4 Conclusions

We have defined a notion of delay limited capacity for traffic with stringent delay requirements. This can be accomplished by a centralized power control to completely mitigate the fading. We also define a delay limited capacity when there is no power control, or when there is a decentralized power control. In this case, we use statistical multiplexing; a large number of users are required.

A crucial feature of all delay limited capacities is that they are achieved by spread spectrum schemes. Indeed, we have shown that the delay limited capacity with power control is achieved by an “successive decoding type” scheme. The main reason is that spread spectrum allows the instantaneous total mutual information in the channel to be split equitably amongst the users. Spread spectrum is also necessary to obtain statistical multiplexing.

We have also characterized the Shannon capacities of the channels and the power control schemes that achieve these capacities. These capacities may be relevant for certain types of data traffic such as emails or faxes in which delay is not a concern. A feature of the optimizing schemes here is that users are not permitted to send data all the time, but only at certain instants when the channel is good.

References


Theorem 3 \textit{The delay limited capacity} $C_d(M)$ \textit{is the largest} $R$ \textit{such that for some power control} $\mathcal{P}$,

1. $\forall \tilde{h} = (h_1, \ldots, h_M) \mid \mathcal{L}|R \leq MW_0 \log \left(1 + \frac{\sum_{i \in \mathcal{L}} P_i h_i}{M \eta W_0} \right), \mathcal{L} \subseteq \{1, 2, \ldots, M\}$

2. $\mathbb{E}_{\mathcal{H}}[P_i(\tilde{h})] = \bar{P}, \quad i = 1, 2, \ldots, M$.

It can be shown that for any joint fading state $\tilde{h}$, the optimal power allocation $\mathcal{P}^*(\tilde{h})$ is the solution of the following problem:

$$\min \sum_{i=1}^{M} P_i \text{ s.t.} |\mathcal{L}|C_d(M) \leq MW_0 \log \left(1 + \frac{\sum_{i \in \mathcal{L}} h_i}{M \eta W_0} \right), \mathcal{L} \subseteq \{1, \ldots, M\}$$

Using Lemma (1) again, we obtain the following characterization of $C_d(M)$ and the optimal control.

Theorem 4 \textit{1. For all joint fading states} $\tilde{h}$, \textit{the delay limited capacity} $C_d(M)$ \textit{and the optimal power allocation} $\mathcal{P}^*(\tilde{h})$ \textit{satisfy:}

$$C_d(M) = MW_0 \log \left(1 + \frac{h_{u_1} \mathcal{P}^*_{u_1}}{M \eta W_0 + \sum_{j=1}^{i-1} h_{u_j} \mathcal{P}^*_{u_j}} \right), \quad i = 1, 2, \ldots, M$$

where $h_{u_1} \leq h_{u_2} \leq \ldots h_{u_M}$.

2. $C_d(M)$ \textit{is uniquely characterized by the equation}

$$\mathbb{E}_{\mathcal{H}}[\mathcal{P}^*_i(\tilde{h})] = \bar{P}$$

Note that if the fading is sufficiently slow that it does not change during the codeword length, then $C_d(M)$ can be achieved by successive decoding, starting from the user at the best fading level.

If users take on the same fading level, the optimal allocation of powers is not unique. Denote the set of users at level $\gamma_k$ by $\mathcal{M}_k(\gamma)$, $k = 1, 2, \ldots, K$. Given a joint fading state $\tilde{h}$, define

$$Q^*_k(\tilde{h}) \equiv \frac{1}{|\mathcal{M}_k|} \sum_{i \in \mathcal{M}_k} \mathcal{P}^*_{i}(\tilde{h}).$$

We can assign an averaged power allocation $\tilde{P}_{i}(\tilde{h}) = Q^*_k(\tilde{h})$ if user $i$ is at fading level $k$. It can be shown that this power allocation is also optimal. Note that under this control, the power allocated to a user depends only on its own fading level and the empirical distribution of users in the different fading levels at that time. As the number of users gets large, the empirical distribution converges to the stationary distribution of the fading process, so it can be expected that decentralized power control is asymptotically optimal. Indeed, this is so.

