Class of stabilising decentralised controllers for interconnected dynamical systems

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Abstract: A method is presented for the systematic design of stabilising decentralised controllers for large-scale interconnected dynamical systems. The design method is based on (i) decentralised implementation of global controllers obtained by using existing global controller design methods, (ii) model reduction of dynamical systems, and (iii) modelling of the interactions among the subsystems comprising the global closed-loop control system. This is used for generating local corrective control signals, which would account for these interactions. The resulting decentralised controller uses local information only to generate local control inputs. It comprises local state feedback controllers and feedforward compensators. Three distinct schemes for the implementation of the controller are proposed. They all guarantee near optimal performance. A four-station interconnected dynamical system is considered. Results of time simulation studies demonstrate that the overall performance of the system under the proposed decentralised controller is near that of the global optimal controller.

1 Introduction

To date, the task of effectively controlling large-scale interconnected dynamical systems is one of the most challenging problems in control engineering. Centralised control of such systems requires the establishment of an extensive communication network for the transfer of information between each part of the interconnection and the central facility. The function of the central facility is to acquire information from all parts of the interconnection, process the information, and send back stabilising control signals to the corresponding parts. All these activities are carried out on-line and in real time. In many practical cases, economical as well as technical considerations constrain the amount of information transfer among the subsystems comprising the interconnected dynamical systems. The economic constraint stems from the need for the establishment of communication channels between each part of the interconnection and the central facility. This becomes a prime consideration when each subsystem of the interconnection is geographically separated from the rest by long distances.

Technical constraint stems from the computational difficulties arising from processing very large amounts of information in real time, especially in large-scale systems comprising the global closed-loop control system. This is used for generating local corrective control signals, which would account for these interactions. The resulting decentralised controller uses local information only to generate local control inputs. It comprises local state feedback controllers and feedforward compensators. Three distinct schemes for the implementation of the controller are proposed. They all guarantee near optimal performance. A four-station interconnected dynamical system is considered. Results of time simulation studies demonstrate that the overall performance of the system under the proposed decentralised controller is near that of the global optimal controller.

Although these methods provide a means of establishing whether or not a given decentralised controller stabilises the global system, they stop short of addressing the problem of developing decentralised controller design methods which would result in the stabilisation of the overall global system and, at the same time, meet some global system performance criteria. The second category [5–10] attempts to develop methods for the design of decentralised stabilising controllers. To date, these attempts have been based on either trial and error [5, 6], or design of state observers [7–10]. In the former case, local state feedback controllers are arbitrarily chosen and a criterion is applied to test the overall stability. If the criterion holds, the design is finished, otherwise a new choice is made. The whole process is repeated until a solution is found. There is no guarantee, however, that this iterative process will converge to a solution. In the latter case, state observers are designed at the local level for the construction of the entire state vector, which is then used to generate local control signals. An overview of decentralised control methods is given in Reference 11.

In this paper, a novel method is presented for the design of simple and effective decentralised controllers for interconnected dynamical systems. The design method is an extension to, and generalisation of, earlier attempts reported in References 9 and 12. The philosophy of the design approach is centred on the idea of decentralised implementation of global controllers, obtained by using existing global controller design methods. Among the features of the proposed decentralised control method are...
(i) complete decentralisation of the control task (i.e. local information only is used to generate the necessary local inputs)
(ii) no constraints are imposed on the structure of the global controller
(iii) local controllers and local compensators are calculated off line
(iv) simple and straightforward controller design algorithm is involved.

2 Problem statement

Consider the following linear time-invariant dynamical system:

\[ \dot{x}(t) = Ax(t) + Bu(t) \]
\[ y(t) = Cx(t) \]

with

\[ x(0) = x_0 \]

where \( x, u \) and \( y \) are \( n, I \) and \( m \)-dimensional state, control and output vectors, respectively. The matrices \( A, B \) and \( C \) are constants with appropriate dimensions.

Let the system described by eqns. 1 be composed of \( N \) subsystems with the \( i \)th subsystem having \( x_i \) and \( u_i \) as its state and control vectors, respectively. Let the dimensions of \( x_i \) and \( u_i \) be \( n_i \) and \( l_i \), respectively, so that: \( \sum_{i=1}^{N} n_i = n \) and \( \sum_{i=1}^{N} l_i = l \). Assume that each subsystem depends only on its own set of control variables, i.e. the control matrix \( B \) is block-diagonal. Accordingly, we write \( x = \begin{bmatrix} x_1^T & x_2^T & \cdots & x_N^T \end{bmatrix}^T, u = \begin{bmatrix} u_1^T & u_2^T & \cdots & u_N^T \end{bmatrix}^T, A = \begin{bmatrix} A_1 & 0 & \cdots & 0 \\ 0 & A_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_N \end{bmatrix} \) and \( B = \begin{bmatrix} B_1 & B_2 & \cdots & B_N \end{bmatrix} \) where \( A_i \) (\( i = 1, 2, \ldots, N \)) are \( (n_i \times n_i) \)-dimensional submatrices of \( A \), and \( B_i \) (\( i = 1, 2, \ldots, N \)) are \( (n_i \times l_i) \)-dimensional submatrices of \( B \).

