On the Task Specific Evaluation and Optimisation of Cable-Driven Manipulators

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Abstract Cable-driven manipulators are traditionally designed for general performance objectives, such as maximisation of workspace. To take advantage of the reconfigurability of cable-driven mechanisms, the optimisation of cable-configurations for specific tasks is presented. Specifically, two types of task specific objectives are explored, the minimisation of cable forces over a desired trajectory and the maximisation of workspace about a desired pose. The formulation and incorporation to the optimisation problem for both task specific objectives are presented. Illustrated using a 3-DoF manipulator example, the results clearly demonstrated the advantages of optimising cable configurations for specific tasks. The potential ease of relocation in cable attachments makes task dependent reconfiguration feasible.

Keywords Cable-driven manipulators, task-specific objectives, optimisation

1 Introduction

Cable-driven parallel manipulators refer to mechanisms where the end-effector is controlled through cables. Cables are attached to the end effector on one end and to the actuator located at the base platform on the other. Desirable characteristics over traditional parallel mechanisms include: reduced weight and inertia, simplified dynamics modelling and the ease of transportation. The unique property of cable driven mechanisms is that its cables can only be actuated unilaterally through tension and not compression (positive cable force).

Due to the potential ease in relocating the cable attachment locations (cable configuration) on the end effector and base platform, cable-driven manipulators can be regarded as a reconfigurable mechanism. A wide range of applications exist for cable-driven mechanisms, such as such as manufacturing [2], rehabilitation [13] and exoskeletons [14]. For a given application, cable configuration can be either determined by the designer or through an optimisation process [1, 9, 12].

Naturally, the selection of an appropriate objective function is crucial in the determination of a desired cable configuration. In [1], the workspace volume was maximised for a two-link upper arm exoskeleton. Maximisation of tension-closure workspace volume and Global Conditioning Index (GCI) was performed on a cable-
driven universal joint module [9]. In [12], the configuration for a locomotion interface actuated by 16 cables was determined. The objectives involved the maximisation of mechanism workspace volume while minimising cable interference.

Objective functions such as workspace volume and GCI are similar to those employed in the design of rigid link mechanisms [5]. The goal is to achieve desirable general performance, as opposed to the efficiency of specific tasks. To take advantage of the potential reconfigurability of cable-driven manipulators, optimisation for specific tasks should be considered. For example, manipulators designed with a large workspace may not be energy efficient for a prescribed trajectory. This saving is particularly significant for highly repetitive tasks, such as pick-and-place in manufacturing or rehabilitation treatment for stroke patients.

In this paper, objective functions to evaluate the desirability of cable-driven manipulator configurations for specific tasks are investigated. Two classes of objective functions are presented, one based on the minimisation of cable forces for a desired trajectory, and the second is based on desired regions of the manipulator workspace. It is shown how workspace properties can be efficiently evaluated by extending the workspace analysis technique proposed in [8]. The impact of the proposed objective functions is illustrated through the optimisation of a 3-DoF manipulator, using a standard Particle Swarm Optimisation (PSO) algorithm. The results highlight the potential of task specific cable arrangement over a manipulator that is optimised for general or global performance.

The remainder of this paper is organised as follows: Section 2 presents the manipulator model, inverse dynamics problem and workspace analysis. The task specific objective functions and optimisation problem are formulated in Section 3. The results for the optimisation of an example 3-DOF manipulator are presented and discussed in Section 4. Finally, Section 5 concludes the paper and presents areas of future work.

2 Manipulator Model and Background

Consider the model shown in Figure 1, where \( \mathbf{r}_{A_i} \) and \( \mathbf{r}_{B_i} \) represents the cable attachments at the base and the end effector for cable \( i \), respectively. The vector \( \mathbf{r}_{0E} \) represents the translation from the origin of the inertial frame \( \{F_0\} \) to the origin of the end effector frame \( \{F_E\} \).

![Fig. 1 General Model for Cable Manipulator](image-url)
The equations of motion of a general \( n \)-DOF manipulator actuated by \( m \)-cables can be expressed in the form

\[
M(q)\ddot{q} + C(q,\dot{q}) + G(q) = -J^T(q)f,
\]

where \( q = [q_1, q_2, \ldots, q_n]^T \in \mathbb{R}^n \) represents the manipulator pose, \( M, C, \) and \( G \) are the mass inertia matrix, centrifugal and Coriolis force vector, and gravitational vector, respectively. The resultant of the cable wrenches is denoted by \(-J^Tf\), where \( J^T(q) \in \mathbb{R}^{n \times m} \) is the transpose of the Jacobian matrix. For a 6-DOF spatial manipulator