Theorem 5 \textit{As the number of users} $M$ \textit{grows,} $C_d(M)$ \textit{converges to the delay limited capacity under optimal decentralized power control, as characterized by Theorem 2.}$

Thus, when there are large number of users, it is nearly optimal for each user to allocate power as a function of its own fading level only. This is of particular interest, since the rate of power updates under optimal centralized power control increases linearly with the number of users, and this is an undesirable feature.
3.3 Centralized power control

In this section, we allow joint power control, in which each user’s power level $P_i(t)$ at time $t$ is a function of all the fading levels $H_i(t), \ldots, H_M(t)$ in the system, i.e. $P_i(t) = P_i(H_i(t), \ldots, H_M(t))$ for a control policy $\mathcal{P}$.

3.3.1 Shannon capacity

Using (2), the Shannon capacity of the channel under optimal centralized power control, subject to the long-term average power constraint $\bar{P}$, can be shown to be:

$$C_M = \max_{\mathcal{P}} \mathbb{E}_H \left[ \log \left( 1 + \frac{\sum_{i=1}^{M} P_i(H_i)}{M\eta W_0} \right) \right]$$

s.t. $\mathbb{E}_H [P_i(H)] = \bar{P} \quad i = 1, 2, \ldots, M$ (17)

where the maximization is over all power control policies $\mathcal{P}$.

The optimal power control for this problem is given by

$$\mathcal{P}^*_i(h_1, \ldots, h_M) = \begin{cases} \frac{1}{\lambda} - \frac{1}{h_i} & \text{if } h_i > \lambda, h_i = \max_j h_j \\ 0 & \text{else} \end{cases}$$

and $\lambda$ is chosen such that the average power constraint is satisfied. Thus, at any one time, only the user with the best channel transmits and the power he uses is determined by a water-filling solution. This result is due to Knopp and Humblet [7].

As $M$ grows large, it can be shown that the limiting capacity under this optimal control is given by:

$$\lim_{M \to \infty} C_M = W_0 \log \left( 1 + \frac{\bar{P}}{\lambda} \right)$$

This is due to the fact that when there are many sources, with high probability there will be a user at the best fading level $\gamma_K$ at any time. Comparing this with eqn. (10), we see that this is also the limiting capacity under the optimal decentralized power control. Thus for large $M$, decentralized power control performs as well as centralized control in achieving Shannon capacity.

3.3.2 Delay limited capacity

The Shannon capacity obtained above is not delay limited, since each user has to wait until it has the best fading level among all users before it can send any information. We now turn to the characterization of the delay limited capacity under centralized power control.

Recall that in the cases when there is no power control or when the power control is decentralized, our notion of delay limited capacity is defined only in the asymptotic regime of large number of users. This is because statistical multiplexing is needed to assure that there is sufficient mutual information most of the time. In contrast, with centralized control, we can coordinate the powers of all users so that there is sufficient mutual information at all times. Thus, we can define the delay limited capacity $C_d(M)$ for a fixed number of users $M$: it is the best equal rate that one can achieve in the sense of getting arbitrarily small decoding error probability with delays that are independent of the rate of change of the fading process.
3.2 Example: two fading levels

To gain more insight into these results, we consider the simple case of two fading levels: \( \gamma_1 \) and \( \gamma_2 \). The delay limited capacity, \( C_d \), occurs precisely when

\[
W_0 \log_2 \left( 1 + \frac{\phi \gamma_1 Q_1(Q_2) + (1 - \phi) \gamma_2 Q_2}{\eta W_0} \right) = \frac{W_0}{\phi} \log \left( 1 + \frac{\phi \gamma_1 Q_1(Q_2)}{\eta W_0} \right)
\]

by Theorem 2. We denote the optimal power control to achieve \( C_d \) by \( Q^* \).

To provide insight into this result, let us consider a different model, without fading, but with the same power control as above. There are \( M \) users, and a deterministic proportion \( \phi \) of these users are always in state \( \gamma_1 \), with the rest always in state \( \gamma_2 \). All users in the first group achieve a common rate denoted by \( R_1 \), and those with the higher gain achieve \( R_2 \). We consider three different cases of \( Q_2 \), defined by the levels \( Q_2^{(a)} \), \( Q_2^{(b)} \), \( Q_2^{(c)} \).

