Observer-based fault detection and identification scheme for power systems

M. Aldeen and F. Crusca

Abstract: A scheme suitable for the detection and identification of faults in power systems is presented. Two notable contributions are made: a re-modelling of faulty components of power systems that is applicable to both normal and faulty conditions, and a fault detection scheme for power systems. The faults are modelled as unknown inputs, decoupled from the state and output measurements through coordinate transformations, and then estimated through the use of observer theory. The proposed scheme is applied to a power system consisting of a synchronous generator, an exciter, a turbine and speed-governing system, and a network of lines and loads. The case where faults occur on the transmission network is considered. It is shown that the proposed fault detection procedure allows for the real-time identification of the occurrence of the faults and determines their exact locations. Results of detailed simulation studies involving disturbances and faults occurring in linear and nonlinear models of the power system are presented.

1 Introduction

Power systems are no different to any other large-scale interconnected system in that they are susceptible to various forms of faults which could occur in any of the components that make up the system. For example, faults can occur in generating units, transformers, the transmission network and/or loads. Faults that take place in any of these components can cause significant disruption of supply and in some cases may have the undesirable effect of destabilising the entire system, and in extreme cases leading to brownouts and blackouts. It is therefore important to be able to detect and isolate such faults as quickly as possible, in order that remedial action can be taken.

In this work the focus is shifted from studying the effect of external disturbances on the power system in terms of load angle swings and effects on voltage profiles and tie-line power flows, to the detection and identification of the fault itself. Although this approach offers considerable benefits per se, especially when incorporated with protection systems, its full potential can be realised when it is incorporated into a wider active fault tolerant control scheme, which is currently the subject of ongoing research.

Over the past decade considerable advances have been made in the area of fault detection and isolation [1, 2] particularly in the areas of aerospace, nuclear reactors, and process control systems. However, a literature survey reveals that the application of analytical model-based fault detection techniques to power systems is presently at its infancy, although a few applications of neural networks to fault detection in power systems have been reported [3, 4]. The fact that conventional dynamic models of power

List of symbols

\( \psi \) flux linkage
\( x \) state vector
\( y \) output vector
\( f \) fault parameters
\( A, B, C, D, E, G, W \) constant parameter matrices
\( \delta \) rotor shaft angle
\( \Omega \) rotor shaft speed
\( \Omega_b \) speed base quantity
\( v, V \) machine and network voltage
\( i, I \) machine and network current
\( \| \cdot \| \) magnitude of a vector
\( E_{FD} \) field winding excitation voltage
\( H \) synchronous machine inertia constant
\( k_d \) rotor damping coefficient
\( T_m, P_m \) mechanical torque and power
\( M \) machine-network coordinate transformation constants

Superscript and subscripts

\( \circ \) superscript refers to nominal value
\( \psi \) subscript refers to the quadrature axis
\( g, q \) subscript refers to damper windings on the quadrature axis
\( d \) subscript refers to the direct axis
\( D \) subscript refers to damper winding on the direct axis
\( F \) subscript refers to field winding

-\( qd \) subscript refers to a 2 vector; \( [q, d]^T \)
-\( SE \) subscript refers to swing equation
-\( e, \Omega \) subscript refers to exciter
-\( N \) subscript refers to network
-\( G \) subscript refers to governor
-\( \text{REF} \) subscript refers to terminal voltage
-\( \hat{\cdot} \) estimate of
systems as reported in the literature [5, 6] are not directly amenable to existing fault detection techniques may be the main reason behind the lack of any major contributions in this area.

A prerequisite for fault detection in power systems is the derivation of an appropriate model. The approach to the modelling of power systems for fault detection purposes is considerably different from that adopted in conventional stability studies. The difference stems from the need for the satisfaction of additional requirements concerning the occurrence and detection of faults as outlined below:

(a) each component of the system should be modelled with fault conditions effectuated internally such that both normal and fault conditions can be simulated uniformly and systematically;
(b) the ability to isolate and remove the fault from the network;
(c) the ability to differentiate between fault signals and normal signals.

These requirements must be satisfied without detracting from the integrity and complexity of the power system model, and in this work they are met as follows. Requirements (a) and (b) can be achieved by introducing different switching mechanisms in the appropriate places of the component being modelled, where as requirement (c) is achieved by treating fault conditions as external unknown signals.