\[
J^T = \begin{bmatrix} \hat{l}_1 \times \hat{l}_1 & \hat{l}_2 \times \hat{l}_2 & \ldots & \hat{l}_m \times \hat{l}_m \end{bmatrix},
\]

where \( \hat{l}_i \) is the cable vector of cable \( i \), defined as \( \hat{l}_i = r_{0E} + r_{Bi} - r_{Ai}. \) The cable force vector is denoted by \( f = [f_1, f_2, \ldots, f_m]^T \in \mathbb{R}^m \), where \( f_i \) is the cable force in cable \( i \). The allowable cable force range in cable \( i \) can be defined as

\[
0 < f_{i,\text{min}} \leq f_i \leq f_{i,\text{max}},
\]

where \( f_{i,\text{min}} \) ensures positive cable force and prevents cable slackness, while \( f_{i,\text{max}} \) provides an upper limit on allowable actuation through the cable.

### 2.1 Inverse Dynamics Problem

For a desired manipulator trajectory \( q_r(t), \dot{q}_r(t) \) and \( \ddot{q}_r(t) \), the system dynamics from (1) at time \( t \) can be expressed as a linear equation

\[
-J^Tf = w,
\]

where \( w = M(q)\ddot{q} + C(q,\dot{q}) + G(q) \). The inverse dynamics problem refers to the determination of cable forces, \( f(t) \), subject to the constraints from (3) to satisfy the system dynamics in (4). For completely and redundantly restrained cable systems \( (m \geq n + 1) \) an infinite number of solutions exist [10]. One approach to resolve the cable force redundancy is to introduce an objective function for cable forces

\[
\begin{align*}
\mathbf{f}^* &= \arg\min_f f^T H f \\
\text{s.t.} & -J^Tf = w \\
f_{\text{min}} \leq f \leq f_{\text{max}},
\end{align*}
\]

where \( H \) is a positive definite weighting matrix and \( \mathbf{f}^* \) is the cable force solution. The cable force constraint refers to the allowable cable forces described in (3).
2.2 Wrench-Closure Workspace

The Wrench-Closure Workspace (WCW) is defined as the poses in which the manipulator can sustain any arbitrary wrench without any upper cable force bounds

\[ W = \{ \mathbf{q} : \forall \mathbf{q} \in \mathbb{R}^n, \exists \mathbf{f} > 0, -J(q)^T \mathbf{f} = \mathbf{w} \}, \tag{6} \]

where \( \mathbf{f} > 0 \) denotes the positive cable force constraint. Point-based evaluation techniques to determine the WCW have been studied in [4, 6]. In [8], a hybrid numerical-analytical approach was proposed to generate the WCW with increased accuracy and efficiency compared to point-wise methods. In this approach, the WCW from (6) is reduced to a set of univariate polynomial inequalities and analytically solved. From this approach, the workspace is represented by a set of linear regions defined by \( q_r \in \mathbb{R}^{n-1} \), the constant pose for \( n-1 \) variables, and the continuous region \( (q_l, q_u) \), where \( q_l \) and \( q_u \) are the lower and upper bounds for variable \( q \), respectively. This approach is extended to evaluate the workspace in Section 3.2.

3 Optimisation Problem and Evaluation Methods

The cable configuration optimisation problem can be expressed as

\[ x^* = \{ r_A^*, r_B^* \} = \arg \min_{r_A, r_B} C(r_A, r_B) \]
\[ \text{s.t} \quad r_A \in \mathcal{A}, r_B \in \mathcal{B}, \tag{7} \]

where \( r_A^* \) and \( r_B^* \) denote the optimal cable attachment locations at the base and end effector, respectively. The sets \( \mathcal{A} \) and \( \mathcal{B} \) represent the possible attachment locations at the base and end effector, respectively, and form the constraints on the optimisation variables. The objective function, \( C(r_A, r_B) \), represents desired properties of the manipulator and is dependent on cable configuration. Two classes of task specific objectives appropriate to cable-driven manipulators are presented in this section.

3.1 Cable Force Characteristics

The evaluation of cable force is a direct and meaningful measure on the configuration’s performance in executing a desired trajectory, \( \dot{\mathbf{q}}(t), \ddot{\mathbf{q}}(t) \) and \( \dddot{\mathbf{q}}(t) \), for a time period of \( 0 \leq t \leq t_{\text{max}} \). Denoting a penalty function for cable forces at time \( t \) as \( h(f^*, t) \), the objective function to minimise the penalty over the entire trajectory can be expressed as

\[ C(r_A, r_B) = \int_0^{t_{\text{max}}} h(f^*, t) \, dt, \tag{8} \]

where \( f^* \) represents the cable forces from the inverse dynamics problem in (5). One possible cable force penalty function at time \( t \) is
\[ h(f,t) = \begin{cases} Q(f) , & f_{\text{min}} \leq f \leq f_{\text{max}} , \\ Q(f_b) , & \text{otherwise} \end{cases} \]

where \( Q \) is the function from (5), and \( f_b > f_{\text{max}} \) represents penalty cable forces when there are no solutions for (5), penalising configurations in which the trajectory cannot be executed.

