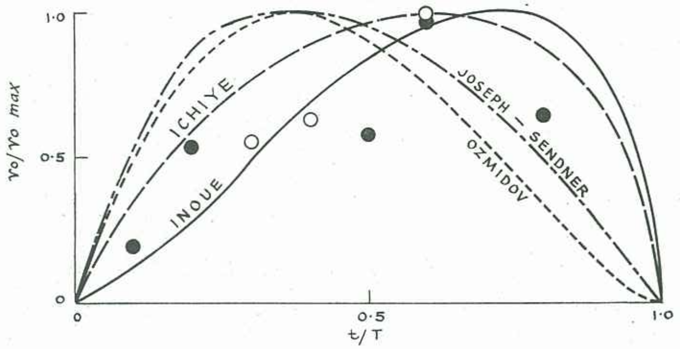

FIG. 1. Four theoretical curves and experimental data (● and ○).

For such intermediate scales of dye patches the distribution of colour intensity is given by

$$\chi_h = \frac{Q}{2\pi A^2}\exp\left(-\frac{r^2}{2A^2}\right)$$

where $r^2 = x^2 + y^2$ and r denotes the radius of the statistical circle of patches. Representing the critical concentration at the visible outline as χ_*, the radius r_0 of the visible domain is given by

$$r_0^2 = \frac{4}{3}\varepsilon t^3 \ln \frac{Q}{\frac{4}{3}\pi\varepsilon t^3 \chi_*}$$

and the relation between r_0 and t is illustrated in Fig. 1.

The above relationship clearly shows that the radius vanishes at the time of

$$T = \left\{\frac{Q}{(4/3)\pi\varepsilon\chi_*}\right\}^{1/3}$$

that is to say, after the time of T the dye patches disappear and that at the characteristic time of $t = T/e^{1/3} = 0{\cdot}714T$ the visible area is the maximum. Making use of the time T the expression of the radius is transformed to

$$r_0 = \sqrt{\left(\frac{Q}{2\pi\chi_*}\right)}\left(\frac{t}{T}\right)^{3/2}\left\{\ln\left(\frac{T}{t}\right)\right\}^{1/2}$$

Recently, a few other formulations of r_0 have been presented which are in general given by

$$r_0 = B\left(\frac{t}{T}\right)^m\left(\ln\frac{T}{t}\right)^n$$

where B is a characteristic radius and m and n denote specific numbers, respectively. Several examples of the numerical values are tabulated as follows:

Authors	m	n	n/m	$e^{-n/m}$
Inoue[1]	3/2	1/2	1/3	0·714
Ichiye[7, 8]	1	1/2	1/2	0·606
Ozmidov[13]	3/2	3/2	1	0·368
Joseph and Sendner[14]	1	1	1	0·368

The values of $e^{-n/m}$ are especially added in the table, since at $t = Te^{-n/m}$ the radius attains to the maximum.

The four curves corresponding to four different formulations are given in Fig. 1, together with the observed data presented by Ichiye.(7, 8)

It seems rather difficult to decide which gives the best fit to the observations. More data as well as further theoretical considerations are desirable.

4. VISIBLE OUTLINE OF CONTINUOUS DYE PLUMES

The practical application of the theoretical results of continuous fixed-source-type diffusion to the diffusion phenomena over sea surfaces have not yet been carried out so frequently, except Hanzawa's treatment in Japan of volcanic pumices dispersed in the Pacific Ocean. In a similar manner, the lateral width of diffusing dye plumes is to be given by the well-known Taylor's expression as follows:

$$<Y^2> = 2<v^2> \int_0^t \int_0^n R_L(\xi)\, d\xi\, dn$$

where $<v^2>^{1/2}$ is the lateral velocity of oceanic turbulence and $R_L(\xi)$ denotes the Lagrangian correlation function.

In the atmosphere, the above expression has been used both to estimate the contaminated area by air pollutant and to characterize the atmospheric turbulence.(2, 3, 15)

To the present author, the same methods appear to be applicable even to the oceanic turbulence. For example, the water pollutants continuously emitted from steaming ships might be dealt with successfully by such methods. The meandering problems of contaminated trails left behind steaming ships are also to be dealt with, taking into account particularly the life-time of eddies in oceanic turbulence fields.(2) The meandering will be also of importance to estimate the persisting time at a certain point of the highly polluted water masses.

5. SIMILITUDE OF DIFFUSION PHENOMENA

If we can estimate precisely in oceans the energy dissipation ε, the velocity $< v^2 >^{1/2}$, the life-time τ of eddies and so on, the similitude of diffusion phenomena will be obtained fairly easily. For example, the diffusion angle can be set equal under the condition of the same intensity of turbulence, $< v^2 >^{1/2}/U$, and the same ratio of Δ/L, where Δ is the scale of eddy and L the scale of object, will further add the similarity of the overall diffusion area.

One thing to be considered particularly is the similitude of the meandering time, which may characterize the life-time of oceanic phenomena. Since the life-time of eddies is characterized by

$$\tau = \frac{<v^2>}{\varepsilon} = \frac{\Delta}{<v^2>^{1/2}} = \varepsilon^{-1/3}\Delta^{2/3}$$

the similitude of meandering periods might be expressed by such a simple relation as

$$\frac{\tau_{\rm I}}{\tau_{\rm II}} = \left(\frac{\Delta_{\rm I}}{\Delta_{\rm II}}\right)^{2/3} \cdot \left(\frac{\varepsilon_{\rm II}}{\varepsilon_{\rm I}}\right)^{1/3}$$

The present author has already discussed the possible application of this relation to estimate the difference in time scale of geometrically similar phenomena in both atmosphere and ocean.(16)

CONCLUSION

Making use of several data on turbulent diffusion phenomena which have been obtained in the atmosphere, the present author has shown possible applications of diffusion theories to oceans. These practical applications should be of great importance to check and improve the foundations of turbulence theory, as has been stressed by Frenkiel(4) at the recent International Symposium on Turbulence and Geophysics, 1961.

The estimation of dissipation energy in oceans, for example, by means of diffusion experiments should be examined by more direct methods such as proposed by Stewart and Grant(12).

The present author is looking forward to closer co-operation between oceanographic and atmospheric scientists which will help to develop the turbulence theories further.

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HYDRAULIC ANALOGY STUDIES OF SHOCK ATTACHMENT TO WEDGES

G. SAIVA*

University of New South Wales,
Kensington, N. S. W.

Synopsis – Transonic flow over two-dimensional wedge profiles may be investigated by the use of the hydraulic analogy. Keeping the water depth small the physical water flow around the wedge can be treated as a distorted dissimilar model of a corresponding prototype gas flow about the same profile and the transfer of data is effected by the application of transonic similarity laws.

Hydraulic analogy studies of bow shock wave detachment distances have indicated that, although there is a considerable disagreement with published gas dynamics data for shocks that are well detached, the conditions at the point of shock attachment can be predicted. This property of the analogy has been used to determine the Mach numbers at shock attachment for wedges moving with finite incidence to the free stream and it is shown that the results so obtained also satisfy transonic similarity laws. The influence of the afterbody on the attachment Mach number is also discussed.

LIST OF SYMBOLS

C_w	Velocity of surface wave propagation
F	Free stream Froude number
K	Transonic similarity parameter
l	Length of wedge upstream of sonic point (chord)
M	Free stream Mach number
r	Half thickness of wedge at sonic point
α	Angle of incidence
γ	Ratio of specific heats
δ	Displacement thickness of boundary layer
ν	Kinematic viscosity of water
θ	Total angle of wedge
τ	Thickness ratio

* Senior Tutor, University of New South Wales.