Let us assume that the global system of eqns. 1 is completely controllable and completely observable and that a satisfactory global state feedback control law of the form

\[ u(t) = Fx(t) + \tau(t) \]

has been found, using conventional state feedback control methods such as pole placement or optimal control, so that the eigenvalues of the closed-loop system lie in preassigned locations in the \( s \)-plane, where \( F \) is an \((l \times n)\)-dimensional global state feedback control matrix and \( \tau \) is an \( m \)-dimensional reference input vector. For simplicity, but without loss of generality, let us assume that the resulting closed-loop eigenvalues are distinct. The decentralised control problem can now be stated as that of finding a set of decentralised local controllers the combined performance of which is equivalent to or near that of the global controller described by eqn. 2.

3 Global control system

If the global control law expressed in eqn. 2 is applied to the global system in eqns. 1, then the closed-loop feedback control system may be expressed as

\[ \dot{x}(t) = Ax(t) + Bu(t) \]
\[ y(t) = Cx(t) \]

with

\[ x(0) = x_0 \]

where \( A = (A + BF) \) is the closed-loop system matrix.

Let the feedback control matrix \( F \) be partitioned in the following way:

\[ F = \begin{bmatrix} F_{11} & \cdots & F_{1N} \\ \vdots & \ddots & \vdots \\ F_{N1} & \cdots & F_{NN} \end{bmatrix} \]

where \( F_{ij} \) are \((l_i \times n_j)\)-dimensional submatrices of \( F \). In view of this definition, the global feedback control system, described by eqn. 3, may be decomposed into \( N \)-subsystems of the following description:

\[ \dot{x}_i(t) = \begin{bmatrix} \hat{A}_i \end{bmatrix} x_i(t) + \sum_{j=1, j \neq i}^{N} \hat{A}_{ij} x_j(t) + \hat{B}_i u_i(t) \\ \quad + B_i \{ u_{ij}(t) + u_{ik}(t) \} \]

The bracketed term \[ \{ \] is the open-loop description of the \( i \)th subsystem, and the braced term \[ \} \] represents its control input, which is composed of two components. The first is the local state feedback control component, \( u_{ij} \), generated locally from

\[ u_{ij}(t) = F_{ij} x_i(t) \]

The second is the corrective state feedback component, \( u_{ik} \), generated from

\[ u_{ik}(t) = \sum_{j=1, j \neq i}^{N} F_{ji} x_j(t) \]

Eqn. 7 implies that, in order to generate the required corrective control inputs, \( u_{ik} \), information about the state of each subsystem must be shared among all the remaining subsystems. This may prove to be prohibitive, especially for large systems. It is the aim of this paper to provide alternative means of generating the required corrective control signals locally, and therefore significantly reducing the overall information transfer burden.

In the following Sections, a three-stage design procedure is developed for the determination of local dynamic and static compensators. The function of these compensators is to generate collectively the required corrective control signals, using local information only. These stages are (i) the isolation of the dominant and nondominant modes of the global closed-loop control system, (ii) the determination of a reduced-order model of the global closed-loop control system, and (iii) the modelling of the interactions among the global closed-loop control system, which will be used for the purpose of generating local corrective control signals. Discussion of these three stages is given next.

4 Isolation of dominant modes

In this Section, an algorithm is presented for the identification and isolation of the dominant and nondominant modes of linear time-invariant dynamical system. The algorithm is based on the notion of combined controllability and observability measure. This will now be developed.

Let the similarity transformation

\[ x(t) = Mz(t) \]

be applied to the closed-loop system expressed in eqn. 3, where \( z \) is an \( n \)-dimensional dummy state vector. Since the closed-feedback system matrix \( \hat{A} \) is assumed to have distinct eigenvalues, a nonsingular modal matrix \( M \) exists so that application of eqn. 8 to the closed-feedback
of each eigenvalue in each output, i.e. participation measure matrix.

From eqn. 9a the steady-state step response of the ith output is determined from the eigenvalue in all the outputs, i.e. participation measure is used:

\[ z_m = \frac{1}{\lambda_i} \sum_{k=1}^{n} y_k \beta_k r_k \quad i = 1, 2, \ldots, n \]  

(10)

where \( y_k \) is the element standing in the ith row and kth column of the matrix \( y \). The term on the right-hand side of eqn. 10 represents the controllability measure of the ith mode from the reference input \( r \).

The steady-state step response of the ith output can be determined by substituting eqn. 10 into the output eqn. 9b to give:

\[ y_m = \sum_{j=1}^{n} \sum_{i=1}^{m} \frac{w_j}{\lambda_i} \sum_{k=1}^{n} y_k \beta_k r_k \quad i = 1, 2, \ldots, n \]  

(11)

In order to determine the relative contribution of the jth eigenvalue in all the outputs, the following participation measure is used:

\[ o_{ij} = \sum_{k=1}^{n} \frac{w_j}{\lambda_i} \sum_{k=1}^{n} y_k \beta_k r_k \]  

(12)

In order to determine the relative dominance of the jth eigenvalue in the ith output, the following expression is used:

\[ o_j = \sum_{i=1}^{m} o_{ij} \]  

(13)

The relative contribution of \( \zeta \), \( \zeta = 1, 2, \ldots, n \), eigenvalues in the ith output is determined from:

\[ \Psi_i = \sum_{j=1}^{n} \phi_{ij} \]  

(15)

Based on the above analysis, the following algorithm is used for the retention of the most dominant modes of the system:

**Algorithm**

1. From eqn. 12, calculate the measure of participation of each eigenvalue in each output, i.e. \( o_{ij} \) \( i = 1, 2, \ldots, m \); \( k = 1, 2, \ldots, n \). This gives \( m \times n \) table of eigenvalue participation measure matrix.

