Passification of nonlinear controllers via a suitable time-regular reset map

Fulvio Forni [a,1] Dragan Nesic [b,2] Luca Zaccarian [a,1]

[a] *DISP, University of Roma, Tor Vergata, Via del Politecnico 1, 00133 Roma, Italy*

[b] *EEE Department, University of Melbourne, Australia.*

Abstract

For a class of square continuous time nonlinear controllers we design a suitable resetting rule inspired by the resetting rule for Clegg integrators and First Order Reset Elements (FORE). With this rule, we prove that the arising hybrid system with temporal regularization is passive in the conventional continuous time sense with a small shortage of input passivity decreasing with the temporal regularization constant. Based on the passivity property, we then investigate the finite gain stability of the interconnection between this passive controller and a passive nonlinear plant.

Key words: passivity, reset control system, hybrid system

1 Introduction

In recent years, much attention has been given to the analysis and design problem of control systems in the hybrid context, namely when the closed-loop dynamics obeys either a continuous law imposing a constraint on the pointwise derivative of the solution when it belongs to the so-called flow set, and/or a discrete law imposing a constraint on the jump that the solution undertakes when it belongs to the so-called jump set. This type of interpretation of hybrid systems, thereby merging classical discrete- and continous-time concepts in a unifying framework has been pursued in the past years by providing a specific mathematical characterization of the underlying mathematical theory. An extensive survey of the corresponding results can be found in [8].

A specific instance of hybrid systems corresponds to the case analysed here of continuous-time plants controlled by a hybrid controller, namely a hybrid closed-loop where the jumps only affect the controller states. Within this class of systems a relevant example consists in the reset control systems first introduced in [5], where a jump linear system (the "Clegg integrator") generalizing a linear integrator was proposed. This generalization was then further developed in [10] where it was extended to first order linear filters, and therein called First Order Reset Elements (FORE). FORE received much attention in recent years and have been proven to overcome some intrinsic limitations of linear controller [1]. Moreover, by relying on Lyapunov approaches, suitable analysis and synthesis tools for the stability of a class of reset systems generalizing control systems with FORE have been proposed in [2,14] and references therein. Moreover, in the recent paper [4] the $\mathcal{L}_2$ stability of reset control systems has been addressed in the passivity context, by showing interesting properties of the reset system under the assumption that the continuous-time part of the reset controller is passive before resets and that a suitable non-increase condition is satisfied by the storage function at jumps. In [4] it was also shown by a simulation example that resets do help closed-loop performance in passivity-based closed-loops.

In this paper we further develop over the ideas of [4] by using a specific temporally regularized reset strategy for the reset controller. The reset strategy generalizes the new interpretation of FOREs and Clegg integrators proposed in [17,14] and references therein. We show that, with the proposed reset strategy, passification is possible for any continuous-time underlying dynamics under some sector growth assumption on the right hand side of the continuous-time dynamics of the controller. The obtained passivity property is characterized by an excess of output passivity and a lack of input passivity whose size

[1] Work supported in part by ENEA-Euratom and MIUR.
`forni|zack@disp.uniroma2.it`
[2] Work supported by the Australian Research Council under the Future Fellowship. `d.nesic@ee.unimelb.edu.au`

can be made arbitrarily small by suitably adjusting the reset rule. As an example, the proposed reset strategy allows to establish a passivity property for any FORE, including those characterized by an exponentially unstable pole, while the results in [4] only allow to establish passivity of FOREs with stable poles. This increased potential of the reset rule proposed here is illustrated on a nonlinear simulation example.

The paper is organized as follows. In Section 2 we describe the class of controllers under consideration and the proposed reset rule, together with some notation and preliminaries characterizing the hybrid systems framework of [8]. In Section 3 we first state our main passivity result and then establish finite $\mathcal{L}_2$ gain properties of interconnected systems involving the proposed reset controller. Finally, in Section 4 we discuss a simulation example.

2 A class of nonlinear reset controllers

Consider the following nonlinear controller mapping the input v to the output u,

$$\begin{aligned} \dot{x_c} &= f(x_c) + g(x_c, v) \\ u &= h(x_c), \end{aligned} \tag{1}$$

where $u \in \mathbb{R}^q$, $v \in \mathbb{R}^q$, so that the controller is square and where the following regularity assumption is satisfied by the right hand side.

Assumption 1 *The functions $f(\cdot)$ and $h(\cdot)$ are continuous and sector bounded, namely there exist two constants L_f and L_h such that for all x_c, $|f(x_c)| \le L_f|x|$ and $|h(x_c)| \le L_h|x_c|$.*

Moreover, $g(\cdot,\cdot)$ is continuous in both its arguments and uniformly sector bounded in the second argument, namely there exists a constant L_g such that for all x_c and all v, $|g(x_c, v)| \le L_g|v|$.

In this paper we propose a hybrid modification of the controller (1) aimed at making it passive from v to u, regardless of the properties of the original dynamics in (1). In particular, the modified controller follows the continuous-time dynamics of (1) at times when the input/output pair belongs to a certain subset of the input/output space. When the input/output pair exits that subset, the state of the controller is reset to zero, intuitively re-initializing the controller within the set where it is allowed to flow.

To avoid Zeno solutions, namely solutions that exhibit infinitely many jumps in a bounded time interval, we also embed the hybrid modification with a temporal regularization clock, imposing that the controller cannot be reset to zero before ρ times after the previous reset (see also [14,11].

