2.3 OPCA

As in the EFPA case, we assume that events related to the N different servers are mutually independent and that the servers are statistically equivalent. Also as in the EFPA case, we let \( a > 0 \) be the arrival rate of new calls at a server and \( a_n \) the arrival rate of \( n \)-calls at a server. Thus, \( a_0 = a \). The stream formed by \( n \)-calls, \( N - 1 \geq n \geq 0 \), arriving at a server, is assumed to follow a Poisson stream with rate \( a_n \). The preemptive priority regime defined by OPCA gives priority to “junior” \( n_h \)-calls over “senior” \( n_l \)-calls for any \( 0 \leq n_h < n_l \leq N - 1 \). Accordingly, the \( a_n \) values can be obtained recursively by

\[
a_{n+1} = E_1 \left( \sum_{i=0}^{n} a_i \right) \sum_{i=0}^{n} a_i - \sum_{i=1}^{n} a_i,
\]

for all \( n = 0, \ldots, N - 1 \), where \( a_N \) is defined as the rate of the stream formed by calls that are blocked and cleared.

The OPCA blocking probability approximation is given by

\[
P_{OPCA} = \frac{a_N}{a},
\]

Eq. (6) has a clear physical interpretation. The \( a_N/a \) ratio is the proportion of calls that are promoted to \( N \)-calls (blocked calls) which is the OPCA blocking probability approximation.

Graphs are shown in Fig. 1 demonstrating the tightness of \( P_{OPCA} \) and \( P_{EFPA} \) as lower bounds for \( P_{exact} \).

We are now able to state the main theorem of this paper.

**Theorem 1.**

\[
P_{EFPA} \leq P_{OPCA} \leq P_{exact}.
\]

The first inequality, namely \( P_{EFPA} \leq P_{OPCA} \), was proved in [54], where the second inequality is stated as the **the OPC Conjecture**. It is the purpose of this paper to provide a mathematical proof of the OPC conjecture.