The need for the re-modelling of the power system as outlined above stems from the fact that existing fault detection techniques [7–12] are based on the principle of the fault being modelled as an external unknown input that enters affinely into the system, and partially from the requirement that the model should be capable of exhibiting the power system behaviour under both normal and abnormal operating conditions.

Conventional fault detection algorithms are typically based on observer theory in which a dynamic model of the plant is generated, and from this an observer is constructed. The residual error between the observer and plant states then serves as an indicator for fault detection. For example, unknown input observer theory is used in [8, 13, 14], and H-infinity is used in [15, 16]. The residual generation algorithms generally required for these approaches, are not required in more recent approaches such as [17], in which unknown input observer theory is directly employed, and [9], in which sliding mode observer theory is used.

In this study we introduce an alternative approach to fault detection and identification in power systems. Firstly, we present models for those components of power systems that may experience faults. The models are developed so as to provide accurate representations of the components under normal and fault conditions without the need to make any changes. Secondly, we present a scheme for the detection and isolation of faults. Unlike existing theory, the scheme is capable of detecting multiple faults and identifying their exact locations in real-time and on-line, as demonstrated in the design and simulation example of Section 4. Therefore, the approach to fault detection presented in this work is twofold, and involves firstly the re-modelling of the power system to decouple the fault from the system as outlined above, and secondly, the application of a suitable observer theory for fault detection in power systems.

The abovementioned technique can be used to identify faults in all of the components of the power system. However, for the sake of brevity and due to space limitations, the emphasis in this study is restricted to the specific case of faults occurring in the transmission network. For this purpose, an application example of a power system comprising a generating unit connected to an infinite bus-bar via a double-circuit transmission line and local load is presented.

2 Power system model

The studied power system is a generating unit connected to an infinite bus-bar through transmission lines. The generating unit comprises a synchronous generator, an exciter [5], and a steam-turbine speed governor [18]. A detailed analysis of the modelling of each these components is given in [19]. Although standard models of power systems components are available in the literature, (see [5] and [6], for example), models of power system components incorporating fault scenarios are not currently available.

Our focus is therefore shifted to the modelling and incorporation of faults in a power system. For this purpose we adopt the eighth-order generator model, third-order IEEE ST1 exciter model, and third-order IEEE governor model derived in [17] and re-model the transmission system incorporating fault conditions. In the derivation of the transmission system, we allow for faults to take place anywhere in the system. We claim that this is the first time such an approach to modelling, simulation, and design of fault detection schemes, which takes into account various fault scenarios, has been reported.

2.1 Model of generating unit

A set of linearised equations for an eighth-order synchronous machine model with one damper winding on the q-axis and two damper windings on the d-axis, an IEEE ST1 exciter and an IEEE speed governing system are derived in [17]. For ease of reference, the state space models for each of these components is provided below.

2.1.1 Synchronous machine model:

\[ \Delta \dot{x}_M = A_M \Delta x_M + B_M \Delta v_{qd} + B_{FD} \Delta E_{FD} + B_{m} \Delta P_m \]  

(1)

where \( x_M = [\psi_d \; \psi_q \; \psi_d \; \psi_q \; \psi_d \; \psi_q \; \psi_d \; \psi_q \]  and \( v_{qd} = [v_q \; v_d \]  .

2.1.2 Exciter model:

\[ \Delta \dot{x}_e = A_e \Delta x_e + B_e \Delta u_e \]  

(2)

\[ \Delta E_{FD} = C_{FD} \Delta x_e \]  

(3)

where \( x_e = [V_1 \; V_3 \; E_{FD}]^T \) are the exciter states (as defined in Fig. 2.14 of [20]), and \( u_e = V_{REF} \).

2.1.3 Speed governor model:

\[ \Delta \dot{x}_G = A_G \Delta x_G + B_G \Delta u_G \]  

(4)

\[ \Delta P_m = C_G \Delta x_G \]  

(5)

where \( x_G = [P_m \; P_{GV} \; P_{SR}]^T \) and \( u_G = P_r \).