2. From eqn. 13, determine the dominance of each eigenvalue in all the outputs, i.e. \( o_j \) \( j = 1, 2, \ldots, n \).

3. Use the result of step 2 above to sort the eigenvalues in order of dominance, ranging from the most dominant to the least dominant. Let these be denoted as \( \lambda_1, \lambda_2, \ldots, \lambda_n \) so that \( \lambda_1 \) is the most dominant and \( \lambda_n \) is the least dominant.

4. From eqn. 14, calculate the relative participation of each eigenvalue in each output variable, i.e. \( \phi_{ij} \) \( i = 1, 2, \ldots, m \); \( j = 1, 2, \ldots, n \).

5. From eqn. 15, calculate the combined relative participation of \( \zeta \) eigenvalues in each output, starting from \( \zeta = 1 \), and finishing with \( \zeta = n \).

6. Test the condition: \( \sum_{j=1}^{n} \phi_{ij} \geq \Psi \).

When this last condition (6) is satisfied, the first \( \zeta \) eigenvalues are the dominant ones. The rest are nondominant. The value \( \Psi \) is chosen, arbitrarily, between 80 and 100% according to the required degree of approximation. A value of 90% and above would result in a reasonably accurate approximation of the original system. It should be noted that the higher the value of \( \Psi \), the higher the order of the reduced model. With \( \Psi = 100\% \), the reduced-order model is the same as the original system.

**5 Model reduction**

Model reduction techniques that have been proposed in the literature over the past two decades may be categorised as (i) those which retain the most dominant eigenvalues or the most important states of the system [13–15], and (ii) those which derive optimal approximation of the system [16–18]. In the former category, the physical meaning of the state is preserved, but in the latter it is lost.

Let us assume that the identification of the dominant (or slow) and nondominant (or fast) modes of the system of eqns. 9 has been carried out, according to the algorithm outlined in Section 4. Let these be grouped into \( q_1 \) dominant and \( q_2 \) nondominant modes, \( q_1 + q_2 = n \), so that eqns. 9 are rewritten in the following way:

\[
\begin{bmatrix}
\tilde{z}_f(t) \\
\tilde{y}_f(t)
\end{bmatrix} =
\begin{bmatrix}
\Lambda_1 & 0 \\
0 & \Lambda_2
\end{bmatrix}
\begin{bmatrix}
\tilde{z}_f(t) \\
\tilde{y}_f(t)
\end{bmatrix} +
\begin{bmatrix}
\Gamma_1 \\
\Gamma_2
\end{bmatrix}r(t)
\]  

(16)

where \( \Lambda_1 \) is a \((q_1 \times q_1)\)-dimensional diagonal matrix containing \( q_1 \) slow (dominant) modes of the system and \( \Lambda_2 \) is a \((q_2 \times q_2)\)-dimensional diagonal matrix containing the fast (nondominant) modes of the system. Accordingly, the transformation equation (eqn. 8) may be partitioned in the following way:

\[
\begin{bmatrix}
x_f(t) \\
x_s(t)
\end{bmatrix} =
\begin{bmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{bmatrix}
\begin{bmatrix}
\tilde{z}_f(t) \\
\tilde{y}_f(t)
\end{bmatrix}
\]  

(17)

where \( M_{11} \) and \( M_{22} \) for \( j = 1, 2, \ldots, N \) are \((n_1 \times q_1)\)- and \((n_2 \times q_2)\)-dimensional submatrices of \( M \), respectively.

From eqns. 16 and 17, the state of each subsystem, \( x_f(t) \), may be represented by:

\[
x_f(t) = M_{1j}z_f(t) + M_{12}z_s(t) \quad j = 1, 2, \ldots, N
\]

(18)

where \( z_f \) and \( z_s \) are defined, from eqn. 16, by the following dynamical equations:

\[
\tilde{z}_f(t) = \Lambda_1\tilde{z}_f(t) + \Gamma_1r(t)
\]

(19)

with \( \tilde{z}_f(0) = \tilde{z}_{f0} \), and

\[
\tilde{z}_s(t) = \Lambda_2\tilde{z}_s(t) + \Gamma_2r(t)
\]

(20)

with \( \tilde{z}_s(0) = \tilde{z}_{s0} \).
Eqns. 18–20 can be viewed as models of the state of each subsystem. These are linear combinations of the dominant and nondominant modes of the global system and form the basis for the design of local compensation schemes. The development of these schemes is given next.

