Power Control and Transmission Scheduling for Network Utility Maximization in Wireless Networks

Min Cao, Vivek Raghunathan, Stephen Hanly, Vinod Sharma and P. R. Kumar

Abstract—We consider a joint power control and transmission scheduling problem in wireless networks with average power constraints. While the capacity region of a wireless network is convex, a characterization of this region is a hard problem. We formulate a network utility optimization problem involving time-sharing across different “transmission modes,” where each mode corresponds to the set of power levels used in the network. The structure of the optimal solution is a time-sharing across a small set of such modes. We use this structure to develop an efficient heuristic approach to finding a suboptimal solution through column generation generations. This heuristic approach converges quite fast in simulations, and provides a tool for wireless network planning.

Index Terms—Network utility maximization, power control, transmission scheduling, column generation

I. INTRODUCTION

In wireless networks, it is well known that the traditional layers of the communication network cannot be considered in isolation. Many authors have proposed joint approaches to issues such as power control, scheduling, and routing [2], [3], [5], [6]. There are many reasons for this. In the present paper, we focus on the following particular characteristics of wireless channels, namely, that they tend to be half-duplex, and of a broadcast nature. The primary half-duplex constraint that a node cannot be simultaneously transmitting and receiving implies that links must be carefully scheduled. The broadcast characteristic of wireless communications brings with it the fundamental issue of interference management, and scheduling is an important mechanism to alleviate interference between links. Clearly scheduling decisions are strongly connected to the lower layer functions of power control, and link layer rate allocation, since scheduling and power control together determine the quality of the communication links. But routing decisions (and, eventually, end-to-end transport capacity) are also determined by the quality of links, and hence transport, routing, scheduling and power control need to be considered jointly: all the layers are intimately connected in a wireless network.

In this paper we formulate and tackle the joint problems of bit rate allocation, power control, scheduling, and routing, using an optimization framework where power consumption is taken into account. We begin with the simplest case of a set of parallel links, where routing is not an issue, and consider the problem of maximizing the sum of the rates of all the links, subject to average power constraints on the transmit nodes. Even here, the half duplex constraints, and the possibility of scheduling the links to mitigate interference, provide a combinatorial aspect to the problem, and it is in fact well known that the general problem we tackle is NP-complete [2]. Our main contribution is a linear programming formulation which expresses the optimal schedules and power levels in terms of linear combinations of basic modes of the system. A characterization of the dimensionality of the search space, leads to an iterative approach in which old modes are pivoted out, and new modes are pivoted in, using a column generation technique. This approach does not circumvent the fundamental complexity of the problem, but we provide numerical evidence that it converges rapidly and obtains “good” (but suboptimal) solutions for reasonably large-sized networks. Then, we extend the method to maximize end-to-end utilities across general wireless mesh networks, using a multi-commodity flow to handle routing and flow control constraints.

There have recently been many works on cross-layer design of wireless networks, in which a resource allocation approach is used to formulate and solve a network-wide utility maximization (NUM) problem [2], [3], [5], [6]. In this framework, the solution to the optimization problem automatically decomposes into a number of subproblems at each layer, and the algorithms at each of these layers interact with each other via dual variables, which act to provide pricing information for cross-layer coordination. However, the PHY/MAC subproblem of wireless networks is typically NP-Complete. There have been efforts to find approximation schemes to solve this problem [4]. However, a simplified interference model is assumed in [4], which does not apply for general wireless networks. In [7], the authors consider the problem of finding the jointly optimal end-to-end rates, routing, power allocation and transmission scheduling for wireless networks. Nonlinear column generation is used to solve the cross-layer problem, and to prove convergence to the global optimal solution. To converge to the global optimum, this procedure needs to find the optimal solution of each column generation subproblem. This generation subproblem is also typically NP-Complete. The authors address the nonlinearity, but not the computational complexity, of the cross-layer solutions using column generation. In this paper, we further establish a dimensionality bound for the optimal solution of the problem. This provides an approach to identify good PHY/MAC modes to be used by the higher
layer operations. An alternative formulation would be to find the optimal allocation of power spectrum, and analogous results can easily be obtained. This approach has been taken in [8] where a similar dimensionality bound was obtained.