We choose these values such that \( 0 < Q_2^{(a)} < Q_2^* \), \( Q_2^{(b)} = Q_2^* \), \( Q_2^* < Q_2^{(c)} < \frac{P}{1-\phi} \) and depict the feasible rate regions \( \mathcal{R}^{(a)} \), \( \mathcal{R}^{(b)} \), \( \mathcal{R}^{(c)} \) in Figure 1. Note that in the limit as \( M \uparrow \infty \), \( \frac{P}{1-\phi} \) is the optimal power control to achieve the limiting Shannon capacity \( C \) of the channel. This follows from (9).

Let us focus first on equal rate points on the equal rate line depicted in the figure. For equal rates, capacity increases from (a) to (b), but decreases from (b) to (c), because the constraint corresponding to just the users in state \( \gamma_1 \) has started to bite. Case (b) represents the maximum equal rate capacity, which coincides with \( C_d \). By increasing \( Q_2 \) from \( Q_2^{(b)} \) to \( Q_2^{(c)} \), users in state \( \gamma_1 \) suffer a decrease in rate, but a rate pair \( (R_1^{(c)}, R_2^{(c)}) \) can be achieved for which \( R^{(c)} \equiv \phi R_1^{(c)} + (1 - \phi) R_2^{(c)} > C_d \) With power allocation \( Q^{(c)} \) in our fading model, users switch between \( \gamma_1 \) and \( \gamma_2 \). All users can achieve the common rate \( R^{(c)} \), which we have noted to be greater than \( C_d \), by time-sharing between \( R_1^{(c)} \) and \( R_2^{(c)} \). An important point is that if the fading is slow, then the delay can be considerable. The limiting Shannon capacity \( C \) is also depicted in the figure.

![Figure 1: Rate regions as a function of \( Q_2 \)](image-url)
The interpretation is that each user waits until its fading conditions are best possible before sending any information.

This Shannon capacity is not delay limited, since a user must wait until its own fading level is best possible. To compute the delay limited capacity for an average power constraint \( \bar{P} \), we first obtain the delay limited capacity \( C_d(\mathcal{Q}) \) for an arbitrary power control policy \( \mathcal{Q} \). This is a generalization of the result in (6), and can be shown to be:

\[
C_d(\mathcal{Q}) = \min_{\not\emptyset \neq \mathcal{L} \subseteq \{1, 2, \ldots, K\}} \frac{W_0}{\sum_{k \in \mathcal{L}} \phi_k} \log \left( 1 + \frac{\sum_{k \in \mathcal{L}} \phi_k \gamma_k Q_k}{\eta W_0} \right),
\]

(11)

To compute the optimal power control which minimizes this capacity subject to an average power constraint, we look at the equivalent problem of minimizing the average power to achieve a given delay limited capacity \( R \):

\[
\min_{\mathcal{Q}, C_d(\mathcal{Q}) \geq R} \sum_{k=1}^{K} \phi_k Q_k
\]

(12)

To solve this optimization problem, we need the following lemma.

**Lemma 1** Let \( c_1 \geq c_2 \geq \ldots \geq c_n \) and \( R_1, R_2, \ldots, R_n \) be given positive numbers. Consider the problem:

\[
\min \sum_{j=1}^{n} c_j x_j \quad \text{s.t.} \sum_{j \in \mathcal{L}} R_j \leq W_0 \log \left( 1 + \frac{\sum_{j \in \mathcal{L}} x_j}{\eta W_0} \right) \quad \forall \mathcal{L} \subseteq \{1, 2, \ldots, n\}
\]

(13)

Then the optimizing \( x^* \) is specified recursively by the equations

\[
R_i = W_0 \log \left( 1 + \frac{x_i^*}{\eta W_0 + \sum_{j=1}^{i-1} x_j^*} \right) \quad i = 1, 2, \ldots, n
\]

(14)