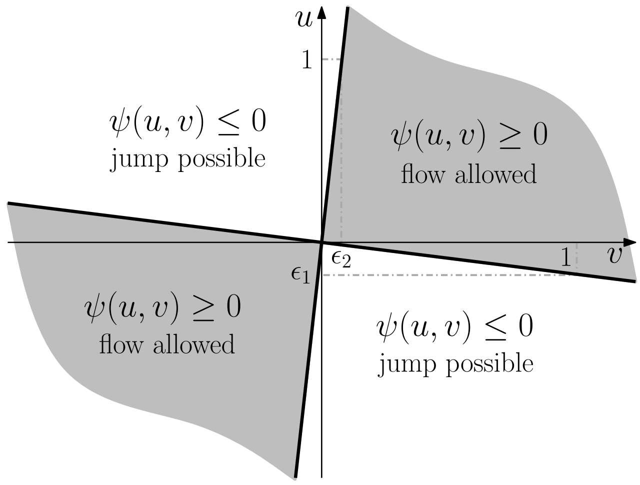


Figure 1. Input/output space of the controller (2) and subsets where $\psi(u,v) \gtreqless 0$.

The proposed hybrid controller is given by

$$\begin{cases} \dot{x_c} = f(x_c) + g(x_c, v) \\ \dot{\tau} = 1 \end{cases} \quad \textit{if } \tau \le \rho \textit{ or } \psi(u,v) \ge 0$$
$$\begin{cases} x_c^+ = 0 \\ \tau^+ = 0 \end{cases} \quad \textit{if } \tau \ge \rho \textit{ and } \psi(u,v) \le 0$$
$$u = h(x_c) \tag{2a}$$

where $\psi(u,v)$ is defined as

$$\psi(u,v) = (u + \epsilon_1 v)^T (v - \epsilon_2 u) \tag{2b}$$

and ϵ_1 and ϵ_2 are some (typically small) non-negative scalars. As usual in the hybrid system framework, we call C the set $\{(x_c, \tau, v) : \tau \le \rho \textit{ or } \psi(h(x_c), v) \ge 0\}$ and D the set $\{(x_c, \tau, v) : \tau \ge \rho \textit{ and } \psi(h(x_c), v) \le 0\}$.

The rationale behind the reset controller (1) is illustrated in Figure 1 where the input/output space of (2) is represented for the case $q = 1$. In the figure, the shaded region corresponds to the set $\psi(u,v) \ge 0$ where the system always flows, regardless of the value of τ. Instead, in the remaining region, where $\psi(u,v) \le 0$, the system will jump provided that $\tau \ge \rho$. Note also that when $\epsilon_1 = \epsilon_2 = 0$, the shaded region reduces to the first and third quadrant, resembling the resetting rule characterized for the first order reset element (FORE) in [17,14]. When the reset occurs, since $h(0) = 0$, the u component of the input/output pair will jump at zero thus resulting in a vertical jump to the horizontal axis. Moreover, ϵ_1 and ϵ_2 allow to have extra degrees of freedom in the resetting rule. In particular, the goal of ϵ_1 is to guarantee that the reset rule maps the new input/output pair in the interior of the shaded set whenever $v \neq 0$. Instead, as it will be clear next, the goal of ϵ_2 is to modify the resetting rule to obtain some strict output passivity for the reset controller (2).

Controller (2) will be dealt with in this paper following the framework of [9,8,3]. In particular, by Assumption 1, controller (2) satisfies the hybrid basic assumptions (see, e.g., [3]), which ensure desirable regularity properties of the solutions, such as existence, and robustness to arbitrarily small perturbations (see [8] for details).

As usual in the hybrid system framework, the evolution of the state $\xi = (\xi_x, \xi_\tau)$ either continuously flows through C, by following the dynamic given by $f(\xi_x) + g(\xi_x, v)$ and 1, or jumps from D to $(0, 0)$. Such alternation of jumps and flow intervals can be conveniently characterized by using a generalized notion of time, called *hybrid time*. By following [9], a set $E \subseteq \mathbb{R}_{\geq 0} \times \mathbb{N}$ is a *hybrid time domain* if it is the union of infinitely many intervals of the form $[t_j, t_{j+1}] \times \{j\}$ where $0 = t_0 \leq t_1 \leq t_2 \leq, \ldots,$ or of finitely many such intervals, with the last one possibly of the form $[t_j, t_{j+1}] \times \{j\}$, $[t_j, t_{j+1}) \times \{j\}$, or $[t_j, \infty] \times \{j\}$.

The evolution of the state ξ of a hybrid system (2), depends on the input signal v, so that both ξ and v must be defined on hybrid time domain. By following [3], we call *hybrid signal* each function defined on a hybrid time domain. A hybrid signal $v : \mathrm{dom}\, v \to \mathcal{V}$ is a *hybrid input* if $v(\cdot, j)$ is Lebesgue measurable and locally essentially bounded for each j. A hybrid signal $\xi : \mathrm{dom}\, \xi \to \mathbb{R}^n \times \mathbb{R}_{\geq 0}$ is a *hybrid arc* if $\xi(\cdot, j)$ is locally absolutely continuous, for each j. With the basic assumptions satisfied, a hybrid arc $\xi = (\xi_x, \xi_\tau)$ and a hybrid input v is a *solution pair* (ξ, v) to the hybrid system (2) if $\mathrm{dom}\, \xi = \mathrm{dom}\, v$, $(\xi(0, 0), v(0, 0)) \in C \cup D$, and

s.1 for all $j \in \mathbb{N}$ and almost all t such that $(t, j) \in \mathrm{dom}\, \xi$,

$$\begin{aligned} &(\xi(t, j), v(t, j)) \in C \\ &\dot{\xi}_x(t, j) = f(\xi_x(t, j)) + g(\xi_x(t, j), v(t, j)); \\ &\dot{\xi}_\tau(t, j) = 1; \end{aligned} \tag{3}$$

s.2 for all $(t, j) \in \mathrm{dom}\, \xi$ such that $(t, j + 1) \in \mathrm{dom}\, \xi$,

$$\begin{aligned} &(\xi(t, j), v(t, j)) \in D \\ &\xi_x(t, j + 1) = 0; \\ &\xi_\tau(t, j + 1) = 0; \end{aligned} \tag{4}$$