The rest of this paper is organized as follows: in Section II, we describe the network model. In Section III we consider the problem of maximizing the sum of link utilities. In Section IV, we consider the problem of maximizing the sum of end-to-end utilities. We describe numerical results in Section V, and extensions in Section VI. Finally, we conclude in Section VII.

II. NETWORK MODEL AND ASSUMPTIONS

We make the following assumptions about the wireless transceiver, reflecting characteristics of IEEE 802.11 wireless card: 1) it is half-duplex, so a node cannot transmit and receive simultaneously; 2) a single-user radio is used, hence, a node can communicate with exactly one another node at any time. These are referred to as primary constraints. We refer to a transmitter-receiver pair as a link. We consider a wireless network with $N$ nodes and $L = N(N-1)$ possible links. Denote the set of all the nodes and links by $\mathcal{N}$ and $\mathcal{L}$, respectively. Let $\mathcal{O}(n)$ and $\mathcal{I}(n)$ denote the set of outgoing and incoming links of node $n$, respectively; and $I\{\cdot\}$ be the indicator function. A transmission mode $m$ can be represented as a transmit power vector $P^m = (P^m_l, \ldots, P^m_L)$ that satisfies the primary constraints $\sum_{l \in \mathcal{O}(n) \cup \mathcal{I}(n)} I\{P^m_l > 0\} = 1$. Due to primary constraints, not all links can be simultaneously active in a transmission mode, and thus many of the $P^m_l$’s might need to be 0. Thus, different transmission modes need to be activated at different times to provide long term end-to-end connectivity. The specific procedure by which transmission modes are selected and activated affects both throughput and delay performances of end-to-end flows.

Denote the channel gain from the transmitter node of link $k$ to the receiver node of link $l$ by $G_{kl}$. For any transmission mode $m$, the SINR achieved at the receiver of link $l$ is

$$\gamma^m_l(P^m) = \frac{G_{kl}P^m_l}{\sum_{k \neq l} G_{kl}P^m_k + \sigma^2},$$

where $\sigma^2$ is the noise power. Thus, for each transmission mode $m$, there is an achievable rate vector $R^m = (R^m_l, \ldots, R^m_L)$ corresponding to $P^m$. $R^m_l$ is assumed to be a non-decreasing function of $\gamma^m_l$, depending on the modulation and coding schemes used. Theoretically, it can be determined by the Shannon function as

$$R^m_l(P^m) = W \log_2(1 + \gamma^m_l(P^m)),$$

where $W$ is the bandwidth of the wireless channel. Note that when $P^m_l = 0$, $R^m_l = 0$; thus inactive links have zero data rates. We assume that the channel gains are fixed in what follows.

In this paper, we are interested in finding power control and transmission scheduling (PC-TS) schemes that maximize the system utility. With the transmission modes defined as above, the joint PC-TS problem consists of selecting the transmission mode at each time instant so as to optimize the system performance objective. In finding the optimal solution, we impose average power constraints on the nodes. However, as discussed above, interference between nearby links requires some scheduling of link activations. A joint PC-TS scheme can be represented as $\{P(t), t \in (0, T)\}$, where $P(t) \in \mathbb{P}, \forall t \in (0, T)$, $\mathbb{P}$ is the set of allowable transmission modes, and $T$ is the duration of the transmission. Note that scheduling is subsumed in such a power control scheme since switching off a link is accomplished by setting its power level to zero. Our goal is to find the optimal PC-TS scheme that maximizes the system utility. The system utility can be defined in various ways to meet different objectives. In Section III, we consider the problem of maximizing the information-transport capability which can be expressed as the sum of link utilities. In Section IV, we consider the problem of maximizing the sum of end-to-end user utilities.

III. POWER CONTROL AND TRANSMISSION SCHEDULING FOR UTILITY MAXIMIZATION

A. Parallel Links Case

We first formulate the problem for maximizing the information-transport capability of $L$ parallel links where there are no primary constraints. Assume that the reward received for transporting one bit over link $l$ is $r_l$, then the reward we get per unit time with a fixed transmission mode vector $P$ is $V(P) = \sum_{l=1}^{L} r_l R_l(P)$.

