

works on any binary architecture, provided only that  $s < 2^w$ . For example, assuming that  $1 \leq s < 2^4 = 16$ ,  $size(4)$  and  $size(10)$  are calculated to be 4 and 2 respectively:

	$s = 4$	$s = 10$
$s$ in binary:	0100	1010
$16 - s$ in binary:	1100	0110
result after AND:	0100	0010
result in decimal:	4	2

The cost of calculating  $size()$  is thus  $O(1)$ , and the overall time taken to process symbol  $s$  is  $O(\log n)$ . Compared to the sorted array method of Witten et al., this mechanism represents a rare instance of algorithmic development in which both space and time are saved. Using it, the overall cost of adaptively maintaining statistics for message  $M$  containing  $m$  symbols is  $O(m \log n)$  time.

One further function is required in the decoder, illustrated in Algorithm 6.5. The *target* value returned by *arithmetic\_decode\_target()* (described in Algorithm 5.4 on page 104) must be located in the array *fen\_prob*. This is accomplished by a binary search variant based around powers of two. The first location inspected is the largest power of two less than or equal to  $n$ , the current alphabet size. That is, the search starts at position  $2^{\lfloor \log_2 n \rfloor}$ . If the value stored at this position is greater than *target*, then the desired symbol cannot have a greater index, and the search focuses on the first section of the array. On the other hand, if the *target* is larger than this middle value the search can move right, looking for a diminished *target*. Once the desired symbol number  $s$  has been determined, function *fenwick\_get\_lbound()* is used to determine the bound  $l$  for the arithmetic decoding function, and *fenwick\_get\_and\_increment\_count()* is used to determine  $c$  and to increment the frequency of symbol  $s$ . Both of these latter two functions are shared with the encoder.

The attentive reader might still, however, be disappointed, as  $O(m \log n)$  time can still be superlinear in the number of bits emitted. Consider, for example, the probabilities

$$P = [1/2, 1/4, \dots, 1/2^i, \dots, 1/2^{n-1}, 1/2^{n-1}].$$

Then a message of length  $m$  has an expected coded length of approximately  $2m$  bits irrespective of the value of  $n$ , yet the time taken to generate that bitstream using a Fenwick tree is  $\Theta(m \log n)$ . In particular, the reader may recall that adaptive Huffman coding does take time linear in the number of bits produced. Can similar linear-time behavior be achieved for adaptive arithmetic coding?

Pleasingly, the answer is “yes” – it is possible to modify the Fenwick tree data structure and obtain asymptotic linearity [Moffat, 1999]. Consider again