2.2 State space model of generating unit

Combining the synchronous generator, exciter and governing models, leads to a complete state space model for the generating unit. This model can be expressed as:

\[ \Delta \dot{x}_{gu} = A_{gu} \Delta x_{gu} + B_{gu} \Delta u_{gu} + B_{gu} \Delta v_{qd} \]  

(6)
where \( x_{gu} = [x_M \ x_e \ x_G]^T \), \( u_{gu} = [V_{REF} \ P_i]^T \), and
\[
A_{gu} = \begin{bmatrix} A_M & B_{FD} & C_{FD} & B_m & C_g \\ 0 & A_e & 0 & 0 & A_G \\ 0 & 0 & A_G & 0 & 0 \end{bmatrix}, \quad B_{gu} = \begin{bmatrix} 0 & 0 \\ B_e & 0 \\ 0 & B_G \end{bmatrix}.
\]

\[
B_{gu,s} = \begin{bmatrix} B_s \\ 0 \\ 0 \end{bmatrix}.
\]

### 2.3 Machine-network reference frame transformation

In order to interface the generating unit with the transmission system, the following linearised network-machine transformations for both the voltage and current, based on Park’s coordinate transformation [21], are required:

\[
\Delta v_{gd} = \Gamma_N \Delta V_N + \Gamma_d \Delta x_{SE} \tag{7}
\]

\[
\Delta I_N = \Phi_{gd} \Delta I_{gd} + \Phi_d \Delta x_{SE} \tag{8}
\]

where \( x_{SE} = [\delta \ \dot{\delta}]^T \), \( \Gamma_N = M_e T (\delta_0) \), \( \Gamma_d = [-M_e T_d (\delta_0)]^T \), \( \Phi_{gd} = M_e T (\delta_0)^T \), \( \Phi_d = [-M_e T_d (\delta_0)]^T \) \( \phi_{gd} \triangleq [v_q \ v_d]^T \), \( \Phi_{gd} \triangleq [I_\text{gq} \ I_\text{gd}]^T \), \( \phi_d \triangleq [i_q \ i_d]^T \). The Park’s transformation matrix and its derivative are defined as:

\[
T(d_0) \triangleq \begin{bmatrix} \cos d_0 & \sin d_0 \\ -\sin d_0 & \cos d_0 \end{bmatrix}, \quad T_d(d_0) \triangleq \begin{bmatrix} \sin d_0 & -\cos d_0 \\ \cos d_0 & \sin d_0 \end{bmatrix}\]

Using these definitions, and after some manipulation, eqns. (7) and (8) may be expressed in terms of the generating unit state vector as:

\[
\Delta v_{gd} = \Gamma_N \Delta V_N + \Gamma_d \Delta x_{gu} \tag{9}
\]

\[
\Delta I_N = \Phi_{gd} \Delta I_{gd} + \Phi_d \Delta x_{gu} \tag{10}
\]

Taking into account that \( i_{gd} = S L^{-1} Q x_{gu} \), where \( S \) is a \( 2 \times 8 \) selection matrix with all elements zero except \( S(1,1) = 1 \) and \( S(2,4) = 1 \), \( L = (L_{gQQ}, \ L_{gFD}) \) with:

\[
L_{gQQ} = \begin{bmatrix} L_q & L_AQ & L_{AQ} & L_{AQ} \\ L_{AQ} & L_AQ & L_{AQ} & L_{AQ} \\ L_{AQ} & L_{AQ} & L_{AQ} & L_{AQ} \end{bmatrix}, \quad L_{gFD} = \begin{bmatrix} L_d & L_{AD} & L_{AD} & L_{AD} \\ L_{AD} & L_d & L_{AD} & L_{AD} \end{bmatrix}
\]

and \( Q \triangleq [1_{1 \times 6} \ 0_{1 \times 8}] \), the generating unit model of equation (6) together with the output equation (10) becomes:

\[
\Delta x_{gu} = (A_{gu} + B_{gu} \Gamma_d C_d) \Delta x_{gu} + B_{gu} \Delta u_{gu} \tag{11}
\]

\[
\Delta I_N = C_{N,gu} \Delta x_{gu} \tag{12}
\]

where \( C_{N,gu} \triangleq \Phi_{gd} SL^{-1} Q + \Phi_d C_d \).

### 2.4 Network model

Here we consider a transmission network comprising two parallel lines connected to an infinite bus and a local line supplying a local load, as shown in Fig. 1.