In practice, there are discrete power levels available to each link, and we will assume that each link can select a power level from the following set of $K$ power levels:

$$\{0, \frac{P_{\text{max}}}{K-1}, \ldots, \frac{(K-2)P_{\text{max}}}{K-1}, \frac{P_{\text{max}}}{K-1}, P_{\text{max}}\}$$

where $P_{\text{max}}$ denotes the maximum possible power level. Let $\mathbb{P}$ denote the set of all $K^L$ power vectors available to the network, and let $\mathbb{M} = \{1, 2, \ldots, K^L\}$ index this set. We refer to the $m$th element in the set, $P^m$, as the $m$th transmission mode.

We allow a schedule to determine the activation of the transmission modes. Thus, over any transmission interval $(0, T)$ we allocate a fraction of time, $\alpha_m$, to the $m$th transmission mode; the time fraction can of course be zero. The utility obtained is then $\sum_{m \in \mathbb{M}} \alpha_m V(P^m)$. We assume that the links are constrained by long-term average power constraints. Thus, the optimization problem is the following.
The column generation subproblem is then be used to identify a new mode to enter the basis. Constraints (4) and (5), respectively. These variables can be dual variables \( \{ \lambda_l \} \). When solving the master problem, we obtain the optimal solution via somewhat heuristic methods. "Good" solutions via somewhat heuristic methods.

An approach that is widely used for solving linear programming problems with a large number of variables, but few constraints, is the method of column generation. The columns correspond to the transmission modes in the present problem. Column generation of the LP provides a decomposition of the problem into a master problem and a subproblem, and it identifies a "good" subset of modes by iteratively solving the master problem and subproblem. The master problem is the same as (3) except that we replace \( M \) by a small subset of modes \( M' \subseteq M \). Note that the dimensionality of the problem is such that basic feasible solutions have at most \( L + 1 \) basic variables, since there are only \( L + 1 \) constraints. Thus, the optimal solution will be a time-sharing amongst at most \( L + 1 \) transmission modes. Thus, we choose to set \( |M'| = L + 2 \), and solve the reduced LP as the master program.

Initially we can randomly pick \( L + 2 \) transmission modes. When solving the master problem, we obtain the optimal dual variables \( \{ \lambda_l, l \in L \} \) and \( \beta \), corresponding to the constraints (4) and (5), respectively. These variables can then be used to identify a new mode to enter the basis. The best mode would be obtained by solving the following subproblem, which is a separation problem for the dual LP. The column generation subproblem is

\[
\begin{align*}
\min_{m \in M \setminus M'} & \sum_{l \in L} \lambda_l P_l^m - V(P^m) + \beta \\
\text{s.t.} & \sum_{l \in L} \lambda_l P_l^m - V(P^m) + \beta < 0.
\end{align*}
\]

We cannot solve this problem exactly, because the size of the set \( M \setminus M' \) is huge; an exhaustive search over all the modes is prohibitive. However, it provides motivation for heuristic approaches for seeking a new transmission mode. To improve the objective of the master program, it is enough to find just one transmission mode \( m \in M \setminus M' \) such that \( \sum_{l \in L} \lambda_l P_l^m - V(P^m) + \beta < 0 \). The objective of the master problem can be improved by adding such a mode into the active set \( M' \). Typically, the corresponding time-sharing variable will increase from zero, and another basic variable will go to zero (become non-basic) unless we have reached a degenerate basic feasible solution. Repeating the procedure in this way, we can obtain a monotonically improving sequence of solutions as the column generation iteration evolves. If at some iteration, \( \sum_{l \in L} \lambda_l P_l^m - V(P^m) + \beta \geq 0 \) for all the modes \( m \in M \setminus M' \), then we have achieved the optimum of the original problem by the primal-dual relation.

At each column generation iteration, we need to find a new mode \( m \) from the set \( M \setminus M' \) that satisfies (7). Note that \( \lambda_l \) can be interpreted as the marginal price for increasing the power level \( P_l \) of link \( l \), and \( \frac{\partial V(P)}{\partial P_l} - \lambda_l \) is the marginal utility gained by increasing \( P_l \). We define \( \pi_l = \frac{\partial V(P)}{\partial P_l} - \lambda_l \), and \( \pi = (\pi_1, \cdots, \pi_L) \). We use a simple heuristic algorithm as follows:

\[\text{Algorithm 1: [Mode generating algorithm (parallel greedy case)]}\]

1) Initially, choose \( P = 0 \), and compute \( \pi \).

2) If \( \max_{l \in L} \pi_l > 0 \), let \( l' = \arg \max_{l \in L} \pi_l \), and raise the power \( P_{l'} \) of link \( l' \) to the next level. Update \( \pi \).