Note that any continuous-time signal $\overline{v} : \mathbb{R}_{\geq 0} \to \mathbb{R}^q$ can be rewritten as hybrid signal with domain E, for any given hybrid domain E. In fact, suppose that $E = \bigcup_{j=0}^{\infty} [t_j, t_{j+1}] \times \{j\}$ is an hybrid time domain. Then, we can define a hybrid signal v *lifted* from $\overline{v}$ on E as follows: $v(t, j) = \overline{v}(t)$ for each $(t, j) \in E$. Conversely, suppose that (ξ, v) is a solution pair to the hybrid system (2). Then, the output signal $u = h(\xi_x)$ is a hybrid signal and $\mathrm{dom}\, u = \mathrm{dom}\, \xi$. From u we can construct an continuous-time signal $\overline{u} : \mathbb{R}_{\geq 0} \to \mathbb{R}^q$ *projected* from u on $\mathbb{R}_{\geq 0}$ as follows: $\overline{u}(t) = u(t, j)$ for each $(t, j) \in \mathrm{dom}\, u$ such that $(t, j + 1) \notin \mathrm{dom}\, u$, and $\overline{u}(t) = u(t, j + 1)$ otherwise.

We denote with $\|\overline{v}\|_p$ the $\mathcal{L}_p$ gain of a continuous-time signal $\overline{v}$. The $\mathcal{L}_p$ gain of a hybrid signal v, related to the continuous part of its domain, will be denoted by $\|v\|_{c,p} = \left(\sum_{j=0}^{J} \int_{t_j}^{t_{j+1}} |v(t, j)|^p dt \right)^{1/p}$. Note that for any continuous-time signal $\overline{v}$ projected from a hybrid signal $\overline{v}$ on $\mathbb{R}_{\geq 0}$, we have that $\|\overline{v}\|_q = \|v\|_{c,p}$. Conversely, for any hybrid signal v lifted from a continuous-time signal $\overline{v}$ on a given hybrid time domain E, we have that $\|v\|_{c,p} = \|\overline{v}\|_p$.

Finally, the following lemma characterizes regularity of solutions. to (2).

Lemma 1 *Under Assumption 1, all the solutions of (2) are uniformly non-Zeno. Moreover, for each $\mathcal{L}_p$ integrable input signal $\overline{v}$, a solution pair (ξ, v) where v is the hybrid input signal lifted from $\overline{v}$ on* $\mathrm{dom}\, \xi$, *is a complete solution pair.*

Proof. By the definition of C and D in (2), given any solution pair $(\xi, v) = ((\xi_x, \xi_\tau), v)$ of (2), $t_j - t_{j-1} \geq \rho$ for all $(t, j) \in \mathrm{dom}(x)$, $j \geq 2$. This implies that the uniformly non-Zeno definition in [9] (see also [6]) is satisfied with $T = \rho$ and $J = 2$.

By $C \cup D = \mathbb{R}^n \times \mathbb{R}_{\geq 0} \times \mathcal{V}$, $\mathrm{dom}\, \xi$ is bounded only if ξ blows up in finite time. Looking at the dynamics of the system in (2a), by Assumption 1, $|\dot{x_c}| \leq |f(x_c) + g(x_c, v)| \leq L_1 |x_c| + K|v|$ and $|\dot{\tau}| = 1$. Therefore, if $|v|$ is $\mathcal{L}_p$ integrable, $|\xi|$ is bounded in any given compact subset of $\mathbb{R}_{\geq 0} \times \mathbb{N}$. $\square$

3 Main results

3.1 Passivity of the reset controller

The following theorem shows that the hybrid controller (2) is almost passive with a shortage of input passivity proportional to the temporal regularization constant ρ plus ϵ_1. Moreover, the slight modification of the function $\psi(\cdot, \cdot)$ enforced by ϵ_2 induces some excess of output passivity.

Theorem 1 *Consider the hybrid controller* (2) *satisfying Assumption 1. Define*

$$\begin{aligned} \varepsilon_1 := \frac{\epsilon_1}{1 - \epsilon_1 \epsilon_2}\,, \qquad \varepsilon_2 := \frac{\epsilon_2}{1 - \epsilon_1 \epsilon_2} \\ \overline{k}(\rho) := \rho \max\{1, \rho e^{L_1}\}(1 + \varepsilon_2 \rho \max\{1, \rho e^{L_1}\}). \end{aligned} \tag{5}$$

Given a $\mathcal{L}_2$ integrable input signal $\overline{v} \in \mathbb{R}_{\geq 0} \to \mathcal{V}$ and a solution pair (ξ, v) to (2), with v lifted from $\overline{v}$ on $\operatorname{dom} \xi$, *then*

$$\int_0^\infty \overline{u}(t)^T \overline{v}(t) \geq -\left(\varepsilon_1 + \overline{k}(\rho)\right) \|\overline{v}(\cdot)\|_2^2 + \varepsilon_2 \|\overline{u}(\cdot)\|_2^2 \quad (6)$$

where the output signal $\overline{u} \in \mathbb{R}_{\geq 0} \to \mathbb{R}^q$ is projected from the hybrid output signal $u : \operatorname{dom} u \to \mathbb{R}^q$ corresponding to the solution pair (ξ, v),

Remark 1 Note that Theorem 1 establishes the passivity of (2) based on the norm $\|\cdot\|_{c,2}$, namely only taking into account the continuous-time nature of the hybrid solutions. This type of passivity concept is relevant because of Lemma 1 and, moreover, allows to rely on standard passivity results [15] to conclude properties of the closed loop between (2) and a plant, ad detailed in section 3.2.