We study the case where the power system is initially operating under normal conditions and then either or both of lines 1 and 2 may undergo three-phase line-to-ground faults at locations \( \gamma_i, i = \{1,2\} \) from the generator terminals. The local line is assumed to be fault-free. Lines 1 and 2 may therefore be modelled as follows:

\[
I_{N1} = (1 - f_1) Y_1 (V_N - V_B) + \rho_1 Y_1 V_N \tag{13}
\]

\[
I_{N2} = (1 - f_2) Y_2 (V_N - V_B) + \rho_2 Y_2 V_N \tag{14}
\]

where \( Y_1 = Z_1^{-1}, \ Y_2 = Z_2^{-1} \), denote the admittance of lines 1 and 2. Here, \( f_i \) and \( \rho_i \) represent the fault signals, where \( f_i = 0 \) signifies no fault condition and \( f_i = 1 \) signifies a fault condition on line \( i \). The variables \( \gamma_i, i = \{1,2\} \) represent the location of the faults along the lines, measured from the generator terminals. For notation convenience, we further define \( \rho_1 = \hat{f}_1 / \gamma_1 \). The local load (assumed to be fault-free) is modelled as:

\[
I_{N3} = Y_L V_N \tag{15}
\]

Using these definitions, (13), (14) and (15) are linearised as follows:

\[
\Delta I_{N1} = (1 - f_1^2 + \rho_1^2) Y_1 \Delta V_N + (V_B - V_N) Y_1 \Delta f_1 + Y_1 V_N \Delta \rho_1 \tag{16}
\]

\[
\Delta I_{N2} = (1 - f_2^2 + \rho_2^2) Y_2 \Delta V_N + (V_B - V_N) Y_2 \Delta f_2 + Y_2 V_N \Delta \rho_2 \tag{17}
\]

\[
\Delta I_{N3} = Y_L \Delta V_N \tag{18}
\]

Thus, the total terminal current is:

\[
\Delta I_N = \Delta I_{N1} + \Delta I_{N2} + \Delta I_{N3}
\]

\[
\Delta I_N = \{(1 - f_1^2 + \rho_1^2) Y_1 (1 - f_2^2 + \rho_2^2) Y_2 + Y_1 \} \Delta V_N + (V_B - V_N) Y_1 \Delta f_1 + (V_B - V_N) Y_2 \Delta f_2 + Y_1 V_N \Delta \rho_1 + Y_2 V_N \Delta \rho_2 \tag{19}
\]

Let \( f = [f_1 \ \rho_1 \ f_2 \ \rho_2]^T \), then (19) may be rewritten as:

\[
\Delta I_N = A_f \Delta f + A_{fp} \Delta f \tag{20}
\]

where

\[
A_f = (1 - f_1^2 + \rho_1^2) Y_1 + (1 - f_2^2 + \rho_2^2) Y_2 + Y_1, \quad \rho_1 = (V_B - V_N) Y_1, \quad A_{fp} = A_f \rho_1, A_{fp} = A_{fp} \rho_2, \quad \rho_2 = Y_2 V_N.
\]

Equation (19) may be rearranged as follows:

\[
\Delta V_N = Z_N \Delta I_N + Z_f \Delta f \tag{21}
\]

where \( Z_N = A_f^{-1} \) and \( Z_f = -A_f^{-1} A_{fp} \).

### 2.5 State space model of power system

Combining the equations for the generating unit model, (11) and (12) with the network model of (21), and substituting \( x \) for \( x_{gu} \) and \( u \) for \( u_{gu} \) leads to the following...
state-equation model for the power system:

$$\Delta \dot{x} = A \Delta x + B \Delta u + \Delta \dot{f}$$

where $A = A_{\text{em}} + B_{\text{em}} \frac{\Gamma_0 C_0}{\Gamma_0 Z_N} Z_N C_{\text{gu}}, B = B_{\text{gu}},$ and $E = B_{\text{gu}} \frac{\Gamma_0}{\Gamma_0 Z_N} Z_f.$

Next we derive linearised equations for the outputs of interest. These are defined above as the load angle, network model and machine-network transformation, and signal to the excitation system and also as an input to the generator output active power is derived from linearisation of the swing equation:

$$\Delta |V_N| = K_{\text{FN}} \Delta V_N$$

where $K_{\text{FN}} \triangleq \frac{(V_N^0)^T}{|V_N^0|^2}.$ Substituting (21) into (23) leads to:

$$\Delta |V_N| = K_{\text{FN}} Z_N \Delta I_N + K_{\text{FN}} Z_f \Delta f$$

However, from (8) and, after some straightforward manipulation, $\Delta |V_N|$ can be represented as:

$$\Delta |V_N| = K_{\text{FN}} Z_N C_{\text{gu}} \Delta x + K_{\text{FN}} Z_f \Delta f$$