3) Continue until \( \max_{l \in L} \pi_l \leq 0 \).

Algorithm 1 greedily attempts to maximize the function \( V(P) - \sum_{l \in L} \lambda_l P_l \), but it is not guaranteed to reach the global maximum since the function is not concave, and the power levels are not continuous. Nevertheless, if it provides a mode \( P^m \) that satisfies (7) then this provides a new column to improve the objective in the master program. If (7) is not satisfied, then the whole column generation procedure terminates with a basic feasible solution. In this case it is possible, but not guaranteed and certainly not easily checked, that the final point will be the optimal solution to the original LP. In practice, it is also possible that the column generation procedure will need to be terminated when the computational budget is reached, given the sheer size of the LP when \( L \) is large.

\[\text{B. General Case}\]

Now we extend to the general case where each node \( n \) can transmit to any other node in the network. In this case, we place the average power constraints on the nodes, and the set of feasible transmission modes now has to take account of the primary constraints. Recall that the primary constraints require that

\[
\sum_{l \in O(n) \cup I(n)} I\{P_l^m > 0\} = 1. 
\]

Thus we now define \( P \) to be the set of all discrete power mode vectors that satisfy (8), and we let \( M \) index the elements of \( P \). Let \( P^{av} = (P_1^{av}, \cdots, P_N^{av}) \) be the vector of average power constraints on the nodes.

As before, we define the utility \( V(P^m) = \sum_{l=1}^L \pi_l R_l^m(P^m) \) associated with each mode \( P^m \in P \). The
The resource allocation problem becomes:

$$\max \sum_{m \in M} \alpha_m V(P^m)$$  \hspace{1cm} (9)

s.t. \hspace{1cm} \sum_{m \in M} \alpha_m \sum_{l \in O(n)} P^m_l - V(P^m) + \beta \geq 0$$

which is a linear program with \(N + 1\) constraints. Again we employ the column generation method to identify a good subset of modes \(M'\). The master problem replaces \(M\) with \(M'\) in (9), where \(M' \subseteq M\) is a small subset of modes with \(|M'| = N + 2\). Assume that by solving the master problem, we obtain the optimal dual variables \(\{\lambda_n, n \in N\} \) and \(\beta\) corresponding to the constraints (10) and (11), respectively. Then the column generation problem is

$$\min_{m \in M | M' \subseteq M} \sum_{n \in N} \lambda_n \sum_{l \in O(n)} P^m_l - V(P^m) + \beta$$

s.t. \hspace{1cm} \sum_{n \in N} \lambda_n \sum_{l \in O(n)} P^m_l - V(P^m) + \beta < 0.$$  \hspace{1cm} (13)

Instead of solving (12) for the optimal mode at each column generation step, we can employ a heuristic algorithm to find a “good” mode guided by the dual price information, but we need to consider the primary constraints in the general case. As before, we define \(\pi_l = \frac{\partial V(P)}{\partial P_l} - \lambda_n, l \in O(n)\). We adopt the following simple heuristic:

**Algorithm 2: [Mode generating algorithm (general case)]**

1. Initially, let \(T = \mathcal{L}, P = 0\), compute \(\pi\).
2. If \(\max_{l \in T} \pi_l > 0\), let \(l^* = \arg \max_{l \in T} \pi_l\), and raise the power \(P_{l^*}\) of link \(l^*\) to the next level. Update \(\pi\).
3. Let \(T(\pi^*)\) be the set of links in primary conflict with \(l^*\), and let \(T = T \setminus T(\pi^*)\).
4. Continue until \(T = \emptyset\) or \(\max_{l \in T} \pi_l \leq 0\).

IV. PROVIDING END-TO-END UTILITIES

Now we proceed to consider network utility maximization for end-to-end flows. Assume that there are \(F\) source-destination flows in the network. Let \(s_f\) denote the flow function. \(U(s_f)\) be the utility that the user gets by achieving this rate. \(U(s_f)\) is assumed to be a concave function, defined according to the objective. Denote the source and destination nodes of flow \(f\) by \(b(f)\) and \(e(f)\), respectively, and the set of all flows by \(\mathcal{F}\). The objective is to maximize the sum of the utilities of all the flows \(\sum_{f=1}^{F} U(s_f)\).