This result has an interesting interpretation. Suppose that there are \( n \) users and user \( i \) requires bit rate \( R_i \). Suppose in addition that the fading level for user \( i \) is \( \gamma_i \) and that this is fixed for all time. Since it is fixed, it is probably better to call it the path gain from user \( i \) to the receiver. Then under the correspondence \( c_i = \frac{1}{\gamma_i} \) \( i = 1, 2, \ldots, n \), the above problem is precisely the problem of minimizing the sum of the transmit powers subject to the bit rate constraints. Moreover, Lemma (14) shows that the optimal solution is to do successive decoding in descending order of the path gains, i.e. user \( i \) is decoded treating users \( 1, 2, \ldots, i-1 \) as noise, its signal then subtracted from the received waveform before decoding user \( i-1 \). An interesting observation is that the optimal decoding order is independent of the bit rate requirements of the users, depending only on the path gains.

Applying the lemma, we can solve the optimization problem (12). The required delay limited capacity can then be solved for by setting \( R \) in (12) such that the average power equals the \( \bar{P} \). Thus, the delay limited capacity is a solution to a system of equations which can be solved iteratively.

**Theorem 2** For an average power constraint \( \bar{P} \), the delay limited capacity \( C_d \) and the associated optimal power control \( \mathcal{Q}^* \) are unique solutions to the equations:

\[
\begin{align*}
C_d &= W_0 \log \left( 1 + \frac{\phi_k \gamma_k Q_k^*}{\eta W_0 + \sum_{j=1}^{k-1} \phi_j \gamma_j Q_j^*} \right) \quad k = 1, 2, \ldots, K \\
\sum_{k=1}^{K} \phi_k Q_k^* &= \bar{P}
\end{align*}
\]

(15)
We can give another interpretation of the quantity \( C_d \): it can be shown that \( C_d \) is also the equal-rate capacity of a multi-access system where a fraction \( \phi \) of the users are always at fading level \( \gamma_1 \) and \( (1 - \phi) \) of the users are always at level \( \gamma_2 \) (Theorem 6.4 of [5]). The conclusion is therefore that if the number of sources is large and the delay constraint is stringent, we can essentially view the original system as one in which the fading levels of individual sources never change but the fraction of sources at each level is the same as that in the equilibrium distribution of the original system.

The above result can be generalized to processes of arbitrary number of fading levels, \( \gamma_1 < \ldots < \gamma_K \). The delay limited capacity is given by:

\[
C_d = \min_{k=1,\ldots,K} \frac{W_0}{\sum_{j=1}^{k} \phi_j} \log \left( 1 + \frac{\bar{P} \sum_{j=1}^{k} \phi_j \gamma_j}{\eta W_0} \right)
\]

Again, this has the interpretation of the equal-rate capacity of a system where \( \phi_k M \) users are always at fading level \( \gamma_k \).

## 3 Power control from channel feedback

### 3.1 Decentralized power control

In the single user channel studied in Goldsmith [4], a user is allowed to adjust its transmitter power based on the current level of fading it is experiencing. In the present section, we extend this to the multi-user context, but still only allow a user to power control based on its own fading level. We consider the centralized approach in which the power control is a joint optimization over all users’ fading levels in Section 3.3. The present power control is motivated by the open loop power control for CDMA in the IS95 standard [10]. Each mobile measures the strength of the cell site pilot, of known transmit power, and deduces from this the level of fading.

Let the power control of each transmitter be given by \( Q(\gamma_k) = Q_k, k = 1, \ldots, K \). (By symmetry, we can assume that all transmitters use the same control.) We impose the long term power constraint

\[
\sum_{k=1}^{K} \phi_k Q_k = \bar{P}
\]

We can specify any power control by the associated vector \((Q_1, Q_2, \ldots, Q_K)\).

Given any power control \( Q \), there is a corresponding Shannon capacity.

\[
C_M(Q) = \mathbb{E}_M \left[ W_0 \log_2 \left( 1 + \frac{\sum_{k=1}^{K} M_k \gamma_k Q_k}{\eta W} \right) \right]
\]

where \( M = (M_1, \ldots, M_K) \) is a Multinomial random vector with parameters \((M, \phi_1, \ldots, \phi_M)\).