2.5.2 Terminal power: An expression for the machine output active power is derived from linearisation of $P_N = V_N^0 I_N$ as:

$$\Delta P_N = (I_N^0)^T \Delta V_N + (V_N^0)^T \Delta I_N$$

which together with (12) and (21) leads to:

$$\Delta P_N = C_{\text{gu}} \Delta x + (I_N^0)^T Z_f \Delta f$$

where $C_{\text{gu}} \triangleq \frac{((V_N^0)^T + (I_N^0)^T Z_N) C_{\text{gu}}}{|V_N^0|^2}.$

2.5.3 Acceleration: The acceleration output is obtained from the swing equation:

$$\ddot{\Delta} = \frac{\alpha B}{2H} (T_m - T_e - \frac{k_d}{\alpha B} \Delta \dot{\delta})$$

where $T_e = M_T M_i T_{eq}$, and $T_{eq} = p d i_q - p q i_d.$ From the current-flux relations for the $q$- and $d$-axes, we obtain:

$$\dot{i}_q = L_q \dot{\psi}_q + L_{AQ} \dot{\psi}_G + L_{AQ} \dot{\psi}_Q$$

$$\dot{i}_d = L_d \dot{\psi}_d + L_{AD} \dot{\psi}_F + L_{AD} \dot{\psi}_D$$

where

$$\begin{bmatrix} L_q & L_{AQ} & L_{AQ} \\ L_{AQ} & L_G & L_{AQ} \\ L_{AQ} & L_{AQ} & L_Q \end{bmatrix} \begin{bmatrix} \dot{\psi}_q \\ \dot{\psi}_G \\ \dot{\psi}_Q \end{bmatrix} = \begin{bmatrix} L_q & L_{AQ} & L_{AQ} \\ L_{AQ} & L_G & L_{AQ} \\ L_{AQ} & L_{AQ} & L_Q \end{bmatrix}^{-1} \begin{bmatrix} L_q & L_{AQ} & L_{AQ} \\ L_{AQ} & L_G & L_{AQ} \\ L_{AQ} & L_{AQ} & L_Q \end{bmatrix} \begin{bmatrix} \dot{\psi}_q \\ \dot{\psi}_G \\ \dot{\psi}_Q \end{bmatrix}$$

Linearising, we obtain:

$$\Delta \dot{\psi}_G = (L_q \dot{\psi}_q - \dot{\psi}_q) \dot{\psi}_q + L_{AQ} \psi_G \dot{\psi}_d + L_{AQ} \psi_G \dot{\psi}_d$$

$$\Delta \dot{\psi}_d = (L_d \psi_d - \dot{\psi}_q) \dot{\psi}_d - L_{AD} \psi_F \dot{\psi}_d - L_{AD} \psi_F \dot{\psi}_d$$

Therefore, for small changes in speed, $(\alpha B + \dot{\delta})/\alpha B \simeq 1,$ for the linearised system it follows that $\Delta T_m = \Delta P_m.$ From (5) it follows that $\Delta T_m = C_G \Delta \dot{\delta},$ and therefore (31) becomes:

$$\Delta \ddot{\delta} = C_{\text{acc}} \Delta \dot{x} + \frac{\alpha B}{2H} \Delta T_m$$

3 Development of unknown input observer-based fault scheme

In this Section we develop an unknown input-observer-based fault detection scheme suitable for multivariable systems, such as power systems. The scheme is based on the idea of decoupling the state and output equations into fault-free and fault-dependent parts. The fault-free part is then used to design an observer that would guarantee estimation of the magnitude and nature of any fault signals. Once this task is achieved, the observer is used to provide estimates of the state vector and output signals. This development is outlined in the following Sections.