We allow multi-path routes, and use a multi-commodity flow model for the routing of data packets in the network. Such a model is widely used in the literature of network routing and optimization. Each source-destination flow \(f\) corresponds to a commodity in the network. Let \(x^f_l\) denote the traffic flow that is assigned to link \(l\) by the routing scheme corresponding to flow \(f\). The flow assignment given by the routing layer must satisfy the flow conservation constraints at each node \(n\):

$$\sum_{l \in O(n)} x^f_l - \sum_{l \in \mathcal{I}(n)} x^f_l = 0, n \in N \setminus \{b(f), e(f)\}\), \hspace{1cm} f \in \mathcal{F},$$

$$\sum_{l \in O(\mathcal{I}(f))} x^f_l = \sum_{l \in \mathcal{I}(\mathcal{I}(f))} x^f_l = s_f, f \in \mathcal{F},$$

This can be compactly written as

$$Ax^f = s^f, f \in \mathcal{F},$$  \hspace{1cm} (14)

where \(A\) is the node-link incidence matrix.

The set of feasible transmission modes is the set \(\mathcal{P}\) defined in Section III-B, which consists of \(|M|\) power vectors that satisfy the primary constraints. The problem we are interested in is: given the per node power budget, what is the optimal joint power control, MAC scheduling and routing scheme that maximizes the sum of the end-to-end utilities. This can be formulated as

$$\max U(s) = \sum_{f \in \mathcal{F}} U(s_f)$$  \hspace{1cm} (15)

s.t. \hspace{1cm} \begin{align*}
Ax^f &= s^f, f \in \mathcal{F}, \\
\sum_{f \in \mathcal{F}} x^f_l &\leq \sum_{m \in \mathcal{M}} R(P^m) \forall l \in \mathcal{L}, \\
\sum_{m \in \mathcal{M}} \alpha_m &\sum_{l \in O(n)} P^m_l = \bar{P}^{av}_n \forall n \in \mathcal{N}, \\
\sum_{m \in \mathcal{M}} \alpha_m &\leq 1.
\end{align*}$$

which is the maximization of a concave function over a convex region defined by a set of intersecting linear regions. Inequality (17) states that the effective capacity of link \(l\) provided by time sharing the modes must be greater than or equal to the sum of the flow rate through the link assigned by the routing layer; (18) corresponds to the average power constraint.

Note that the optimal vectors \(\alpha\) and \(P\) define a vector of link capacities, \(c\), and a vector of node average power levels, \(P^{ave}\), respectively, via:

$$c = \sum_{m \in \mathcal{M}} \alpha_m R(P^m)$$  \hspace{1cm} (20)

$$P^{ave} = \sum_{m \in \mathcal{M}} \alpha_m BP^m$$  \hspace{1cm} (21)

where \(B\) is the transmitter-link incidence matrix, whose \((n, l)\) entry is given by

$$B(n, l) = \begin{cases} 1, & \text{if } n \text{ is the transmitter of link } l, \\ 0, & \text{otherwise}. \end{cases}$$

Given the optimal link capacities, the optimal \(s\) and \(\alpha\) are
characterized as the solution of the problem:

$$\max U(s) = \sum_{f \in F} U(s_f)$$

s.t. \(Ax^f = s^{(f)}, f \in F, \sum_{f \in F} x^f_l \leq c_l, \forall l \in L\)

This provides a natural decoupling of the network operations: the higher layers decide the optimal source rates \(s\) and flow assignments \(x\) that achieve the maximum utility, while the physical and MAC layers find the optimal operating point \(c\) through time sharing among a set of transmission modes.

Note that (28) is identical to (12) if we let \(U^\alpha = \lambda\). This provides a natural decoupling of the network operations: the higher layers decide the optimal source rates \(s\) and flow assignments \(x\) that achieve the maximum utility, while the physical and MAC layers find the optimal operating point \(c\) through time sharing among a set of transmission modes.