### 3.2 Workspace Evaluation

Upon determination of the workspace, for example, WCW, existence of desired regions can be evaluated. This is particularly to ensure that the designed manipulator is able to operate within a specifically prescribed region for a desired task. By prescribing a desirability of manipulator pose \( \mathbf{q} \) within the workspace \( W \) as \( v(\mathbf{q}) \), the objective function to achieve maximum workspace desirability can be expressed as

\[ C(\mathbf{r}_A, \mathbf{r}_B) = -\int_W v(\mathbf{q}) \, dW . \]  

One of the simplest measures of workspace is its volume, where the entire workspace is equally weighted \( v_1(\mathbf{q}) = 1 \). The disadvantage in such a general performance measure is that the maximisation of volume does not imply that the manipulator can operate in a desired region. To accommodate for this, \( v(\mathbf{q}) \) can be constructed to favour desired regions. For example, if the desired workspace region is the region about a manipulator pose \( \mathbf{q}_s = \{q_{s_1}, q_{s_2}, \ldots, q_{s_n}\} \), where \( q_{s_i} \) corresponds to the pose variable \( q_i \), respectively, one weighting function can be

\[ v_2(\mathbf{q}) = \frac{1}{1 + a_1(q_1 - q_{s_1})^2 + \cdots + a_{n-1}(q_{n-1} - q_{s_{n-1}})^2 + a_n(q_n - q_{s_n})^2} . \]

The properties of (11) are \( v_2(\mathbf{q}_s) = 1 \), \( v_2(\mathbf{q}) \leq 1 \), and \( v_2(\mathbf{q}) \to 0, ||\mathbf{q} - \mathbf{q}_s|| \to \infty \). The constant \( a_i \) corresponds weighting in dimension \( i \).

Taking advantage of the workspace generation approach proposed in [8], the workspace desirability from (10) can be expressed as

\[ C = -\sum_{q_1, q_2} \cdots \sum_{q_{n-1}} \int_{q_l}^{q_u} v(\mathbf{q}_r, q) dq_1 \cdots dq_{n-1} dq_r , \]  

where \( \mathbf{q}_r = \{q_{r_1}, q_{r_2}, \ldots, q_{r_{n-1}}\} \) represents the constant variables and \( q \) is the analytical variable. It is important to note that if the definite integral of \( v(\mathbf{q}_r, q) \) with respect to \( q \) can be determined analytically, then (12) becomes

\[ C = -\sum_{q_1, q_2} \cdots \sum_{q_{n-1}} (V(\mathbf{q}_r) - V(q_l)) \cdot dq_r \cdots dq_{r_{n-1}} dq_{r_1} , \]  

where \( V \) is the definite integral of \( v(\mathbf{q}_r, q) \) with respect to \( q \). It is worth noting that for both the volume and proposed task specific workspace region function from (11), \( V(q) = \int v(q) \, dq \) can be determined. Evaluation of workspace in such a manner preserves the advantages of the workspace generation from [8], increased efficiency and accuracy.
4 Simulation and Results

To illustrate and compare the different task specific objective functions, the optimisation of a completely restrained 3-DoF manipulator driven by 4-cables is presented. As shown in Figure 2, the manipulator is constrained at the base through a ball joint, and the pose of the manipulator can be defined as \( q = [\alpha \ \beta \ \gamma]^T \), where \( \alpha \), \( \beta \), and \( \gamma \) are the \( xyz \)-Euler angles of rotation, respectively.

For the presented manipulator, three task specific objective functions \( C_1 \) to \( C_3 \) and one general measure \( C_4 \) have been selected for optimisation. These objective functions are described in Table 1. Trajectories are defined by a starting pose \( q_s \) and ending pose \( q_e \), assuming zero starting and ending velocities and accelerations \( \dot{q}_s = \dot{q}_e = \ddot{q}_s = \ddot{q}_e = 0 \). The trajectory \( q(t) \) is then generated from \( t = 0 \) to \( t = t_{\text{max}} \) by quaternion interpolation. The evaluation function for a desired workspace region from (11) for the 3-DoF manipulator is

\[
V_2(q) = \frac{1}{1 + a_\alpha (\alpha - \alpha_s)^2 + a_\beta (\beta - \beta_s)^2 + a_\gamma (\gamma - \gamma_s)^2} .
\] (14)