The Shannon capacity of the channel subject to the power constraint (7) is obtained by maximizing (8) over all \( Q \) satisfying (7). The optimal power control in the limit as \( M \to \infty \) can be computed explicitly and is given by:

\[
Q_k^* = \begin{cases} 
\bar{P}/\phi_K & k = K \\
0 & k \neq K 
\end{cases}
\]

The corresponding limiting capacity is given by

\[
C = W_0 \log \left( 1 + \frac{\gamma_K \bar{P}}{\eta W_0} \right)
\]
From (4), the rate of flow of joint mutual information between the sources and the receiver at any time \( t \) is given by:

\[
MW_0 \log(1 + \frac{\tilde{P}(\gamma_1 \phi + \gamma_2(1 - \phi))}{\eta W_0}) + o(M) = MC + o(M)
\]

This suggests the possibility of each user transmitting reliably at rate close to \( C \), by “splitting” the mutual information equally between the sources. Large delays need not incurred since we do not have to average the fades over time to get this amount of mutual information. We will make precise below when this can be done.

The continuous-time baseband system (1) is sampled at the Nyquist rate \( \frac{1}{2MW_0} \) and the resulting discrete-time system is:

\[
Y_n = \sum_{i=1}^{M} \sqrt{H_{in}} X_{in} + Z_n
\]

where \( y_n = y(\frac{n}{2MW_0}) \), etc. Suppose each source \( i \) codes over a block length of \( 2MW_0T \) symbols, where \( T \) is the delay in seconds, using a codebook \( C_i \) of size \( 2^{RT} \) (i.e. at rate \( R \) bits per second). The following result hold.

**Theorem 1** For any rate \( R < C_d \), where

\[
C_d \equiv \min\left\{ W_0 \frac{\phi}{\sqrt{\gamma_1 \phi}} \log(1 + \frac{\tilde{P}(\gamma_1 \phi + \gamma_2(1 - \phi))}{\eta W_0}), W_0 \log(1 + \frac{\tilde{P}(\gamma_1 \phi + \gamma_2(1 - \phi))}{\eta W_0}) \right\}
\]

and for any \( \epsilon > 0 \), one can choose \( M,T \) and codebooks \( C_1, \ldots, C_M \) of size \( 2^{RT} \), independent of the correlation structure of the fading processes \( H_i(\cdot) \)'s but only dependent on the stationary distribution, such that the probability of decoding some user incorrectly is less than \( \epsilon \). Conversely, if a rate \( R \) is achievable in the above sense, then necessarily \( R \leq C_d \).

Thus, the coding delay to get a small probability of error is independent of how slow the rate of change of the fading is. We call \( C_d \) the delay limited capacity, to distinguish it from the Shannon capacity. We note that if

\[
W_0 \frac{\phi}{\sqrt{\gamma_1 \phi}} \log(1 + \frac{\tilde{P}(\gamma_1 \phi + \gamma_2(1 - \phi))}{\eta W_0}) \geq W_0 \log(1 + \frac{\tilde{P}(\gamma_1 \phi + \gamma_2(1 - \phi))}{\eta W_0})
\]

then \( C_d = C_{\infty} \) and we can transmit at rates close to Shannon capacity without the need of time averaging over fades.

Although we shall not present proofs in this paper, it is instructive to look at the form of the bound we obtained on the error probability:

\[
p_e \leq \frac{M \exp(-T E(\delta, R) + 1)}{1 - M \exp(-T E(\delta, R) + 1)} + 2MW_0T \exp(-MI(\delta)).
\]

and this bounds holds for all \( \delta > 0 \). The exponent \( E(\delta, R) \) is positive for \( R < C_d - \delta \). The first term decays exponentially with \( T \) and is a consequence of averaging over the white noise. It is derived from random coding exponents associated with certain Gaussian channels. The second term decays exponentially with \( M \) and is the large deviations probability of not getting sufficient mutual information over the time interval \([0, T]\).
to transmit at equal rate; by symmetry, the equal-rate capacity (per user) is given by:

$$C = \mathbb{E}_H [W_0 \log(1 + \frac{\bar{P}}{M\eta W_0} \sum_{i=1}^{M} H_i)]$$