At each column generation iteration, we choose a small set of transmission modes \(M' \subset M\) with \(|M'| = L + N + 1\), and solve the following restricted problem:

$$\max U(s) = \sum_{f \in F} U(s_f) \quad (23)$$

s.t. \(Ax^f = s^{(f)}, f \in F, \sum_{f \in F} x^f_l \leq c_l, \forall l \in L\)

$$\sum_{l \in O(n)} \sum_{m \in M'} \alpha_m R_l(P^m), \forall l \in L \quad (25)$$

$$\sum_{l \in O(n)} \sum_{m \in M'} \alpha_m P^m_l \leq \bar{P}^n, \forall n \in N \quad (26)$$

$$\sum_{m \in M'} \alpha_m = 1. \quad (27)$$

Since \(U(s_f)\) is a concave function, (23) is a concave program with linear constraints, which can be easily solved when \(M'\) is a small set. Furthermore, there is no duality gap, and we can find the optimal dual variables corresponding to the constraints (25), (26) and (27), denoted by \(\{\mu_l, l \in L\}\), \(\{\lambda_n, n \in N\}\) and \(\beta\), respectively. The corresponding column generation subproblem is:

$$\min_{m \in M \setminus M'} \sum_{n \in N} \lambda_n \sum_{l \in O(n)} P_l^m - \sum_{l \in L} \mu_l R_l^m + \beta < 0. \quad (29)$$

We generate the network topology by randomly placing \(N\) nodes in a 1000 \times 1000 m square. The path loss between node \(i\) and \(j\) is \(G_{ij} = 10^{-4} \cdot d_{ij}^{-3.5}\), where \(d_{ij}\) is the distance between node \(i\) and \(j\). The bandwidth is assumed to be \(W = 20\) MHz. We do not assume any specific modulation and coding scheme, but assume that the rate function on the SINR is given by the Shannon function as in (2) where the SINR \(\gamma^n(P^m)\) is given by (1), and the noise power is \(\sigma^2 = 10^{-12}\). Assume that each node has a maximum power \(P^\text{max} = 100\) mW, and average power budget \(P^\text{av} = 40\) mW. We assume that each node has 5 power levels, from 20 mW to 100 mW, in increments of 20 mW.

We construct a simple network with \(N = 14\) nodes, with the locations shown in Fig. 1. We consider two end-to-end flows with source and destination nodes denoted by \(s1, s2\) and \(d1, d2\), respectively. There are altogether \(L = 182\) possible links. Theoretically the size of the transmission mode set is \(|M'| \leq L + N + 1 = 197\). Practically the number of transmission modes needed is often much less. In this example, we set \(M' = 30\), and start by randomly choosing \(M'\) transmission modes. At each iteration, we solve the problem (23), and based on the optimal dual variables, find a new transmission mode according to Algorithm 2. Simultaneously, a transmission mode with \(\alpha_m = 0\) is pivoted out. The evolution of the achieved utility with the column generation iterations is shown in Fig. 2.

Solving problem (23) also gives the corresponding multi-path routes. The multi-path routes for flow \(s1 \to d1\) and \(s2 \to d2\) are shown in Fig. 3 and Fig. 4, respectively, where the thickness of the lines represents the flow rates. It can be seen that due to the power budget and interference,
the multi-path flows tend to use “short” rather than “long” links.

When the column generation iterations converge, we see that there are only 20 transmission modes with \( \alpha_m > 0 \). Due to space, we only show a sample transmission mode in Figure 5, where the active links are marked by lines with arrows pointed from the transmitter to the receiver, and the number of red circles represents the transmit power level. As can be seen from the results, the utility maximization tends to choose high transmit power levels but well spaced active links.

VI. CONTINUOUS POWER LEVELS

The discretization of the space of power vectors provides us with a combinatorial approach, which suffers from a corresponding explosion in complexity as the size of the network grows. It might be wondered if a more tractable approach would be to treat the power vectors as continuous variables, in a continuous relaxation.

Certainly, many of the observations made in this paper hold in the continuous case. For example, let \( \mathbb{P} \) be any measurable, compact subset of \( \mathbb{R}^L_+ \) that satisfies the primary constraints. We can consider any time interval \((0, T)\), and any measurable power allocation to the links, \( P(t), t \in (0, T) \) for which \( P(t) \in \mathbb{P} \) for all \( t \), and which satisfies the node power constraints:

\[
\frac{1}{T} \int_0^T B P(t) dt \leq \bar{P}^{av}
\]

where \( \bar{P}^{av} \) is the vector of node average power constraints, and \( B \) is the transmitter-link incidence matrix defined in (22).

