THE INFLUENCE OF ENTROPY IN ONE-DIMENSIONAL NOZZLE FLOWS

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Synopsis — A simple mathematical model of one-dimensional variable area gas flow, which takes heat addition and irreversibility effects into account, is presented. It is shown what influence entropy changes have on flow properties and some light is cast on the effects of heating and cooling on the location of the critical area in nozzles with non-adiabatic walls.

LIST OF SYMBOLS

A	Area of passage
$C=(\gamma RT)^{\frac{1}{2}}=(\gamma p/\varrho)^{\frac{1}{2}}$	Local velocity of sound
$C_p=R\gamma/(\gamma-1)$	Specific heat at constant pressure
$C_v=R/(\gamma-1)$	Specific heat at constant volume
$F=pA(1+\gamma M^2)$	Impulse function
f	Friction factor
$M=V/C$	Local Mach number
$m=\varrho AV=\varrho AMC$	Mass flow rate
p	Static pressure
$p_0=p\left(1+\frac{\gamma-1}{2}M^2\right)^{\gamma/(\gamma-1)}$	Isentropic stagnation pressure
q	Heat added (see Eq. 21)
R	Gas constant in Equation of State, $p=\varrho RT$
S	Entropy
S^1	See Eq. (27)
S_i	See Eq. (27)
T	Static temperature

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$T_0 = T\left(1+\frac{\gamma-1}{2}M^2\right)$	Isentropic stagnation temperature
V	Local velocity
x	Distance along nozzle axis
$\gamma = C_p/C_v$	Ratio of specific heats
ϱ	Density
*	Indicates critical ($M=1$) conditions

Suffixes

t	Conditions at throat
1	Conditions at datum point

1. INTRODUCTION

In order to establish a simple model of one-dimensional gas flow in nozzles which will cast some light on the role played by entropy changes, it is convenient to limit attention to a gas which obeys the perfect gas laws and which has constant chemical state. It is further assumed that heat is added to (or removed from) the gas by direct transfer through the walls. Thus changes in the isentropic stagnation temperature are considered to be due to external sources.

Dissipation, due to both viscosity and the irreversibility of the thermodynamic processes, is included in the analysis by introducing specific entropy as an independent variable. Naturally, adoption of entropy as an independent variable gives rise to some difficulties: for example it is not possible to investigate how entropy is produced. However, by proceeding in this way it is possible to reach some useful conclusions which supplement present knowledge obtained from other models. Because both heat transfer and viscous dissipation give rise to finite entropy gradients it is usual to find, in these other models, that entropy is considered as a dependent variable. Thus in the well known Shapiro–Hawthorne model[1], the entropy gradient for variable area flow with heat transfer and wall friction is found to be,

$$\frac{dS}{dx} = \frac{\gamma}{\gamma-1}R\left\{\left(1+\frac{\gamma-1}{2}M^2\right)\frac{1}{T_0}\frac{dT_0}{dx}+\frac{(\gamma-1)M^2}{2}\cdot\frac{4f}{D}\right\}$$

indicating that dS/dx is due to two quasi-independent sources, related by heat transfer–boundary layer intraction. Since the influence coefficient relating entropy gradient and the rate of change of duct area is zero in the Shapiro–Hawthorn model, the influence of irreversibility effects, manifested by that part of the entropy gradient not due to heat transfer, on duct area cannot be determined directly. The present analysis overcomes this problem and also gives a more realistic picture than is obtained by considering purely diabatic variable area flow along lines suggested by Barrere *et al.*[2]

2. THE BASIC RELATIONSHIPS

Consider one-dimensional steady shock free flow of a perfect gas having constant specific heats and molecular weight, in a passage of variable area, $A = A(x)$. Suppose there is heat exchange from the surroundings such that $T_0 = T_0(x)$ and due to both heating (cooling) and dissipation there is a variation in the specific entropy along the passage, $S = S(x)$. Let suffix 1 refer to conditions at a datum point, $x = x_1$, near the inlet.