(2)

For the purpose of focusing on the essential ideas, we shall first consider a simple class of fading processes, having only two fading levels $\gamma_1$ and $\gamma_2$ ($\gamma_1 < \gamma_2$), and that $\mathbb{P}(H_i = \gamma_1) = \phi$, $\mathbb{P}(H_i = \gamma_2) = 1 - \phi$, The equal-rate capacity for this fading model is

$$C(M) = \mathbb{E}_{M_1} [W_0 \log(1 + \frac{\bar{P}(\gamma_1 M_1 + \gamma_2 (M - M_1)))}{M\eta W_0}$$

(3)

where $M_1$ is a random variable with Binomial distribution $\text{Bin}(M, \phi)$; it is the steady-state number of users who are fading at level $\gamma_1$. An intuitive understanding of this result can be obtained by viewing the capacity in terms of a time average of mutual information ([3]), the rate of flow of which can be viewed as a random process depending on the fading levels of the users. Specifically, if $M_1(t)$ is the number of sources fading at level $\gamma_1$ at time $t$, then the instantaneous rate of flow of joint mutual information between the sources and the receiver can be thought of as:

$$MW_0 \log(1 + \frac{\bar{P}(\gamma_1 M_1(t) + \gamma_2 (M - M_1(t))))}{M\eta W_0}$$

(4)

(This assumes that the transmitted waveforms are independent white Gaussian processes with power $\bar{P}$. ) Thus the amount of mutual information per source averaged over a time interval $[0, T]$ is

$$\frac{1}{T} \int_{0}^{T} W_0 \log(1 + \frac{\bar{P}(\gamma_1 M_1(t) + \gamma_2 (M - M_1(t))))}{M\eta W_0} dt$$

As $T \to \infty$, this quantity converges to (3), since the process $\{M_1(t)\}$ is stationary and ergodic. From this interpretation, one can see that to transmit information reliably at a rate close to the capacity (3), one must code over a duration $T$ significantly larger than the time-scale of the fluctuations of the fading processes $\{H_i(t)\}$. This is so that one can “accumulate” sufficient mutual information per unit time, close to the ergodic average, before decoding.

In many situations, it is unrealistic to code over such a time-scale, especially when there are strict delay requirements (such as for real-time services like voice or video) and some components of the fading are slow.

The capacity (3) is then not a very meaningful measure in those situations. It is then of interest to ask what can be achieved if codeword lengths are not necessarily long enough to average the fading over time. It turns out that if there are sufficiently large number of sources, then in some cases one can still get close to the Shannon capacity (3) without the need to average the fading over time. Instead, the averaging is done over the fading of the different users at a given time.

Let us look more carefully at the situation when there are many sources. By law of large numbers, the fraction of sources at $\gamma_1$ at time $t$, $\frac{M_1(t)}{M}$, converges to $\phi$ as $M$ becomes large. From (3), the equal-rate Shannon capacity $C_M$ converges to

$$C = W_0 \log(1 + \frac{\bar{P}(\gamma_1 \phi + \gamma_2 (1 - \phi))}{\eta W_0})$$
In the context of a single user channel, prior work by Ozarow et al. [9] has considered the issue of delay limited traffic. Cheng [1] considers the issue of fading in the multi-user context. Both papers take an outage approach.

An interesting conclusion we reach is that to achieve Shannon capacity in a multi-user channel with fading, and with a long-term average power constraint on each user, users cannot be transmitting in every time slot. The delay for each user must be a function of the rate of fading in the channel. However, it is clear that a delay limited user must be allocated time slots at a rate independent of the rate of fading. We show that bit rates below the delay limited capacity can be achieved by spread spectrum schemes in which each user is allocated every time and frequency slot. In fact, there must be enough “sharing of spectrum” to allow the total instantaneous mutual information in the channel to be split amongst the users according to their rate requirements.

Knopp and Humblet [7] have recently obtained the multi-user Shannon capacity under optimal centralized power control. We will review this in Section 3.3.1. We become aware of this work while preparing the present paper, and the results in the present paper were obtained independently from those in [7].