<table>
<thead>
<tr>
<th>Function</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 )</td>
<td>WCW region</td>
<td>Function from (14) with weight values of ( a_\alpha = a_\beta = a_\gamma = 100 ) about pose ( \alpha_s = -1, \beta_s = 0.5, \gamma_s = \frac{3\pi}{5} )</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>Trajectory</td>
<td>( q_s = \left[ \frac{-\pi}{10} \ \frac{\pi}{10} \ 0 \right]^T ), ( q_e = \left[ \frac{\pi}{10} \ -\frac{\pi}{10} \ 0 \right]^T ) and ( t_{\text{max}} = 1 )</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>Trajectory</td>
<td>( q_s = \left[ \frac{3\pi}{10} \ -\frac{2\pi}{5} \ -\frac{\pi}{5} \right]^T ), ( q_e = \left[ \frac{\pi}{10} \ \frac{3\pi}{5} \ -\frac{\pi}{5} \right]^T ) and ( t_{\text{max}} = 1 )</td>
</tr>
<tr>
<td>( C_4 )</td>
<td>WCW</td>
<td>WCW volume (general performance measure as a baseline)</td>
</tr>
</tbody>
</table>

For evaluation function \( C_4 \), the optimal cable configuration \( \mathbf{x}_i^* \) was determined through a standard Particle Swarm Optimisation (PSO) algorithm. To allow comparison between task specific evaluation functions, each optimal configuration was also evaluated for \( C_1 \) to \( C_4 \). The results for these evaluations are shown in Table 2, where the minimum value representing the optimal cost is highlighted.

The optimal cable configuration for the WCW volume \( \mathbf{x}_i^* \) is a robust configuration that allows the manipulator to satisfy a wide range of tasks. The disadvantage
Table 2 Comparison of Evaluation Functions

<table>
<thead>
<tr>
<th></th>
<th>$x_1^*$</th>
<th>$x_2^*$</th>
<th>$x_3^*$</th>
<th>$x_4^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>-282.3</td>
<td>-161</td>
<td>-11.05</td>
<td>-216.7</td>
</tr>
<tr>
<td>$C_2$</td>
<td>116.7</td>
<td>3.8</td>
<td>808006.5</td>
<td>13.9</td>
</tr>
<tr>
<td>$C_3$</td>
<td>233536.2</td>
<td>1864.8</td>
<td>282.6</td>
<td>6973</td>
</tr>
<tr>
<td>$C_4$</td>
<td>-35543</td>
<td>-29966</td>
<td>-2884</td>
<td>-36321</td>
</tr>
</tbody>
</table>

of such a configuration is that efficiency for specific tasks are not considered. For example, the $\alpha$-$\beta$ cross section of the WCW at $\gamma = \frac{2\pi}{5}$ is shown in Figure 3(a). It can be observed that workspace does not exist about pose $\alpha = -1$ and $\beta = 0.5$. In comparison, the workspace from configuration $x_1^*$, optimised for the workspace region about $(\alpha, \beta, \gamma) = (-1, 0.5, \frac{2\pi}{5})$, is shown in Figure 3(b), where the workspace about the desired location is satisfied. This is particularly useful to ensure that a manipulator is more robust to operate at a desired workspace region.

![Fig. 3 WCW for optimal configurations $x_4^*$ and $x_1^*$](image)

The benefits of task specific evaluations can be further observed for the minimisation of trajectory cable forces. It can be observed that $x_1^*$ can perform the trajectories $C_2$ and $C_3$, but much less efficiently than the optimal configurations $x_2^*$ and $x_3^*$, respectively. The comparison in cable forces required to generate trajectory $C_3$ between configurations $x_1^*$ and $x_3^*$ is shown in Figure 4. It is clear that $x_3^*$ from Figure 4(b) performs the trajectory much more efficiently, with a maximum force of approximately 20N, compared to the maximum force of 100N from $x_1^*$.

From the comparison, the benefits in task specific evaluation to determine optimal cable configurations can be observed. This study suggests that it is difficult to obtain a robust cable configuration to satisfy a wide range of tasks, while performing them efficiently. In addition, the optimal configuration for one task specific objective typically performs poorly for another. Due to the potential reconfigurability of cable-driven systems, the adaptation of task specific objectives become feasible.
5 Conclusion

A study to compare the effectiveness of different task specific evaluation functions for optimising cable-driven manipulators was presented. Two classes of tasks were considered, minimisation of cable force to perform a trajectory, and maximisation of a workspace region. In addition, an efficient method to evaluate the quality of WCW was demonstrated extending from [8]. Performing optimisation on a 3-DoF cable-driven manipulator, the results indicated the improvement and savings of task specific objectives, as compared to a robust performance measure such as workspace volume. Future work should consider other task specific measures, such as cable interference, and to extend this to the optimisation of multilink manipulators.

References