From thermodynamics, the entropy gradient at any point x along the duct is,

$$\frac{dS}{dx} = \frac{\gamma}{\gamma-1}R\frac{d}{dx}\ln T - R\frac{d}{dx}\ln p \tag{1}$$

The isentropic stagnation pressure and temperature are, by definition,

$$p_0 = p\left(1+\frac{\gamma-1}{2}M^2\right)^{\gamma/(\gamma-1)} \tag{2}$$

$$T_0 = T\left(1+\frac{\gamma-1}{2}M^2\right) \tag{3}$$

from which it is seen that,

$$\frac{d}{dx}\ln p_0 = \frac{d}{dx}\ln p + \frac{\gamma}{\gamma-1}\frac{d}{dx}\ln\left(1+\frac{\gamma-1}{2}M^2\right) \tag{4}$$

and

$$\frac{d}{dx}\ln T_0 = \frac{d}{dx}\ln T + \frac{d}{dx}\ln\left(1+\frac{\gamma-1}{2}M^2\right) \tag{5}$$

Using Eqs. (4) and (5) in Eq. (1) it follows that

$$\frac{1}{R}\frac{dS}{dx} = \frac{\gamma}{\gamma-1}\frac{d}{dx}\ln T_0 - \frac{d}{dx}\ln p_0 \tag{6}$$

This equation can be integrated with respect to distance x to show the entropy change is,

$$\frac{S-S_1}{R}=\frac{\gamma}{\gamma-1}\ln\left(\frac{T_0}{T_{0_1}}\right)-\ln\left(\frac{p_0}{p_{0_1}}\right) \tag{7}$$

or in other words that

$$\frac{p_0}{p_{0_1}}=\left(\frac{T_0}{T_{0_1}}\right)^{\gamma/(\gamma-1)} e^{-(S-S_1)/R} \tag{8}$$

Employing Eq. (8) it is then possible to determine a number of integral relations between the states of the gas at points along the duct, relative to corresponding states at the datum point and in terms of the independent variables $M=M(x)$; $T_0=T_0(x)$ and $S=S(x)$.

3. THE INTEGRAL RELATIONS

Integral relations between the properties at arbitrary point x and datum point x_1 may be obtained by using Eq. (8) in conjunction with other equations which involve only point functions.

For example, using Eq.(2) and Eq. (8) it is seen that

$$\frac{p}{p_1}=\left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M^2}\right)^{\gamma/(\gamma-1)}\left(\frac{T_0}{T_{0_1}}\right)^{\gamma/(\gamma-1)} e^{-(S-S_1)/R} \tag{9}$$

and by Eq. (3), it follows that

$$\frac{T}{T_1}=\frac{T_0}{T_{0_1}}\left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M^2}\right) \tag{10}$$

The ratio of densities is found from the equation of state,

$$p=\varrho RT \tag{11}$$

Thus,

$$\frac{\varrho}{\varrho_1}=\frac{p}{p_1}\frac{T_1}{T}$$

$$=\left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M^2}\right)^{1/(\gamma-1)}\left(\frac{T_0}{T_{0_1}}\right)^{1/(\gamma-1)} e^{-(S-S_1)/R} \tag{12}$$

by Eqs. (9) and (10).

By using the equation of continuity,

$$\varrho AMC=\varrho_1 A_1 M_1 C_1 \tag{13}$$

the area ratio is found to be

$$\frac{A}{A_1}=\frac{\varrho_1}{\varrho}\frac{M_1}{M}\left(\frac{T_1}{T}\right)^{\frac{1}{2}}$$

$$=\frac{M_1}{M}\left(\frac{1+\frac{\gamma-1}{2}M^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{(\gamma+1)/2(\gamma-1)}\left(\frac{T_{0_1}}{T_0}\right)^{(\gamma+1)/2(\gamma-1)} e^{(S-S_1)/R} \tag{14}$$

And by definition of local Mach number,

$$\frac{V}{V_1}=\frac{M}{M_1}\left(\frac{T}{T_1}\right)^{\frac{1}{2}}$$

$$=\frac{M}{M_1}\left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M^2}\right)^{\frac{1}{2}}\left(\frac{T_0}{T_{0_1}}\right)^{\frac{1}{2}} \tag{15}$$

Finally the ratio of impulse functions is found as,

$$\frac{F}{F_1}=\frac{pA}{p_1A_1}\left(\frac{1+\gamma M^2}{1+\gamma M_1^2}\right)$$

$$=\frac{M_1}{M}\left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M^2}\right)^{\frac{1}{2}}\left(\frac{1+\gamma M^2}{1+\gamma M_1^2}\right)\left(\frac{T_0}{T_{0_1}}\right)^{\frac{1}{2}} \tag{16}$$