6 Feedforward compensation scheme

In this Section, three feedforward compensation schemes will be developed. The function of each of these schemes is to generate local corrective control signals equivalent to those generated from remote state feedback, as described by eqn. 7. The structure of each compensation scheme depends on how the nondominant part of the system is treated.

6.1 Decentralised control scheme 1

Since the contribution of the fast modes to the system dynamics is only important at the beginning of the response, early model reduction methods [13] ignored the fast modes altogether, i.e. \( z_j = 0 \). Applying this to eqn. 18 gives the following approximate model of the state of the \( j \)th subsystem:

\[
x_j(t) \approx M_{j1} z_j(t) + M_{j2} E z_j(t) \quad j = 1, 2, \ldots, N
\]

(21)

Substituting this into eqn. 7 yields

\[
u_{j1}(t) = Y_j z_j(t)
\]

(22a)

where

\[
Y_j = \sum_{k=1}^{N} F_{jk} M_{j1}
\]

(22b)

Eqns. 20 and 22 describe a dynamic compensation scheme, the output of which is equivalent to the local corrective signal described by eqn. 7.

6.2 Decentralised control scheme 2

A modified method of model reduction [19] is based on the assumption that the step response of the nondominant part of the system is instantaneous. According to eqn. 19 this gives

\[
z_j(t) = \gamma_j r(t)
\]

(23)

From eqns. 23 and 18, we obtain the following approximate models for the state of the \( j \)th subsystem:

\[
x_j(t) \approx M_{j1} z_j(t) + M_{j2} \gamma_j r(t) \quad j = 1, 2, \ldots, n
\]

(24)

Substituting this model approximation into eqn. 7 gives

\[
u_{j1}(t) = Y_j z_j(t) + Q_j r(t)
\]

(25a)

where

\[
Y_j = \sum_{k=1}^{N} F_{jk} M_{j1}
\]

(25b)

and

\[
Q_j = -\sum_{k=1}^{N} F_{jk} M_{j2} \gamma_j
\]

(25c)

Eqns. 20 and 25 represent a compensation scheme, where the corrective control signal is generated locally by a dynamic and static compensators.

6.3 Decentralised control scheme 3

The nondominant dynamics may be approximated by an optimal linear combination of the dominant modes [14], i.e. \( z_j(t) = E z_j(t) \), where \( \hat{z}_j \) is an estimate of \( z_j \). In view of this approximation and from eqn. 18, we obtain

\[
x_j(t) \approx (M_{j1} + M_{j2} E z_j(t)) \quad j = 1, 2, \ldots, N
\]

(26)

Substituting this in eqn. 7 gives

\[
u_{j1}(t) = \gamma_j z_j(t)
\]

(27a)

where

\[
\gamma_j = \sum_{j=1}^{N} F_{jk} M_{j1} + M_{j2} E
\]

(27b)

Eqns. 20 and 27 represent a compensation scheme that is, basically, similar to that of scheme 1. The difference is in the structure of the output eqn. 27b, where an additional term corresponding to the nondominant modes has been included.

7 Decentralised control

In this Section three decentralised control structures are proposed. The implementation issue concerning the most effective structure for a particular interconnected system is discussed in terms easily determined by quantitative measures.

7.1 Decentralised control structure 1

Let us, for example, consider the decentralised controller described in scheme 2 (Section 6.2) and examine its structure and related implementation issues. According to this scheme, the control input to each subsystem is given by

\[
u(t) = u_{i1}(t) + u_{i2}(t)
\]

(28)

where \( u_{i1} \) is the local state feedback control component, and \( u_{i2} \) is the corrective control component. The local state feedback component is given by eqn. 6 as

\[
u_{i1}(t) = F_{ii} x_i(t)
\]

The corrective control component is composed of two parts, generated from dynamic and static compensators, i.e.

\[
u_{i2}(t) = u_{i2}^d(t) + u_{i2}^s(t)
\]

(29)

The dynamic compensator part is obtained from

\[
u_{i2}^d = Y_j z_j(t)
\]

(30a)

\[
\gamma_j = \gamma_j r_j(t) + \sum_{j=1}^{N} \gamma_j r_j(t)
\]

(30b)

where \( \gamma_j \) is the \( j \)th column of the reference matrix \( \Gamma \). The static compensator part is obtained from

\[
u_{i2}^s = \sum_{j=1}^{N} \theta_{i,j} r_j(t)
\]

(31)

where \( \theta_{i,j} \) is the \( j \)th column of the static compensator \( Q_i \). Obviously, to generate the required corrective control signal, all the reference inputs to the global systems, i.e. \( r_j (j = 1, 2, \ldots, N) \), must be known \textit{a priori}. In most cases these are known and therefore this requirement is conveniently met. In cases such as these, the decentralisation of the control task is achieved as presented above without further consideration. The same argument applies equally to schemes 1 and 3.
This may imply that different parts of the interconnection following local dynamic and static compensators are most appropriate to a given part of the interconnection. As a result, the following analysis highlights this important practical aspect of the design process and its implication with regard to the selection of the most practical and efficient structure for local decentralised controllers.