3.1 Decoupling the output equation

Using singular value decomposition, matrix $W$ can be expressed as:

$$W = U \Sigma W V^T = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_2 & 0 \\ 0 & \Sigma_2 \end{bmatrix} V^T = U_1 \Sigma_2 V^T$$

where $f_e = f_w.$
where \( U \in \mathbb{R}^{m \times m} \), \( \Sigma_W \in \mathbb{R}^{m \times q_e} \), \( V^T \in \mathbb{R}^{n \times q_e} \), \( U_1 \in \mathbb{R}^{m \times q_e} \), \( U_2 \in \mathbb{R}^{n \times (m-q_e)} \), and \( \Sigma_2 \in \mathbb{R}^{n \times q_e} \). Equation (35) may then be written as:

\[
y = CX + U\Sigma_W V^T f_w
\]  

(37)

Define \( f_w = V^T f_w\), \( f_w \in \mathbb{R}^{q_e} \) and pre-multiply (37) by \( U^T \) to obtain:

\[
U^T y = \begin{bmatrix} U_1^T y_1 \\ U_2^T y_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}
\]  

(38)

Then (37) becomes:

\[
\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} U_1^T \\ U_2^T \end{bmatrix} CX + \begin{bmatrix} \Sigma_2 \\ 0 \end{bmatrix} f_w
\]  

(39)

From (39) we have:

\[
y_1 = U_1^T CX + \Sigma_2 f_w
\]  

(40)

\[
y_2 = U_2^T CX = U_2^T y
\]  

(41)

Thus, the output equation has now been decoupled, as shown in (40) and (41). Note that comparing (38) with (41) reveals that \( y_2 \) is not affected by the output fault signal, \( f_w \).

### 3.2 Decoupling the state equation

Singular value decomposition of matrix \( E \in \mathbb{R}^{m \times q_e} \) gives:

\[
E = QS_E R^T = \begin{bmatrix} Q_1 & Q_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 \\ 0 \end{bmatrix} R^T = Q_1 \Sigma_1 R^T
\]  

(42)

where \( Q \in \mathbb{R}^{m \times m} \), \( \Sigma \in \mathbb{R}^{m \times q_e} \), \( R \in \mathbb{R}^{n \times m} \), \( Q_1 \in \mathbb{R}^{m \times q_e} \), \( Q_2 \in \mathbb{R}^{n \times (m-q_e)} \), and \( \Sigma_1 \in \mathbb{R}^{m \times q_e} \). Define:

\[
f_e = R^T f_w; \quad \tilde{f}_e \in \mathbb{R}^{n \times q_e}
\]

so that

\[
A \triangleq Q^T AQ = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}; \quad Q^T B = B \triangleq \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}
\]  

(44)

then (34) becomes:

\[
\begin{align*}
\dot{x}_1 &= A_{11} x_1 + A_{12} \dot{x}_2 + B_2 u + \Sigma \tilde{f}_e \\
\dot{x}_2 &= A_{21} x_1 + A_{22} \dot{x}_2 + B_2 u
\end{align*}
\]  

(45)

(46)

Use this coordinate transformation

\[
x = Q \hat{x} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}
\]

in (41) to obtain:

\[
\dot{y}_2 = CX \hat{x}
\]

(47)

where \( C \in \mathbb{R}^{(m-q_e) \times m} \) is defined as:

\[
C \triangleq U_2^T C = [C_1 \ C_2]
\]

(48)

with \( C_1 \in \mathbb{R}^{(m-q_e) \times q_e} \), and \( C_2 \in \mathbb{R}^{(m-q_e) \times (n-q_e)} \). Then (47) becomes:

\[
\dot{y}_2 = C_1 x_1 + C_2 \dot{x}_2
\]

(49)

Since rank \( C_1 \) is \( q_e \), an \((m-q_e) \times (m-q_e)\) nonsingular matrix may be constructed from:

\[
N = \begin{bmatrix} C_1^T \\ M \end{bmatrix}
\]

(50)

where \( C_1^T \) is the pseudo-inverse of \( C_1 \), defined as \( C_1^T = (C_1^T C_1)^{-1} C_1^T \), and \( M \in \mathbb{R}^{(m-q_e) \times (m-q_e)} \) is an arbitrarily selected matrix so that \( N \in \mathbb{R}^{(m-q_e) \times (m-q_e)} \)

\[
N = \begin{bmatrix} C_1^T \\ M \end{bmatrix}
\]