Thus, providing $T_0(x)$, $S(x)$ and $M(x)$ are known it is possible to use Eqs. (9), (10), (12) and (14) through (16) to find values of the dependent variables $p(x)$, $T(x)$, $\varrho(x)$, $A(x)$, $V(x)$ and $F(x)$ at any point along the duct. In application, however, there is difficulty in that whilst $M(x)$ can be prescribed, $T_0(x)$ can only be determined if the temperature distribution along the passage and the walls is known: and $S(x)$ cannot be readily found. Whilst the integral relations give a qualitative picture of how the dependent variables are affected by both heating and entropy change it is clear the model is not very suitable for design purposes. Nevertheless it is possible to apply the results in some special cases.

Before proceeding there is one additional result which is useful. It is obtained by considering the mass flow rate.

The basic relationship for mass flow rate per unit area is,

$$\frac{m}{A} = \left(\frac{\gamma}{R}\right)^{\frac{1}{2}} Mp \left(\frac{1}{T}\right)^{\frac{1}{2}} \tag{17}$$

which can be written as

$$\frac{m}{A} = \left(\frac{\gamma}{RT_{0_1}}\right)^{\frac{1}{2}} M p_{0_1} \left(\frac{p}{p_0}\right)\left(\frac{p_0}{p_{0_1}}\right)\left(\frac{T_{0_1}}{T_0}\right)^{\frac{1}{2}}\left(\frac{T_0}{T}\right)^{\frac{1}{2}} \tag{18}$$

By using Eqs. (2), (3) and (8) this becomes

$$\frac{m}{A} = \left(\frac{\gamma}{RT_{0_1}}\right)^{\frac{1}{2}} p_0 \frac{M}{\left(1+\frac{\gamma-1}{2}M^2\right)^{(\gamma+1)/2(\gamma-1)}} \left(\frac{T_0}{T_{0_1}}\right)^{(\gamma+1)/2(\gamma-1)} e^{-(S-S_1)/R} \tag{19}$$

or, on rearrangement,

$$\frac{m}{p_{0_1}}\left(\frac{RT_{0_1}}{\gamma}\right)^{\frac{1}{2}} = \frac{MA}{\left(1+\frac{\gamma-1}{2}M^2\right)^{(\gamma+1)/2(\gamma-1)}} \left(\frac{T_0}{T_{0_1}}\right)^{(\gamma+1)/2(\gamma-1)} e^{-(S-S_1)/R} \tag{20}$$

Equation (20) gives a general relation between M, A, T_0 and S for a specified mass flow rate with datum conditions p_{0_1}, T_{0_1}, S_1 at area A_1.

4. SOME SPECIAL CASES

It will be noticed several simple types of flow appear as special cases of the foregoing integral relations.

By putting $T_0 \; T_{0_1}$, $S=S_1$, results identical with those of an isentropic analysis are obtained.

Adiabatic variable area flow is obtained by putting $T_0 = T_{0_1}$ see Ref. 3) and diabatic constant area flow by putting $A = A_1$.

In the latter case it follows from Eq. (14) or Eq. (20) that

$$\frac{M_1}{M}\left(\frac{1+\frac{\gamma-1}{2}M^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{(\gamma+1)/2(\gamma-1)} = \left(\frac{T_0}{T_{0_1}}\right)^{(\gamma+1)/2(\gamma-1)} e^{-(S-S_1)/R}$$
$$= \left(1+\frac{q}{C_P T_{0_1}}\right)^{(\gamma+1)/2(\gamma-1)} e^{-(S-S_1)/R} \tag{21}$$

where q is the heat added between x_1 and x.

The geometry of ducts for adiabatic flow with constant Mach number, and hence constant static temperature may be discovered by using Eqs. (20) and (3) to state,

$$\frac{m}{p_{0_1}}\left(\frac{RT_{0_1}}{\gamma}\right)^{\frac{1}{2}}\left(1+\frac{\gamma-1}{2}M_1^2\right)^{(\gamma+1)/2(\gamma-1)} = M_1 A \quad e^{-(S-S_1)/R} \tag{22}$$

It is seen the area must vary according to

$$A = A_1 e^{(S-S_1)/R} \tag{23}$$

which result) may also be obtained directly from Eqs. (10) and (14).