Owing to the physical characteristics of interconnected dynamical systems, the strongest reaction to a disturbance in any part of the interconnection takes place in that part. The next strongest reaction comes from that part of the system that has the strongest interconnection to the disturbed part, and so on. Therefore different parts of the interconnection react differently to a given disturbance, depending on how strongly coupled these parts are to the disturbed one. To quantify the strength of the interaction among the subsystems of a global system, the following analysis is used:

As a diagonal block in the global system matrix \( A \) represents the dynamics of the corresponding subsystem, and the off diagonal-blocks represent the strength of the interactions \( SI \) between the subsystem and the rest of the interconnection, the \( SI \) may be quantified in terms of the relative norm of the off-diagonal matrices with respect to the diagonal one [20]. For example, the strength of the interaction between subsystem \( i \) \((i = 1, 2, \ldots, N)\) and subsystems \( j \) \((j = 1, 2, \ldots, N)\) and subsystems \( k \) \((k = 1, 2, \ldots, N)\) may be evaluated as

\[
SI_{ij} = \frac{|A_{ij}|}{|A_{ii}|}, \quad i, j = 1, 2, \ldots, N
\] (32a)

where \( |\cdot| \) denote the norm of a matrix, which may be taken as either the largest singular value of the matrix or the square root of the sum of the squared elements of the matrix.

In the case where two subsystems, say \( i \) and \( j \) are not directly coupled to each other, but are indirectly coupled through subsystem \( k \), then the \( SI \) between them may be determined by multiplying the strength of interactions between subsystems \( i \) and \( k \) and subsystems \( k \) and \( j \); i.e.

\[
SI_{ij} = SI_{ik} \times SI_{kj}
\] (32b)

The above analysis forms the basis for the decentralised control structure outlined below. It permits the incorporation of the system behaviour into the controller design. Based on this analysis the controller may turn out to comprise a number of subcontrollers each one of which is most appropriate to a given part of the interconnection. This may imply that different parts of the interconnection have different controller structures. As a result, the following local dynamic and static compensators are obtained:

\[
b_k(t) = \Lambda_k y_k(t) + \sum_{j=1}^{\zeta} \gamma_{k} r_{j}(t)
\] (33a)

\[
u_k(t) = Y_k y_k(t)
\] (33b)

\[
u_k = \sum_{j=1}^{\zeta} \theta_{k, j} r_{j}(t)
\] (33c)

where \( e \) is a \( q \)-dimensional dummy variable, \( \zeta \) is the number of the most strongly coupled subsystems to the disturbed subsystem, and \( k \) is the index to those coupled subsystems. This means that, for example, subsystem 3 is experiencing the disturbance and has the strongest interconnection with subsystem 2, then \( \zeta = 2 \), and \( k = 1 \) and 2, i.e.

\[
b_3(t) = \Lambda_3 y_3(t) + \gamma_3 r_1(t) + \gamma_3 r_2(t)
\]

\[
u_3 = Y_3 y_3(t)
\]

and

\[
u_3 = \theta_{3, 1} r_1(t) + \theta_{3, 2} r_2(t)
\]

The same analysis applies to the remaining subsystems. In this way the overall system is subdivided into clusters, each comprising a number of strongly interconnected subsystems. Implementation of such a decentralised control scheme requires each subsystem in a cluster to acquire knowledge regarding the reference inputs to the other subsystems within the same cluster only. This is not a prohibitive requirement to satisfy, as usually only the neighbouring subsystems need to communicate their reference inputs with each others. A main advantage of this scheme is the obvious saving in the number of communication channels. The saving comes at the expense of a slight deterioration in the steady-state performance of the overall system, owing to the lack of information at the cluster level. In view of the physical nature of interconnected systems, this performance deterioration is not, in general, significant, and therefore does not warrant further consideration.

### 7.3 Decentralised control structure 3

In cases where the subsystems of an interconnection are weakly coupled no information transfer is necessary. For systems of this nature the local state feedback controllers take a prominent role and are capable of damping out the effects of remote disturbances. This leads to the following simply structured set of decentralised dynamic and static local compensators:

\[
b_i(t) = \Lambda_i y_i(t) + \gamma_i r_i(t)
\] (34a)

\[
u_i = Y_i y_i(t)
\] (34b)

and

\[
u_i = \theta_{i, i} r_i(t)
\] (35)

In this way, each subsystem will have its own compensation network which receives local information only. A scheme of this kind means that all components of the local control system, i.e. local state feedback controller, dynamic compensator, and static compensator will respond when the disturbance is local, otherwise only the local state feedback controllers will respond. Such a control scheme is in line with present day practices, such as in the power industry.

As this structure lacks any form of information transfer facility, it is expected that a degree of steady-state offset will appear in the static performance of the overall system. The magnitude of this offset depends on the strength of the interconnection. However, this should not constitute a major concern, as a gain adjustment strategy
could be easily implemented to eliminate such a static offset.