⌟

Proof of Theorem 1. Consider an input signal $\overline{v} : \mathbb{R}_{\geq 0} \to \mathcal{V}$ such that $\|\overline{v}\|_2$ is defined, and consider a solution pair $(\xi, v) = ((\xi_x, \xi_\tau), v)$ to the hybrid system (2), where v is the hybrid signal lifted from $\overline{v}$ on $\operatorname{dom} \xi$. By Lemma 1, $\operatorname{dom} v$ is unbounded.
Define the set $\mathcal{T} = \bigcup_j [t_j, t_j + \rho] \times \{j\}$ where for all j, t_j is such that, for each $\tau \in \mathbb{R}_{>0}$, $(t_j - \tau, j) \notin \operatorname{dom} \xi$. Note that by time regularization, $\mathcal{T} \subseteq \operatorname{dom} \xi$ but $\mathcal{T}$ is not necessarily a hybrid time domain. It follows that $\forall (t, j) \in \operatorname{dom} \xi$ such that $(t, j) \notin \mathcal{T}$ we have $\xi_\tau(t, j) \geq \rho$, therefore

$$u(t, j) v(t, j) + \varepsilon_1 |v(t, j)|^2 - \varepsilon_2 |u(t, j)|^2 \geq 0 \quad (7)$$

where $\varepsilon_1 = \frac{\epsilon_1}{1 - \epsilon_1 \epsilon_2}$ and $\varepsilon_2 = \frac{\epsilon_2}{1 - \epsilon_1 \epsilon_2}$. Therefore

$$\begin{aligned}
\int_0^\infty \overline{u}(t)^T \overline{v}(t) dt &= \sum_j \int_{t_j}^{t_{j+1}} u(t, j)^T v(t, j) dt = \\
&= \sum_j \left(\int_{t_j}^{t_j + \rho} u(t, j)^T v(t, j) dt + \right. \\
&\qquad \left. + \int_{t_j + \rho}^{t_{j+1}} u(t, j)^T v(t, j) dt \right) \\
&\geq \sum_j \left(\int_{t_j}^{t_j + \rho} u(t, j)^T v(t, j) dt \; + \right. \\
&\qquad \left. + \int_{t_j + \rho}^{t_{j+1}} -\varepsilon_1 |v(t, j)|^2 dt + \int_{t_j + \rho}^{t_{j+1}} \varepsilon_2 |u(t, j)|^2 dt \right) \\
&\geq \sum_j \left(\int_{t_j}^{t_j + \rho} u(t, j)^T v(t, j) dt - \varepsilon_2 |u(t, j)|^2 dt + \right. \\
&\qquad \left. + \int_{t_j}^{t_{j+1}} -\varepsilon_1 |v(t, j)|^2 dt + \int_{t_j}^{t_{j+1}} \varepsilon_2 |u(t, j)|^2 dt \right).
\end{aligned} \quad (8)$$

Consider now the continuous dynamics of x_c in (2a). By Assumption 1, we have

$$|\dot{x_c}| \leq |f(x_c) + g(x_c, v)| \leq L_1 |x_c| + K |v| \quad (9)$$

Then, for $(t, j) \in [t_j, t_j + \rho] \times \{j\} \subseteq \mathcal{T}$, we have

$$\begin{aligned}
|u(t, j)| &\leq L_2 \int_{t_j}^{t} e^{L_1 (t - s)} K |v(s, j)| ds \\
&\leq L_2 \int_{t_j}^{t_j + \rho} e^{L_1 (t_j + \rho - s)} K |v(s, j)| ds \\
&\leq L_2 K \max\{1, e^{L_1 \rho}\} \int_{t_j}^{t_j + \rho} |v(s, j)| ds
\end{aligned} \quad (10)$$

Note that in (10) there is no dependence on the initial condition by the fact that $\xi_x(t_j, j) = 0$. It follows that

$$\begin{aligned}
&\int_{t_j}^{t_j + \rho} |u(t, j)|^2 dt \leq \\
&\leq \int_{t_j}^{t_j + \rho} L_2^2 K^2 \max\{1, e^{2 L_1 \rho}\} \left(\int_{t_j}^{t_j + \rho} |v(s, j)| ds \right)^2 dt \\
&= \rho L_2^2 K^2 \max\{1, e^{2 L_1 \rho}\} \left(\int_{t_j}^{t_j + \rho} |v(s, j)| ds \right)^2 dt \\
&\leq \rho^2 L_2^2 K^2 \max\{1, e^{2 L_1 \rho}\} \int_{t_j}^{t_j + \rho} |v(s, j)|^2 ds
\end{aligned} \quad (11)$$

where we used Holder's integral inequality [16, page 274] in the last step of (11).

$$\begin{aligned}
&\int_{t_j}^{t_j + \rho} u(t, j)^T v(t, j) dt \leq \\
&\leq L_2 K \max\{1, e^{L_1 \rho}\} \left(\int_{t_j}^{t_j + \rho} |v(t, j)| dt \right)^2 \\
&\leq \rho L_2 K \max\{1, e^{L_1 \rho}\} \int_{t_j}^{t_j + \rho} |v(t, j)|^2 dt
\end{aligned} \quad (12)$$

where, as above, the last inequality is obtained by using Holder's integral inequality.