From consideration of Eq. (9) it is evident Eq. (23) is equivalent to $pA = \text{const.}$ That is to say Eq. (23) represents the well-known. pA family of ducts which diverge for all Mach numbers.[4] However, in the case of non-adiabatic flow when constant Mach number does not imply constant static temperature, a different result is obtained using either Eq. (22) or Eqs. (10) and (14). It is,

$$\frac{A}{A_1} = \frac{T_1}{T} e^{(S-S_1)/R} \tag{24}$$

Similarly non-adiabatic flow with constant static temperature involves an area ratio,

$$\frac{A}{A_1} = \frac{M_1}{M} e^{(S-S_1)/R} \tag{25}$$

The ducts indicated by Eqs. (24) and (25) will be different from the pA family.

5. THE LOCATION OF THE CRITICAL AREA

To investigate the influence of the independent variables it is more profitable to use the foregoing equations in differential form. As an example let us see what can be said about the location of the critical area.

The differentiation of either Eq. (14) or Eq. (20) yields,

$$\frac{dA}{dx}=\frac{A}{M}\left(\frac{M^2-1}{1+\frac{\gamma-1}{2}M^2}\right)\frac{dM}{dx}-\frac{A}{T_0}\left(\frac{\gamma+1}{2(\gamma-1)}\right)\frac{dT_0}{dx}+\frac{A}{R}\frac{dS}{dx} \qquad (26)$$

in which the entropy gradient can now be split into two parts to account for heating and irreversibility effects separately. This is done by putting

$$\frac{dS}{dx}=\frac{dS'}{dx}+\frac{dS_i}{dx} \qquad (27)$$

where $dS'/dx=(Cp/T)(dT_0/dx)$, by definition of the thermostatic entropy function and dS_i/dx is positive due to the irreversibility of the real thermodynamic process.

It follows that

$$\frac{dS'}{dx}=\frac{C_P}{T_0}\left(1+\frac{\gamma-1}{2}M^2\right)\frac{dT_0}{dx} \text{ and hence,}$$

$$\frac{A}{R}\frac{dS'}{dx}=\frac{A}{T_0}\cdot\frac{\gamma}{\gamma-1}\left(1+\frac{\gamma-1}{2}M^2\right)\frac{dT_0}{dx} \qquad (28)$$

As will be seen, by using Eqs. (27) and (28) the entropy gradient due to irreversibility can be retained and the entropy gradient due to heat transfer incorporated in the second term on the R.H.S. of Eq. (26). The result is,

$$\frac{dA}{dx}=\frac{A}{M}\left(\frac{M^2-1}{1+\frac{\gamma-1}{2}M^2}\right)\frac{dM}{dx}+\frac{A}{T_0}\left(\frac{1+\gamma M^2}{2}\right)\frac{dT_0}{dx}+\frac{A}{R}\frac{dS_i}{dx} \qquad (29)$$

which gives an idea of the influence of heating on the location of the critical area if we put $M=1$. On doing so it is evident that

$$\left(\frac{dA}{dx}\right)^*=\frac{A^*}{T_0^*}\left(\frac{\gamma+1}{2}\right)\left(\frac{dT_0}{dx}\right)^*+\frac{A^*}{R}\left(\frac{dS_i}{dx}\right)^* \qquad (30)$$

which leads to the immediate conclusions, for heating with $(dT_0/dx)^* >0$, that $(dA/dx)^*>0$ and hence A^* must be located downstream of the minimum cross section (throat) of the passage. It is apparent heat addition tends to move the critical area downstream from its location in adiabatic flow: that is to say move it further downstream

from the throat.† On the other hand it appears from Eq. (30) that cooling, $(dT_0/dx)^*<0$, will cause the critical area to shift towards the throat. Barrere *et al.*[(2)] who considered the case of diabatic flow state that cooling can move the critical area upstream of the minimum cross-section. However, in terms of the present analysis, $(dA/dx)^*$ can only be negative if the first term on the R.H.S. predominates. There is already ample evidence from other models[(5)] that cooling due to heat transfer cannot be preponderant and thus it is safe to infer the critical area should always lie between the throat and adiabatic locus when cooling occurs; and it should always be downstream of the adiabatic locus when heatig occurs.