The completely decentralised control structure, described in this Section, may be implemented in the manner shown in Figs. 1 and 2. Fig. 1 represents schemes 1 and 3, and Fig. 2 represents scheme 2. In either case the corrective control component is generated from dynamic, or dynamic and static, compensators. This is then added to the local state feedback control component to form the required control input to each subsystem.

\[
\begin{align*}
\dot{x}_1(t) &= \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix} x_1(t) \\
&+ \begin{bmatrix} 0 & 0.4 \\ 0 & -0.6 \end{bmatrix} x_2(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1(t) \\
y_1(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x_1(t)
\end{align*}
\]

Subsystem 1:

\[
\begin{align*}
\dot{x}_2(t) &= \begin{bmatrix} 0 & 0 \\ -17 & -8 \end{bmatrix} x_2(t) \\
&+ \begin{bmatrix} 0 & 0 \\ 0.3 & 0 \end{bmatrix} x_3(t) + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u_2(t) \\
y_2(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x_2(t)
\end{align*}
\]

Subsystem 2:

\[
\begin{align*}
\dot{x}_3(t) &= \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x_3(t) \\
&+ \begin{bmatrix} 0 & 0 \end{bmatrix} x_4(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_3(t) \\
y_3(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x_3(t)
\end{align*}
\]

Subsystem 3:

\[
\begin{align*}
\dot{x}_4(t) &= \begin{bmatrix} 0 & 1 \\ -9 & -20 \end{bmatrix} x_4(t) \\
&+ \begin{bmatrix} -0.3 & 0 \end{bmatrix} x_4(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_4(t) \\
y_4(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x_4(t)
\end{align*}
\]

**8 Numerical example**

In this Section an interconnected dynamical system comprising four subsystems, as shown in Fig. 3, will be considered for the application of the proposed decentralised control schemes.

**8.1 System description and characteristics**

The 4-subsystems interconnected dynamical system of Fig. 3, comprises a total of 9 states, 4 inputs, and 4 outputs. The dynamical equations governing each sub-

![Fig. 1 Schematic diagram of decentralised control, schemes 1 and 3](image1)

![Fig. 2 Schematic diagram of decentralised control, scheme 2](image2)

![Fig. 3 Interconnected dynamical system comprising four subsystems](image3)
The relative strength of the interactions among the four subsystems is shown in Table 1. Each row in this Table represents the percentage interaction of each of the subsystems with respect to the corresponding subsystem. For example the value 20.71 in the second row represents the strength of the interaction of subsystem 3 with subsystem 2, and so on.

**Table 1: Strength of interaction among the subsystems**

<table>
<thead>
<tr>
<th>Subsystem 1 (%)SI</th>
<th>Subsystem 2 (%)SI</th>
<th>Subsystem 3 (%)SI</th>
<th>Subsystem 4 (%)SI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subsystem 1</td>
<td>100</td>
<td>4.26</td>
<td>0.90</td>
</tr>
<tr>
<td>Subsystem 2</td>
<td>18.43</td>
<td>100</td>
<td>20.71</td>
</tr>
<tr>
<td>Subsystem 3</td>
<td>0.47</td>
<td>25.73</td>
<td>100</td>
</tr>
<tr>
<td>Subsystem 4</td>
<td>0.28</td>
<td>0.54</td>
<td>0.32</td>
</tr>
</tbody>
</table>

From Table 1 it is easy to conclude that the most interaction in the given interconnection takes place with subsystem 2. It also shows that although subsystems 1, 3 and 4 are directly connected to subsystem 2, only subsystems 1 and 3 provide measurable, though weak, interactions with subsystem 2. The rest of the interactions are all almost negligible.

The analysis presented above may be verified on examining the open-loop output responses of the four subsystems to a unit step increase in the reference input $r_1$. These responses are shown in Figs. 4–7, from which it is clear that, among the rest of the subsystems, the most affected by the disturbance is subsystem 2. This is a result which can be easily arrived at by looking at row 1 of Table 1. Examination of this row shows that subsystem 2 has the highest strength of interaction with subsystem 1.

Figs. 4–7 also show that the open-loop system performance is not satisfactory, as some oscillations are exhibited. Therefore an appropriate controller must be designed for it in order to improve its transient response.

8.2 Design of optimal controller

In the following account, a state feedback optimal controller [21] will be designed for the global open-loop system so that, on its application, the oscillations are damped out and a satisfactory system performance is obtained. The optimal control problem may be stated as that of finding the control input $u(t)$ which, subject to the constraints given by the dynamical system equations, minimises the following cost function:

$$J = \int_0^\infty [x^T(t)Sx(t) + u^T(t)Ru(t)] \, dt$$

where $S$ and $R$ are the state and control weighting matrices. The solution to this is given by $u(t) = Fx(t)$, where $F$ is the state feedback optimal control matrix. This is calculated from $F = R^{-1}B^TP_u$, where $P_u$ is the steady-state solution to the matrix Riccati equation [21]. If the state and control weighting matrices are chosen so that

$$S = \text{diagonal}(0, 2, 0, 2, 0, 2, 0, 2)$$

$$R = \text{diagonal}(1, 4, 2, 1)$$
then the following global optimal state feedback control matrix is obtained:

\[
F = \begin{bmatrix}
0.1753 & 0.5061 & 0.1220 & -0.2748 & -0.0575 & -0.1003 & -0.0024 & -0.1095 & -0.0057 \\
-0.0318 & -0.0144 & -0.0173 & 0.1299 & 0.0734 & 0.0184 & -0.0279 & 0.0370 & 0.0019 \\
0.1089 & -0.0012 & -0.6403 & -0.5241 & -0.0557 & 0.0008 & 0.3749 & -0.0814 & -0.0036 \\
-0.0225 & -0.0057 & 0.0123 & 0.0481 & 0.0074 & 0.0063 & -0.0072 & 0.0111 & 0.0050
\end{bmatrix}
\]