Define $k(\rho) = \rho \max\{1, \rho e^{L_1}\}$. By (11), (12), we have that

$$\begin{aligned}
&\left| \int_{t_j}^{t_j + \rho} u(t, j)^T v(t, j) dt - \varepsilon_2 |u(t, j)|^2 dt \right| \leq \\
&\leq k(\rho) (1 + \varepsilon_2 k(\rho)) \int_{t_j}^{t_j + \rho} |v(t, j)|^2 dt
\end{aligned} \quad (13)$$

Define now $\overline{k}(\rho) = k(\rho)(1 + \varepsilon_2 k(\rho))$ then, from (8), we can say that

$$
\begin{aligned}
&\int_0^\infty \overline{u}(t)^T \overline{v}(t) dt \geq \\
&\geq \sum_j \left(-\overline{k}(\rho) \int_{t_j}^{t_j+\rho} |v(t,j)|^2 dt + \right. \\
&\left. + \int_{t_j}^{t_{j+1}} -\varepsilon_1 |v(t,j)|^2 dt + \int_{t_j}^{t_{j+1}} \varepsilon_2 |u(t,j)|^2 dt \right) \\
&\geq - \left(\varepsilon_1 + \overline{k}(\rho)\right) \sum_j \int_{t_j}^{t_{j+1}} |v(t,j)|^2 dt + \\
&+ \sum_j \int_{t_j}^{t_{j+1}} \varepsilon_2 |u(t,j)|^2 dt \\
&= - \left(\varepsilon_1 + \overline{k}(\rho)\right) \|\overline{v}(\cdot)\|_2^2 + \varepsilon_2 \|\overline{u}(\cdot)\|_2^2
\end{aligned}
\tag{14}
$$

□

Remark 2 It is important to underline that the passive behavior of the hybrid controller (2) is strongly related to the definition of the jump and flow sets D and C, more than to the dynamic equations of the controller. Roughly speaking, the passive behavior of the controller can be considered as an effect of the definition of $\psi(u, v)$, that forces a particular shape of the sets C and D. Following this intuition, while $\psi(u, v)$ constrains C and D to induce passivity, time regularization adds some extra constraint on C and D possibly destroying part of this passivity property. This results in a shortage of passivity parameterized with ρ. ⌟

3.2 Application to feedback systems

In this section we use the *passivity theorem* [15] to establish useful stability properties of the reset controller (2) interconnected to any passive nonlinear plant: [3]

$$
\begin{aligned}
\dot{x}_p &= f_p(x_p, u + d) \\
y &= h_p(x, u + d),
\end{aligned}
\tag{15}
$$

via the negative feedback interconnection $v = w - y$, where w is an external signal. In (15), d is an additive disturbance acting at the plant input. The following statement directly follows from the properties of (2) established in Theorem 1.

Proposition 1 *Consider the hybrid controller* (2) *satisfying Assumption 1 in feedback interconnection $v = w - y$ with the plant (15).*

For any $\epsilon_1 \geq 0$, $\epsilon_2 > 0$ and $\rho > 0$, given ε_1 and $\overline{k}(\rho)$ as in (6), if the plant is output strictly passive with excess of output passivity $\delta_P > \varepsilon_1 + \overline{k}(\rho)$, then the closed-loop system (2), (15) with $v = w - y$ is finite-gain $\mathcal{L}_2$ stable from (w, d) to (u, v).

In Proposition 1 we require a specific excess of output passivity from the plant because we assume that the controller requires implementation with certain prescribed selections of ϵ_1 and ρ. In the case where it is possible to reduce arbitrarily these two parameters, it is possible to relax the requirements of Proposition 1 as follows:

Proposition 2 *Consider the hybrid controller* (2) *satisfying Assumption 1 in feedback interconnection $v = -y$ with the plant (15).*

If the plant (15) is output strictly passive, then for any $\epsilon_2 > 0$, there exist small enough positive numbers ϵ_1^* and ρ^* such that for all $\epsilon_1 \leq \epsilon_1^*$ and all $\rho \leq \rho^*$, the closed-loop system (2), (15) with $v = w - y$ is finite-gain $\mathcal{L}_2$ stable from (w, d) to (u, v).

Proof. The proposition is a straightforward consequence of Proposition 1 noting that for a fixed ϵ_2, the lack of output passivity established in Theorem 1 decreases monotonically to zero as ϵ_1 and ρ go to zero. Then it is always possible to reduce the two parameters to match the passivity condition in [15]. □

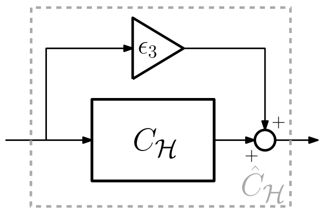


Figure 2. The very strictly passive version (16) of the reset controller ($C_{\mathcal{H}}$ corresponds to (2)).

Both Propositions 1 and 2 either require an explicit bound on the excess of output passivity of the plant or constrain the controller parameters ϵ_1 and ρ to be small enough. An alternative solution to this is to add an extra feedforward loop to the reset controller (2), following the derivations in [12, page 233], to guarantee that the arising reset system is very strictly passive, namely it is both input strictly passive and output strictly passive. To this aim, we modify the output equation of (2) by adding the feedforward term $\epsilon_3 v$, as represented in Figure 2. The corresponding reset controller can then be

[3] See also [4] for a similar application of the passivity theorem to reset controllers.

written as:

$$\begin{cases} \dot{x_c} = f(x_c) + g(x_c, v) \\ \dot{\tau} = 1 \end{cases} \quad \textit{if } \tau \leq \rho \textit{ or } \hat{\psi}(\hat{u}, v) \geq 0$$
$$\begin{cases} x_c^+ = 0 \\ \tau^+ = 0 \end{cases} \quad \textit{if } \tau \geq \rho \textit{ and } \hat{\psi}(\hat{u}, v) \leq 0$$
$$\hat{u} = h(x_c) + \epsilon_3 v \tag{16a}$$

where $\hat{\psi}(\hat{u}, v)$ is defined as

$$\hat{\psi}(\hat{u}, v) = ((\hat{u} + (\epsilon_1 - \epsilon_3)v)^T((1 + \epsilon_2\epsilon_3)v - \epsilon_2\hat{u}) \tag{16b}$$

and $\epsilon_3 > 0$ is suitably selected as specified below. When using the modified reset controller (16), the following result holds.