6. EFFECTS OF HEATING AND COOLING

Entropy change affects the pressure and density distributions as shown in Eqs. 9 and 12. The pressure gradient is found, from Eq. 9, to be

$$\frac{dp}{dx}=-\frac{p}{M}\left(\frac{\gamma M^2}{1+\frac{\gamma-1}{2}M^2}\right)\frac{dM}{dx}-\frac{p}{R}\frac{dS}{dx}+\frac{\gamma}{\gamma-1}\frac{p}{T_0}\frac{dT_0}{dx} \qquad (31)$$

in which

$$\frac{p}{R}\frac{dS}{dx}=\frac{p}{R}\frac{dS'}{dx}+\frac{p}{R}\frac{dS_i}{dx}$$

$$=\frac{p}{T_0}\frac{\gamma}{\gamma-1}\left(1+\frac{\gamma-1}{2}M^2\right)\frac{dT_0}{dx}+\frac{p}{R}\frac{dS_i}{dx} \qquad (32)$$

Thus it is seen that

$$\frac{dp}{dx}=-\frac{p}{M}\left(\frac{\gamma M^2}{1+\frac{\gamma-1}{2}M^2}\right)\frac{dM}{dx}-\frac{\gamma}{\gamma-1}\frac{p}{T_0}\left(\frac{\gamma-1}{2}M^2\right)\frac{dT_0}{dx}-\frac{p}{R}\frac{dS_i}{dx} \qquad (33)$$

Here, with $dS_i/dx>0$ it is evident that, for flow acceleration, $dM/d^x > 0$, heating always tends to increase the negative pressuregradient; on the other hand cooling always tends to reduce the negative pressure gradient.

† It is implicit, here, that ds_i/dx will be identical for both adiabatic and non-adiabatic cases which need not be true. However, the assumption is justifiable in terms of our basic assumptions.

Similarly it is seen from Eq. 12 that,

$$\frac{d\varrho}{dx} = -\frac{\varrho}{M}\left(\frac{M^2}{1+\frac{\gamma-1}{2}M^2}\right)\frac{dM}{dx} - \frac{\varrho}{R}\frac{dS}{dx} + \frac{1}{\gamma-1}\frac{\varrho}{T_0}\frac{dT_0}{dx} \tag{34}$$

where

$$\frac{\varrho}{R}\frac{dS}{dx} = \frac{\varrho}{T_0}\frac{\gamma}{\gamma-1}\left(1+\frac{\gamma-1}{2}M^2\right)\frac{dT_0}{dx} + \frac{\varrho}{R}\frac{dS_i}{dx} \tag{35}$$

That is to say

$$\frac{d\varrho}{dx} = -\frac{\varrho}{M}\left(\frac{M^2}{1+\frac{\gamma-1}{2}M^2}\right)\frac{dM}{dx} - \frac{\varrho}{T_0}\left(1+\frac{\gamma M^2}{2}\right)\frac{dT_0}{dx} - \frac{\varrho}{R}\frac{dS_i}{dx} \tag{36}$$

by which it is seen for flow acceleration, that heating also tends to increase the negative density gradient whilst cooling tends to decrease the negative density gradient.

7. CONDITIONS AT THE THROAT

In isentropic flow the throat and critical areas coincide. And with $M=M^*=1$; $(dA/dx)^*=0$ it follows from Eq. (26) that $(dM/dx)^*$ is indeterminate. Hence, vide Eqs. (31) and (34), $(dp/dx)^*$ and $(d\varrho/dx)^*$ are also indeterminate in isentropic flow.

Such conditions do not occur here as we get from Eq. (29),

$$\frac{dM}{dx} = \left(\frac{1+\frac{\gamma-1}{2}M^2}{M^2-1}\right)\frac{M}{A}\frac{dA}{dx} - \frac{M}{T_0}\left(\frac{1+\frac{\gamma-1}{2}M^2}{M^2-1}\cdot\frac{1+\gamma M^2}{2}\right)\frac{dT_0}{dx}$$

$$-\frac{M}{R}\left(\frac{1+\frac{\gamma-1}{2}M^2}{M^2-1}\right)\frac{dS_i}{dx} \tag{37}$$

which, on putting $(dA/dx)=0$, shows that

$$\left(\frac{dM}{dx}\right)_t = -\frac{M_t}{2T_{0_t}}\left\{\frac{\left(1+\frac{\gamma-1}{2}M_t^2\right)\left(1+\gamma M_t^2\right)}{M_t^2-1}\right\}\left(\frac{dT_0}{dx}\right)^t$$

$$-\frac{M_t}{R}\left(\frac{1+\frac{\gamma-1}{2}M_t^2}{M_t^2-1}\right)\left(\frac{dS_i}{dx}\right)^t \tag{38}$$

at the throat.