Application of this controller to the global system results in the closed-loop control system having the set of eigenvalues given by \( \lambda_1 = -19.5906, \lambda_2 = -3.9608, \lambda_{3,4} = -0.8982 \pm 2.0658, \lambda_5 = -1.0306 \pm 1.6348, \lambda_6 = -0.5152, \lambda_7 = -0.5791 \) and \( \lambda_8 = -1.5016 \).

Figs. 4–7 also show the closed-loop system output responses, overlaid on their corresponding open-loop responses, to a unit step change in the reference input \( r_1 \). It is clear that a degree of improvement in the system performance has been achieved by the global optimal controller.

8.3 Isolation of the dominant modes
Application of the modes identification algorithm, outlined in Section 4, yields the combined controllability and observability measures shown in Table 2.

From Table 2, it is clear that the most dominant modes of the global closed-loop system are \( \lambda_1 = -0.5791, \lambda_2 = -0.5152 \) and \( \lambda_3 = -1.5016 \). Thus only these three eigenvalues will be retained, as they represent the dominant dynamics of the system. The rest will, therefore, be assumed to be nondominant. As a result, a 3rd-order dynamic compensator is obtained, as follows:

\[
\dot{x}(t) = \begin{bmatrix}
-0.5791 & 0 & 0 \\
0 & -0.5152 & 0 \\
0 & 0 & -1.5016
\end{bmatrix} x(t) + \begin{bmatrix}
-0.1520 \\
0.0879 \\
0.1280
\end{bmatrix} r(t)
\]

\[
u_r(t) = \begin{bmatrix}
-0.0068 \\
0.0009 \\
-0.0837
\end{bmatrix} r(t)
\]

8.4 Decentralised controllers
The decentralised control scheme described in Section 7.3 will be chosen for this system. This choice is based on the analysis made in Section 8.1, where it was established that the interaction among the four subsystems is relatively weak and therefore transferring information among the subsystem is not necessary for the control purpose. Although this choice results in a slight degradation in the static performance of the overall system, the advantages of having a simple, practical and cost-effective scheme outweigh those disadvantages. Accordingly, on using the decentralised controller design procedure described in Section 8.1, and on incorporating structure 3, the following set of completely decentralised controllers are obtained.

**Table 2: Controllability and observability measures**

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>% of ( \Psi_1 ) in ( y_1 )</th>
<th>% of ( \Psi_2 ) in ( y_2 )</th>
<th>% of ( \Psi_3 ) in ( y_3 )</th>
<th>% of ( \Psi_4 ) in ( y_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_1 = -0.5791 )</td>
<td>14.77</td>
<td>33.83</td>
<td>44.05</td>
<td>45.34</td>
</tr>
<tr>
<td>( \lambda_2 = -0.5152 )</td>
<td>22.18</td>
<td>51.17</td>
<td>52.16</td>
<td>95.17</td>
</tr>
<tr>
<td>( \lambda_3 = -1.5016 )</td>
<td>33.8</td>
<td>62.2</td>
<td>86.83</td>
<td>96.8</td>
</tr>
<tr>
<td>( \lambda_4 = -1.0306 \pm 1.6248 )</td>
<td>70.24</td>
<td>78.32</td>
<td>93.97</td>
<td>98.3</td>
</tr>
<tr>
<td>( \lambda_5 = -0.8982 \pm 2.0658 )</td>
<td>96.2</td>
<td>97.36</td>
<td>99.81</td>
<td>99.8</td>
</tr>
<tr>
<td>( \lambda_6 = -3.9608 )</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>( \lambda_7 = -19.5906 )</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

**Subsystem 1:**

\[
\dot{x}_1(t) = \begin{bmatrix}
-0.5791 & 0 & 0 \\
0 & -0.5152 & 0 \\
0 & 0 & -1.5016
\end{bmatrix} x_1(t) + \begin{bmatrix}
-0.1520 \\
0.0879 \\
0.1280
\end{bmatrix} r_1(t)
\]

\[
u_{r1}(t) = \begin{bmatrix}
-0.0068 \\
0.0009 \\
-0.0837
\end{bmatrix} r_1(t)
\]

**Subsystem 2:**

\[
\dot{x}_2(t) = \begin{bmatrix}
-0.5791 & 0 & 0 \\
0 & -0.5152 & 0 \\
0 & 0 & -1.5016
\end{bmatrix} x_2(t) + \begin{bmatrix}
0.0874 \\
-0.0618 \\
-0.0515
\end{bmatrix} r_2(t)
\]

\[
u_{r2}(t) = \begin{bmatrix}
-0.0068 \\
0.0009 \\
-0.0837
\end{bmatrix} r_2(t)
\]