Proposition 3 *Consider the hybrid controller* (16) *satisfying Assumption 1 in feedback interconnection $v = w - y$ with a passive plant (15).*

For any $\epsilon_1 \geq 0$, $\epsilon_2 > 0$ and $\rho > 0$, given ε_1 and $\overline{k}(\rho)$ as in (6), if $\epsilon_3 > \varepsilon_1 + \overline{k}(\rho)$, then the closed-loop system (16), (15) with $v = w - y$ is finite-gain $\mathcal{L}_2$ stable from (w, d) to (u, v).

Proof. Define a new output $\hat{u} = u + \epsilon_3 v$ and denote by $\overline{\hat{u}}$ the output signal projected from $\hat{u}$ on $\mathbb{R}_{\geq 0}$. Then, from (6), we have that

$$\int_0^\infty \overline{\hat{u}}(t)^T \overline{v}(t) \geq \epsilon_2 \int_0^\infty \overline{u}^T \overline{u} + \left(\epsilon_3 - \varepsilon_1 - \overline{k}(\rho)\right) \int_0^\infty \overline{v}^T \overline{v}$$
$$\geq \frac{1}{1+2\epsilon_2\epsilon_3} \left(\epsilon_2 \int_0^\infty \overline{\hat{u}}^T \overline{\hat{u}} + \left(\epsilon_3 - \varepsilon_1 - \overline{k}(\rho)\right) \int_0^\infty \overline{v}^T \overline{v}\right).$$

It follows that

$$\int_0^\infty \overline{\hat{u}}(t)^T \overline{v}(t) \geq \eta_1 \|\overline{\hat{u}}\|_2^2 + \eta_2 \|\overline{v}\|_2^2 \tag{17}$$

with $\eta_1 = \frac{\epsilon_2}{1+2\epsilon_2\epsilon_3} > 0$ and $\eta_2 = \frac{\epsilon_3 - \varepsilon_1 - \overline{k}(\rho)}{1+2\epsilon_2\epsilon_3} > 0$.

Replace now the output u of the controller (2) with $\hat{u} = u + \epsilon_3 v = h(x_c) + \epsilon_3 v$. Then, $\hat{\psi}(\hat{u}, v)$ is obtained by substituting $u = \hat{u} - \epsilon_3 v$ in the expression of $\psi(u, v)$ of Equation (2b). By the *passivity theorem* in [15], Proposition 3 follows. □

Remark 3 The results in this section can be seen as a generalization of the results on full reset compensators in [4], where passivity techniques are used to establish finite gain $\mathcal{L}_2$ stability of the closed-loop between passive nonlinear plants and reset controllers. When focusing on linear reset controllers such as Clegg integrators [5] and First Order Reset Elements (FORE) [10,2], the novelty of Theorem 1 as compared to the results in [4] is that those results establish passivity of FORE whose underlying linear dynamics is already passive (namely FORE with stable poles). Conversely, our results of Theorem 1 apply regardless of what the underlying dynamics of the controller is. Therefore, for example, any FORE with arbitrarily large unstable poles would still become passive using the flow and jump sets characterized in by (2). Note however that, as compared to the approach in [4], we are using a different selection of the flow and jump sets. In the example section we illustrate the use of unstable FOREs within (2). ⌟

4 Simulation example

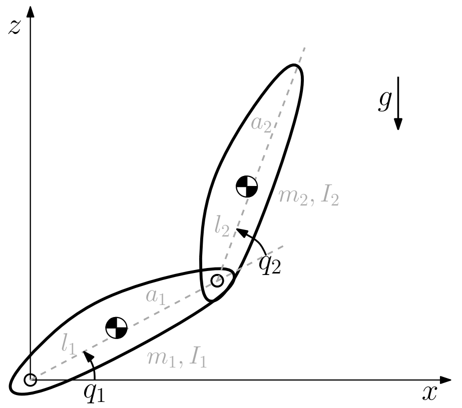
We consider a planar two-link rigid robot manipulator in Figure 3, as modeled in [13]. Denoting by $q \in \mathbb{R}^2$ the two joint positions and by $\dot{q} \in \mathbb{R}^2$ the corresponding velocities, the manipulator is modeled as

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + h(q) = u_p \tag{18}$$

where $D(q)$ is the inertia matrix, $C(q, \dot{q})\dot{q}$ comprises the centrifugal and Coriolis terms, $h(q)$ is the gravitational vector, and u_p represents the external torques applied to the two rotational joints of the robot. In Figure 3, m_1 and m_2 represent the links masses, a_1 and a_2 represent the links lengths, l_1 and l_2 represent the distances of the center of mass of each link from the preceding joint, and I_1 and I_2 represent the rotational inertias at the two joints. The numerical values of the parameters are listed in the table of Figure 3. Denoting $D(q) = \begin{bmatrix} d_{11} & d_{12} \\ d_{12} & d_{22} \end{bmatrix}$, $C(q, \dot{q}) = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & 0 \end{bmatrix}$, and $h(q) = \begin{bmatrix} h_1 & h_2 \end{bmatrix}^T$, we get:

$d_{11} = I_1 + m_1 l_2^2 + I_2 + m_2(a_1^2 + l_2^2 + 2a_1 l_2 \cos(q_2))$,
$d_{12} = I_2 + m_2(l_2^2 + a_1 l_2 \cos(q_2))$,
$d_{22} = I_2 + m_2 l_2^2$,
$c_{11} = -m_2 a_1 l_2 \sin(q_2)\dot{q}_2$,
$c_{12} = -m_2 a_1 l_2 \sin(q_2)(\dot{q}_1 + \dot{q}_2)$,
$c_{21} = m_2 a_1 l_2 \sin(q_2)\dot{q}_1$,
$h_1 = g(m_1 l_1 + m_2 a_1)\cos(q_1) + g m_2 l_2 \cos(q_1 + q_2)$, $h_2 = g m_2 l_2 \cos(q_1 + q_2)$.

Given a reference signal $r \in \mathbb{R}^2$ representing the desired joint position, following a standard passivity based approach, it is possible to close a first control loop around the robot (18) to induce the equilibrium point $(q, \dot{q}) = (r, 0)$ while guaranteeing passivity from a suitable input u to the joint velocity output $\dot{q}$, as shown in Figure 4. In particular, define $V(q, r) = \frac{k_p}{2}(q - r)^T(q - r)$, where the scalar $k_p > 0$ is a weight parameter on the position



Link	l_i $[m]$	m_i $[kg]$	I_i $[kgm^2]$	a_i $[m]$
1	0.5	6	0.2	1
2	0.25	5	0.1	0.5

Figure 3. The robot example and its parameters.

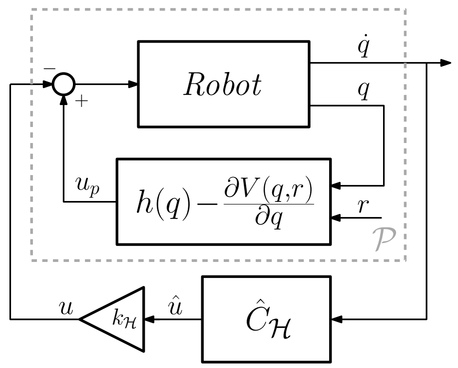


Figure 4. Control loop of the two-links robot.

error, and choose

$$u_p = -\frac{\partial V(q,r)}{\partial q} + h(q) + u. \tag{19}$$

Then, the interconnection (18), (19) corresponds to

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + \frac{\partial V(q,r)}{\partial q} = u \tag{20}$$

and, following similar steps to those in [7], it can be shown to be passive from u to $\dot{q}$. In particular, use the storage function $E = \frac{1}{2}\dot{q}^T D(q)\dot{q} + V(q,r)$ to conclude

$$\begin{aligned}\dot{E} &= \dot{q}^T D(q)\ddot{q} + \frac{1}{2}\dot{q}^T \dot{D}(q)\dot{q} + k_p(q-r)^T\dot{q} \\ &= \dot{q}^T u + \dot{q}^T\left(\frac{1}{2}\dot{D}(q) - C(q,\dot{q})\right)\dot{q} \\ &= \dot{q}^T u\end{aligned} \tag{21}$$

where the second equality follows from (20) and the third equality follows from the well known fact that $z^T(\dot{D}(q) - 2C(q,\dot{q}))z = 0$, for all $z \in \mathbb{R}^2$.

For the outer loop, we rely on the very strictly passive controller (16) where the dynamics in (16a) is selected as a pair of decentralized First Order Reset Elements, namely denoting $x_c = [x_{c1}\ x_{c2}]^T$, we select $f(x_c) = [\lambda_1 x_{c1}\ \lambda_2 x_{c2}]^T$ and $g(x_c, \dot{q}) = \dot{q}$. Moreover, as shown in Figure 4, we choose $u = k_{\mathcal{H}}\hat{u}$, where $k_{\mathcal{H}}$ is a positive constant.

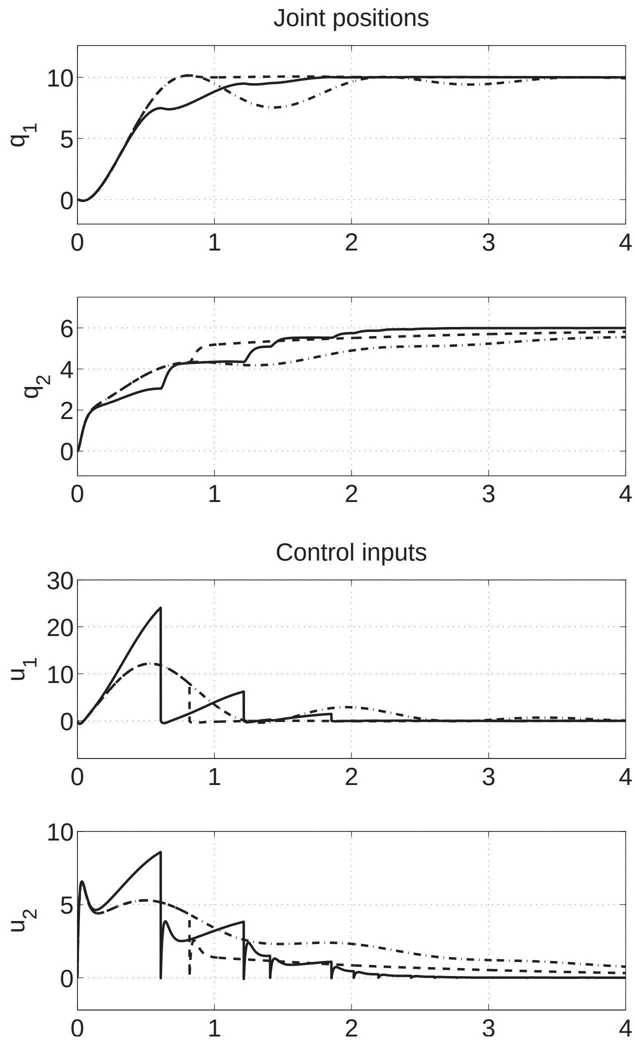


Figure 5. Simulations results. Stable FORE and no resets (dash-dotted), stable FORE with resets (dashed) and unstable FORE with resets (solid).