This indicates for heating with

$$\left(\frac{dT_0}{dx}\right)_t > 0$$

and $M_t<1$

that

$$\left(\frac{dM}{dx}\right)_t > 0$$

On the other hand with cooling,

$$\left(\frac{dT_0}{dx}\right)_t < 0$$

and $M_t<1$

it is clear $(dM/dx)_t$ can be negative only if

$$\left(\frac{dT_0}{dx}\right)_t < \left\{\frac{2T_{0_t}}{R}\left(\frac{1}{1+\gamma M_t^2}\right)\left(\frac{dS_i}{dx}\right)_t\right\} \tag{39}$$

There is no evidence this is possible. In fact the evidence available from experimental studies indicates the contrary, viz.

$$\left(\frac{dT_0}{dx}\right)_t > \left\{\frac{2T_{0_t}}{R}\left(\frac{1}{1+\gamma M_t^2}\right)\left(\frac{dS_i}{dx}\right)_t\right\} \tag{40}$$

This being so, it is possible to obtain explicit relationships for the pressure and density gradients at the throat. The pressure gradient is found, from Eqs. (33) and (38), to be

$$\left(\frac{dp}{dx}\right)_t = \frac{p_t}{T_{0_t}}\left(\frac{1+\gamma M_t^2}{M_t^2-1}\right)\left(\frac{dT_0}{dx}\right)_t + \frac{p_t}{R}\left[\frac{(\gamma-1)M_t^2+1}{M_t^2-1}\right]\left(\frac{dS_i}{dx}\right)_t \tag{41}$$

which, with $M_t<1$ indicates

$$\left(\frac{dp}{dx}\right)_t < 0 \tag{42}$$

Similarly it may be shown from Eqs. (36) and (38) that

$$\left(\frac{d\varrho}{dx}\right)_t = \frac{\varrho_t}{T_{0_t}}\left(\frac{1+\frac{\gamma-1}{2}M_t^2}{M_t^2-1}\right)\left(\frac{dT_0}{dx}\right)_t + \frac{\varrho_t}{R}\left(\frac{1}{M_t^2-1}\right)\left(\frac{dS_i}{dx}\right)_t \tag{43}$$

which leads to the conclusion that

$$\left(\frac{d\varrho}{dx}\right)_t < 0 \tag{44}$$

CONCLUSION

The present model gives a fairly complete picture of conditions likely to be met in real nozzle flows, at least within the perfect gas assumptions.

It is not a particularly useful model; however, it does knit together information which must otherwise be presented in considerably less tractable form. Certainly it presents the subtle differences between isentropic, adiabatic, reversible non-adiabatic and irreversible non-adiabatic flows in a unified fashion. In this regard it may be of some immediate use for instructional purposes.

As it is a one-dimensional model it suffers from the usual inadequacies of one-dimensional theory. Thus before trying to use or extend it some experiments are necessary to discover whether the theory *does* represent real flow conditions adequately.

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AN ANALOGUE COMPUTER FOR THE SOLUTION OF DRAINAGE PROBLEMS

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Synopsis – A hydraulic analogue computer is described which consists of storage tanks interconnected by pipes which represent culverts, creeks and other connections between prototype basins. Resistance in these pipes can be varied by the manipulation of valves. The stages of the river are represented by an outlet tank whose level variation can be made to represent that of a prototype flood.

The outline of an electric analogue computer is given and the comparative economics of this and the hydraulic analogue are discussed.

LIST OF SYMBOLS

A	Surface area of basin at any given height
a	Cross-sectional area of culvert or channel
C_m	Resistance coefficient for model
C_p	Resistance coefficient prototype
h	Head loss
K	Scale factor, used with subscript to denote which scale factor, e.g. K_A is scale factor for area
n	Manning's n
Q	Discharge
R_a	Hydraulic radius
R	Ohmic resistance
S	Water surface slope
t	Time
V	Velocity of flow
y	Height of water surface above some datum.

Subscripts:

m	Model
p	Prototype