**Subsystem 3:**

\[
\dot{x}_3(t) = \begin{bmatrix}
-0.5791 & 0 & 0 \\
0 & -0.5152 & 0 \\
0 & 0 & -1.5016
\end{bmatrix} x_3(t) + \begin{bmatrix}
1.2669 \\
-0.5501 \\
-1.6619
\end{bmatrix} r_3(t)
\]

\[
u_{r3}(t) = \begin{bmatrix}
-0.0536 \\
-0.0495 \\
-0.0470
\end{bmatrix} r_3(t)
\]

\[
u_{r3}(t) = \begin{bmatrix}
0.9293 \\
0.3749
\end{bmatrix} r_3(t)
\]

\[
u_{r4}(t) = \begin{bmatrix}
-0.0173 \\
0.1299 \\
0.0734
\end{bmatrix} x_4(t)
\]
Subsystem 4:

\[
\mathbf{v}_d(t) = \begin{bmatrix}
-0.5791 & 0 & 0 \\
0 & -0.5152 & 0 \\
\end{bmatrix}
\begin{bmatrix}
v_d(t) \\
0 \\
0 \\
-1.5016
\end{bmatrix}
+ \begin{bmatrix}
0.0174 \\
0.0608 \\
-0.0048
\end{bmatrix} \mathbf{r}_d(t)
\]

\[
u_{d4}(t) = [0.0098 \ 0.0143 \ 0.0098] \mathbf{r}_d(t)
\]

\[
u_{d3}(t) = [0.9998] \mathbf{r}_d(t)
\]

\[
u_{d2}(t) = [0.0111 \ 0.0505] \mathbf{x}_d(t)
\]

9 Simulation results

Extensive simulation studies of the four-subsystem interconnection have been carried out. These involved application of each of the three compensation schemes outlined in Section 6, incorporating a number of information transfer patterns, ranging from that where each subsystem receives information regarding the reference settings of all the remaining subsystems to that where no information is transmitted from any subsystem to another. For space limitation reasons, the simulation results of only one case are presented in this paper (interested readers can obtain copies of the full results directly from the author). The reported case involves scheme 2, with no subsystems receiving or transmitting any information about their reference input settings, i.e.

**Fig. 8** Subsystem 1 output response to a unit step increase in subsystem 1

**Fig. 9** Subsystem 2 output response to a unit step increase in subsystem 1

**Fig. 10** Subsystem 3 output response to a unit step increase in subsystem 1

**Fig. 11** Subsystem 4 output response to a unit step increase in subsystem 1

10 Conclusion

In this paper a new method for the design of decentralised controllers for interconnected dynamical systems is presented. Three decentralised controller schemes have been proposed. In schemes 1 and 3, the subsystem controller comprises a local state feedback controller, and a dynamic feedforward compensator, whereas in scheme 2 an additional static compensator is involved. Furthermore, three structures have also been proposed for the implementation of each controller. The choice of the most appropriate controller scheme and structure to be implemented depends on the dynamical behaviour of the system. The characteristics of each of the three proposed structures can now be briefly discussed.

(i) The first structure, described by eqns. 6 and 28–31, is suitable for all types of interconnected dynamical systems. It is, however, most suited to systems with very strong interactions. The operational requirement for this structure is the knowledge, by each part of the system, of all the reference inputs to the remaining parts. In most
systems this is applicable and therefore no realisation problems arise. Under such a structure, both the state feedback controllers and the compensators are operational all the time. The overall system performance under such a decentralised control structure is very close to that under the global one.

(ii) The second structure, described by eqns. 6, 32 and 33, is most suited to interconnections where the interaction is strong only among groups of subsystems, owing to their proximity and ties to each other. In this structure it is necessary for these groups to share information about the nature of the reference settings only within themselves. This is not an unreasonable requirement because transferring some information along relatively short distances is not so prohibitive a task. This implies that, in this structure, the overall system is subdivided into groups, each of which operates under the control structure outlined in (i) above. The overall system performance under such a decentralised control structure is also close to that of the global one, as only a minor static performance degradation could result.

(iii) The third structure, described by eqns. 6, 34 and 35, is most suited to systems with relatively weak interactions among their various parts. A structure of this description requires no transfer of information of any kind. In this case, the state feedback controller is operational all the time, and the dynamic and the static compensators are operational only when local disturbances occur, i.e. the major control activity takes place in that part of the interconnection that experiences the disturbance. The overall system static performance under this controller is expected to show some degradation, but this should not be of a major nature.

Thus, unlike other existing methods, the proposed design method is systematic, simple, and adaptable to systems with varying degrees of interconnections and interactions. The pattern of information transfer can be changed from that where no information transfer network is in place, as is the case with completely decentralised control schemes, to that where some or all parts of the global system inter-change information, as is the case with partially decentralised control schemes.

The simulation results presented in Section 9 illustrate the suitability of the proposed design methods for interconnected dynamical systems. The performance is very comparable to the global optimal one, despite the fact that no information is transmitted by or received from any part of the system.

11 References

12 ALDEEN, M.: ‘Decentralized control of power systems via interaction modelling’. Proceedings of IEAust Control 90, August 1990, Gold Coast, Queensland, Australia