2 Statistical Multiplexing of Fading

Consider the continuous-time multiple-access channel:

\[ Y(t) = \sum_{i=1}^{M} \sqrt{H_i(t)}X_i(t) + Z(t) \]  

(1)

where \( M \) is the number of users, \( X_i(t) \) and \( H_i(t) \) are the transmitted waveform and the fading process of the \( i \)th user respectively, and \( Z(t) \) is white Gaussian noise with power spectral density \( \eta/2 \). Note that in this model, we consider fading effects which are frequency non-selective. We assume that the fading processes for all users are stationary, ergodic, statistically identical and independent of each other. We also assume that there are finite number of fading levels \( \gamma_1 < \gamma_2 < \ldots < \gamma_K \), such that \( \mathbb{P}(H_i(0) = \gamma_k) = \phi_k \). In practice, the fading process takes on a continuum of values and this can be taken as a discrete approximation to simplify the mathematics.

A total bandwidth of \( 2MW_0 \) is available for communication (i.e. \( 2W_0 \) per user.) Since we are dealing exclusively with frequency non-selective effects, it suffices to assume that the transmitted waveforms are bandlimited to \( [-MW_0, MW_0] \). Each user is also subjected to an average transmitted power constraint of \( \bar{P} \). While the decoder is assumed to have complete knowledge of the realizations of the fading processes, the transmitters know only their statistics. (In the next section, we will consider the case when the transmitters also have access to the realizations of the fading processes and can perform power control based on the instantaneous state of the channel.)

The capacity region for this multiaccess channel is well-known; it is given by:

\[ C = \{(R_1, \ldots, R_M) : \sum_{i \in \mathcal{L}} R_i \leq \bar{H} \log\left(1 + \frac{\bar{P}}{M\eta W_0} \sum_{i \in \mathcal{L}} H_i\right), \forall \mathcal{L} \subset \{1, \ldots, M\}\} \]

where \( \bar{H} = (H_1, \ldots, H_M) \) and \( H_i \)'s are i.i.d. random variables having the stationary distribution of the fading processes. We are interested in the case when each user wants
The Multi-access Fading Channel: 
Shannon and Delay Limited Capacities

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Abstract

We use statistical multiplexing and power control to define a notion of delay limited capacity, valid for traffic such as voice and video that cannot tolerate delay. Our capacity bounds the performance of schemes that are robust to changes in the rate of the fading processes.

1 Introduction

Fading poses severe problems in mobile, wireless communications. An important technique currently used to mitigate fading is error control coding. Shannon theory enables us to calculate the maximum possible rate in bits/sec that can be reliably sent on the channel. It is not clear, however, that such Shannon capacities are always useful measures if the traffic is delay limited. With block lengths constrained by the delay requirements of the traffic, it is not clear that the fading will always be sufficiently fast to allow averaging over the block length.

In the present paper, we show that a notion of “delay limited capacity” can be defined. We first consider a white Gaussian noise model in which users suffer fading, but users do not have the facility of power control. Rather than average the fading over time, we use statistical multiplexing to average the fading over users. In a similar manner to the random coding exponent approach (see for example [2]), we compute a bound on error probabilities consisting of two terms: one, which decays exponentially in time, much like the error exponent of a white noise, non-fading channel, and one that decays exponentially in the number of users. Moreover, the bound depends only on the stationary distribution of the fading process, and not on the rate of the fading process. We use this bound to define the notion of delay limited capacity. To achieve rates above the delay limited capacity, block lengths long enough to average out the (possibly slow) fading are then required. In Section 3, we extend this notion to systems with power control, generalizing the single-user work of Goldsmith [4], in which Shannon capacity is calculated assuming feedback from the receiver to the transmitter about the state of the channel. First we consider decentralized power control, in which users allocate power based on their own level of fading. Finally, in Section 3.3 we show that with optimal centralized power control, in which powers are chosen by the base station as a function of the joint vector of all fading levels, we can define delay limited capacity without the need for statistical multiplexing. We calculate both delay limited capacities and Shannon capacities, and characterize the optimal power control for each case.