INFORMATION CAPACITY OF RADIO NETWORKS

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Preface

I hereby declare that this dissertation is not substantially the same as any that I have submitted for a degree or diploma or other qualification at any other University. I further declare that no part of this dissertation has already been or is being concurrently submitted for any such degree, diploma or other qualification.

Part I is based on work done in collaboration with Phil Whiting \(^1\). Part II is the result of my own work and includes nothing which is the outcome of work done in collaboration with others.

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## Contents

1 **Introduction** ................................................................. 1
   1.1 Overview ................................................................. 1
   1.2 Mobile radio, cellular networks and macrodiversity .................. 2
   1.3 Summary of Part I .................................................... 4
   1.4 Summary of Part II .................................................. 4

I **A Shannon model of a radio network** ................................ 7

2 **Introduction to Part I** .................................................. 9
   2.1 Overview ................................................................. 9
   2.2 Assumptions ........................................................... 10
   2.3 Justification for focus on uplink .................................. 11
   2.4 Final remark .......................................................... 11

3 **Shannon's theory of information and multi-user information theory** ......................................................... 13
   3.1 Introduction ............................................................ 13
   3.2 Shannon's theory of information .................................... 13
      3.2.1 Entropy, a measure of information .......................... 13
      3.2.2 Channel capacity ............................................... 14
      3.2.3 Continuous random variables ............................... 16
   3.3 Gaussian channels .................................................. 16
      3.3.1 Discrete time Gaussian channels ............................ 17
      3.3.2 Continuous time processes with power constraints ....... 17
      3.3.3 Bandwidth constraints ....................................... 17
      3.3.4 Band limited white noise ................................... 19
      3.3.5 The white noise channel .................................... 19
   3.4 Multi-user Information Theory .................................... 20
   3.5 The Gaussian multi-user channel ................................... 22
      3.5.1 The discrete time Gaussian channel ....................... 22
      3.5.2 The continuous time white noise channel ................. 22
      3.5.3 Examples of Gaussian, multiple access channels ....... 23
   3.6 Conclusions .......................................................... 25

4 **Shannon capacity of multi-receiver networks and bandwidth partitioning** ................................................. 27
   4.1 Introduction ............................................................ 27
   4.2 A multi-receiver Gaussian channel .................................. 28
   4.3 Multi-user, Multi-receiver ........................................... 30
   4.4 The biting constraint and power control ........................... 31
   4.5 Two cell network capacity ........................................... 32
8.5.4 Large deviations for $\beta = 1$ .................................................. 94
8.6 Voice activity and large deviations .............................................. 95
8.7 Adaptive power control .......................................................... 95
8.7.1 A 1 dimensional manifold ...................................................... 95
8.7.2 An adaptive, decentralised power control algorithm ................. 96
8.7.3 Adaptive power control in two cells ...................................... 97
8.7.4 External noise zero ............................................................. 99
8.7.5 Connections with power control algorithms in the literature ....... 99
8.7.6 Metzler matrices ............................................................... 101
8.8 Expanding and contracting cells .............................................. 101

9 A multi-receiver spread spectrum channel: capacity .................. 103
9.1 Introduction ............................................................................. 103
9.2 Chapter overview .................................................................... 103
9.3 The signal to noise ratio in multi-receiver spread spectrum ......... 104
  9.3.1 Transmission delay and a simple macrodiversity scheme ......... 105
  9.3.2 Further justification for assuming correlations are zero ......... 106
9.4 Capacity and the received power equations ............................... 106
  9.4.1 The nature of $\mathcal{L}$ ......................................................... 111
9.5 An interpretation in terms of costs and capacities ...................... 112
9.6 $J$ classes of user ..................................................................... 113
9.7 Voice activity and the Chernoff bound ...................................... 113
9.8 Listening with 2 receivers in a 3 receiver network ....................... 113
  9.8.1 An approximate model in which user interference is ignored ... 114
  9.8.2 An approximate model in which external noise is ignored ....... 115
9.9 Received power limitations and capacity utilisation ................... 117
  9.9.1 The constant received powers case ...................................... 118
  9.9.2 Sufficiently uniform traffic .................................................. 119
  9.9.3 Nonuniform traffic ............................................................ 120
9.10 Examples of capacity calculations .......................................... 120
  9.10.1 Constant received powers case ......................................... 120
  9.10.2 Nonuniform traffic .......................................................... 121
9.11 Conclusions ........................................................................... 121

10 A multi-receiver spread spectrum channel: adaptive power control 123
10.1 Overview ................................................................................. 123
10.2 A decentralised, adaptive power control algorithm ................. 123
  10.2.1 Divergence of the vector field ........................................... 124
  10.2.2 Global stability when $K = 2$ ............................................ 125
10.3 Compartmental systems and column diagonal dominance ........ 126
10.4 A compartmental power control algorithm ................................ 127
10.5 A question of decentralisation ................................................. 128
10.6 Phase portraits ........................................................................ 128
10.7 A mathematical description of the phase portraits .................. 130
  10.7.1 The vector field on $\mathcal{L}$ .................................................. 130
  10.7.2 The linearised system for $K = 2$ ....................................... 132
10.8 Final remarks on the cellular and macrodiversity algorithms .... 135
10.9 Conclusions ............................................................................ 136

A Tightest constraint sets ............................................................ 137
B Some results on positive definite, symmetric matrices

C Code division multiple access and frequency hopping
   C.1 Direct sequence CDMA ................................................. 145
   C.2 FDMA ................................................................. 147
   C.3 Frequency hopping ..................................................... 147
   C.4 Applicability of the signal to noise model .................. 148

D The cost of interference in the signal to noise model

E Deviations beyond the central limit theorem
   E.1 Deviations of an average of random variables ............... 153
   E.2 Deviations of products of averages of random variables .. 155
      E.2.1 \( \mu_0 \neq 0 \) .................................................. 159
      E.2.2 \( \mu_0 = 0, \mu_1 \neq 0 \) ....................................... 160
      E.2.3 \( \mu_0 = 0, \mu_1 = 0 \) ........................................... 161

F Proof of Theorem 9.6

G Proof of Theorem 9.7

Bibliography
List of Figures

1.1 Hexagonal cell layout ................................................................. 3
3.1 Two user rate region for $Q_1$ and $Q_2$ ........................................ 21
3.2 Two user capacity in white noise ................................................. 23
4.1 Diversity reception for a single user with K receivers ....................... 29
4.2 A network with two receivers ..................................................... 32
4.3 Optimal vs. bandwidth partitioning for the two cell network ............... 34
4.4 K receivers in a circle ............................................................... 40
5.1 Approaching the Shannon limit cancelling in groups ......................... 52
8.1 A two cell network ................................................................. 84
8.2 Comparison of capacity regions .................................................. 89
8.3 Power control under light loading ............................................... 98
8.4 Power control under heavy loading ............................................ 99
8.5 Cell 0 contracts .................................................................. 102
9.1 Capacity region for two groups ................................................... 115
10.1 Decentralised alg. (two receivers) ............................................ 129
10.2 Decentralised alg. (three receivers) 1. .................................... 130
10.3 Decentralised alg. (three receivers) 2. .................................... 131
10.4 Modified two receiver algorithm .................................................. 131