By Proposition 3, the closed loop system (18), (19), (16a) with $u = k_{\mathcal{H}}\hat{u}$ is finite-gain $\mathcal{L}_2$ stable. Figure 5 compares several simulation results for this closed-loop using the constant reference signal $r = [10\,6]^T$ and the following values of the parameters: $k_p = 100$, $k_{\mathcal{H}} = 100$ and $\rho = 0.1$. First, we select stable FORE poles $(\lambda_1, \lambda_2) = (-2, -1)$ so that the closed-loop stability can be concluded also using the results in [4]. For this case, when no resets occur, the position output (namely q) and plant input (namely u) responses correspond to the dash-dotted curves in Figure 5. That response is converging because the system without resets is passive due to the stability of the FORE poles. When introducing resets, the response becomes the dashed curves in the figure, where it can be appreciated that a single reset occurring around $t = 0.8\ s$ significantly improves the closed-loop response. A last simulation is carried out by selecting an unstable FORE with $(\lambda_1, \lambda_2) = (2, 1)$. In this case the speed of convergence of the second joint is faster at the price of a reduction of the speed of convergence of the first joint. Note also that the dwell time imposed by the temporal regularization is never active for this specific simulation, as each jump occurs after more than $\rho = 0.1$ seconds from the previous jump. We don't include a simulation with the unstable FORE without resets because this leads to diverging trajectories.

5 Conclusions

In this paper we proposed a reset rule for nonlinear controllers which ensures a certain type of input/output passivity. Then, relying on the passivity theorem we concluded useful properties of control systems involving this type of reset controller. A simulation example illustrates the effectiveness of the approach.

References

[1] O. Beker, C.V. Hollot, and Y. Chait. Plant with an integrator: an example of reset control overcoming limitations of linear feedback. *IEEE Transactions Automatic Control*, 46:1797–1799, 2001.

[2] O. Beker, C.V. Hollot, Y. Chait, and H. Han. Fundamental properties of reset control systems. *Automatica*, 40:905–915, 2004.

[3] C. Cai and A.R. Teel. Characterizations of input-to-state stability for hybrid systems. *Systems & Control Letters*, 58(1):47–53, 2009.

[4] J. Carrasco, A. Banos, and A. Van der Schaft. A passivity-based approach to reset control systems stability. *Systems & Control Letters*, 59(1):18–24, 2010.

[5] J.C. Clegg. A nonlinear integrator for servomechanisms. *Trans. A. I. E. E.*, 77 (Part II):41–42, 1958.

[6] P. Collins. A trajectory-space approach to hybrid systems. In *International Symposium on the Mathematical Theory of Networks and Systems*, Leuven, Belgium, July 2004.

[7] I. Fantoni, R. Lozano, and M.W. Spong. Energy based control of the pendubot. *Automatic Control, IEEE Transactions on*, 45(4):725–729, Apr 2000.

[8] R. Goebel, R. Sanfelice, and A. Teel. Hybrid dynamical systems. *Control Systems Magazine, IEEE*, 29(2):28–93, April 2009.

[9] R. Goebel and A.R. Teel. Solutions to hybrid inclusions via set and graphical convergence with stability theory applications. *Automatica*, 42(4):573 – 587, 2006.

[10] I. Horowitz and P. Rosenbaum. Non-linear design for cost of feedback reduction in systems with large parameter uncertainty. *Int. J. Contr*, 21:977–1001, 1975.

[11] K.H. Johansson, J. Lygeros, S. Sastry, and M. Egerstedt. Simulation of Zeno hybrid automata. In *Conference on Decision and Control*, pages 3538–3543, Phoenix, Arizona, 1999.

[12] H.K. Khalil. *Nonlinear Systems*. Prentice Hall, USA, 3rd edition, 2002.

[13] F. Morabito, S. Nicosia, A.R. Teel, and L. Zaccarian. Measuring and improving performance in anti-windup laws for robot manipulators. In B. Siciliano, A. De Luca, C. Melchiorri, and G. Casalino, editors, *Advances in Control of Articulated and Mobile Robots*, chapter 3, pages 61–85. Springer Tracts in Advanced Robotics, 2004.

[14] D. Nešić, L. Zaccarian, and A.R. Teel. Stability properties of reset systems. *Automatica (B)*, 44(8):2019–2026, 2008.

[15] A. J. Schaft. *L2-Gain and Passivity in Nonlinear Control*. Springer-Verlag New York, Inc., Secaucus, NJ, USA, 1999.

[16] M. Vidyasagar. *Nonlinear Systems Analysis*. Prentice-Hall, Englewood Cliffs, New Jersey, 2nd edition, 1993.

[17] L. Zaccarian, D. Nešić, and A.R. Teel. First order reset elements and the Clegg integrator revisited. In *American Control Conference*, pages 563–568, Portland (OR), USA, June 2005.