The general continuous version of the end-to-end utility maximization problem can then be written as

\[
\max \ U(s) = \sum_{f \in F} U(s_f) \tag{30}
\]

s.t. \( Ax^f = s(f), \quad f \in F, \tag{31} \)

\[
\sum_{f \in \mathcal{F}} x^f_l \leq \frac{1}{T} \int_0^T R_l(P(t)) dt, \forall l \in \mathcal{L} \tag{32}
\]

\[
\frac{1}{T} \int_0^T B P(t) dt \leq \bar{P}^{av}, \tag{33}
\]

which is an optimization over all feasible, measurable power allocation policies, as well as link end-to-end rates. This
optimization can be written more compactly as
\[
\max U(s) = \sum_{f \in \mathcal{F}} U(s_f) \quad \text{(34)}
\]
subject to
\[
Ax^f = s^{(f)}, \quad f \in \mathcal{F},
\]
\[
\sum_{f \in \mathcal{F}} x^f_l \leq \int R_l(P) \, dp(P), \forall l \in \mathcal{L} \quad \text{(36)}
\]
\[
\int_{\mathcal{P}} BP \, dp(P) \leq P^{ave}, \quad \text{(37)}
\]
where, for any power allocation $P(t)$, $\rho$ is the corresponding probability measure on $\mathcal{P}$, defined for measurable sets $G$ by:
\[
\rho(G) = \frac{1}{T} \int_0^T I[P(t) \in G] \, dt. \quad \text{(38)}
\]

Note that (30)-(33) optimizes over probability measures $\rho$ on $\mathcal{P}$.

Again, Caratheodory’s convexity theorem can be used to simplify the form of the optimal solution to (30)-(33). Let $\rho$ now denote the unique measure that achieves the optimum, and write the link capacities, and average node power levels, respectively, as
\[
c = \int_{\mathcal{P}} R_l(P) \, dp(P) \quad \text{(39)}
\]
\[
P^{ave} = \int_{\mathcal{P}} BP \, dp(P). \quad \text{(40)}
\]
Since $(c, P^{ave})$ lies in the convex hull of the set $(R(P), BP) : P \in \mathcal{P}$, it follows that there are $M$ feasible transmission modes $P^{(1)}, P^{(2)}, \ldots, P^{(M)}$ and non-zero time-sharing variables $\alpha_1, \alpha_2, \ldots, \alpha_M$, where $M \leq L + N + 1$, such that
\[
c = \sum_{m=1}^M \alpha_m R(P^{(m)}) \quad \text{(41)}
\]
\[
P^{ave} = \sum_{m=1}^M \alpha_m BP^{(m)}. \quad \text{(42)}
\]

If a genie were to provide the $M$ transmission mode vectors, we could replace (30)-(33) with the much easier:
\[
\max U(s) = \sum_{f \in \mathcal{F}} U(s_f) \quad \text{(43)}
\]
subject to
\[
Ax^f = s^{(f)}, \quad f \in \mathcal{F},
\]
\[
\sum_{f \in \mathcal{F}} x^f_l \leq \sum_{m=1}^M \alpha_m R_l(P^{(m)}), \forall l \in \mathcal{L} \quad \text{(45)}
\]
\[
\sum_{l \in \Omega(n)} \sum_{m=1}^M \alpha_m P^{(m)}_l \leq \bar{P}^{ave}_n, \forall n \in N \quad \text{(46)}
\]
\[
\sum_{m=1}^M \alpha_m = 1. \quad \text{(47)}
\]

which is the maximization of a concave function over a convex region defined by linear constraints, with only a small number of variables. Unfortunately, we have no insight into how to select the optimal transmission modes $P^{(1)}, P^{(2)}, \ldots, P^{(M)}$. Nevertheless, further work in this direction may be fruitful if some additional structure to the problem can be discovered to identify the optimal transmission modes.

VII. CONCLUSIONS

We considered a joint power control and transmission scheduling problem in wireless networks with average power constraints in the network utility maximization framework. This problem is known to be computationally intractable. We formulate it as an optimization problem involving time-sharing across different “transmission modes”, where each “transmission mode” corresponds to the set of nodal power levels used in the network. We establish the structure of the optimal solution as time-sharing across a small fixed set of such modes. We use this structure to develop a heuristic approach to find a suboptimal solution through column generation iteration. It provides a tool for wireless network planning and slow time scale operations.

REFERENCES