(50)

nonsingular. Premultiplying (47) by (50) gives:

\[
[\hat{C}_1^T M]\dot{y}_2 = [C_1^T M] [C_1 \ C_2] x_1
\]

(51)

Since \( \hat{C}_1 \hat{C}_1 = I_{q_e} \), (51) yields:

\[
x_1 = \hat{C}_1^T (y_2 - \hat{C}_2 \dot{x}_2)
\]

(52)

\[
M y_2 = M C \hat{C}_1 \hat{C}_1^T x_1 + M C \hat{C}_2 \dot{x}_2
\]

(53)

Substituting (52) into (46) yields:

\[
\dot{x}_2 = (A_{22} - \bar{A}_{21} C_1 \hat{C}_1^T) \dot{x}_2 + \bar{B}_2 u + \bar{A}_{21} C_1 \hat{C}_1^T y_2
\]

(54)

Using \( \dot{y}_2 = U_2^T y \) in (54) gives:

\[
\dot{x}_2 = \bar{A}_2 \dot{x}_2 + \bar{B}_2 u + \bar{G}_2 y
\]

(55)

where \( \bar{A}_2 = (A_{22} - \bar{A}_{21} C_1 \hat{C}_1^T) \), \( \bar{B}_2 = \bar{B}_2 \) and \( \bar{G}_2 = \bar{A}_{21} C_1 \hat{C}_1^T \). Substituting (55) into (53) gives:

\[
M(I_{m-q_e} - C_1 \hat{C}_1^T) y_2 = M(I_{m-q_e} - C_1 \hat{C}_1^T) C_2 \dot{x}_2
\]

(56)

Define:

\[
\hat{C} \triangleq M(I_{n-q_e} - \hat{C}_1 \hat{C}_1^T) \hat{C}_2;
\]

\[
\hat{H} \triangleq M(I_{n-q_e} - \hat{C}_1 \hat{C}_1^T) U_2^T
\]

Using the definitions in (56) gives:

\[
\ddot{y}_2 = \hat{C} \dot{x}_2
\]

(58)

Define:

\[
\ddot{y}_2 = \hat{H} y = \hat{C} \dot{x}_2
\]

(59)

Thus, we now have a fault-free system described by the state equation, (55), and the output equation, (59), i.e.:

\[
\dot{x}_2 = \bar{A}_2 \dot{x}_2 + \bar{B}_2 u + \bar{G}_2 y
\]

(60a)

\[
\dot{y}_2 = \bar{C} \dot{x}_2
\]

(60b)

where the dimension of the state vector, \( \dot{x}_2 \), is \( n-q_e \).

### 3.3 Observer design

If the pair \( \{A_2, \hat{C}_1\} \) is observable, then the following system can act as an observer for the system described by (60a) and (60b):

\[
\dot{x}_2 = \bar{A}_2 \dot{x}_2 + \bar{B}_2 u + \bar{G}_2 y + L(\ddot{y}_2 - \hat{C} \dot{x}_2)
\]

(61)

where \( L \) is the observer gain matrix and must be found so that the observer matrix \( \bar{A}_2 - L \bar{C}_2 \) is stable.

Once an estimate of \( \dot{x}_2 \) is obtained from the observer described in (61), the estimate of \( \dot{x}_1 \) can be found from (52) as:

\[
\dot{x}_1 = \hat{C}_1^T (U_2^T y - \hat{C}_2 \dot{x}_2)
\]

(62)

Finally, the estimate, \( \hat{x} \), of the original overall state vector, \( x \), is now constructed from (43) as:

\[
\hat{x} = Q \hat{x} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}
\]

(63)

### 3.4 Estimation of fault signals

In this Section, we derive two fault estimation schemes, one for fault signals that appear in the state equation and the other for the fault signals that appear in the output equation.

#### 3.4.1 Estimation of output fault signals

Substituting (62) into (63) gives the following:

\[
\hat{x} = Q \hat{x}_2 + Q_1 y
\]

(64)
where $\bar{Q}_x \triangleq Q \left[ \begin{bmatrix} -C^T & C_2 \\ I & 0 \end{bmatrix} \right]$, $\bar{Q}_y \triangleq Q \left[ \begin{bmatrix} C^T U_2^T \\ 0 \end{bmatrix} \right]$. From (37), and after replacing $x$ by its estimate, $\hat{x}$, we have:

$$\dot{\hat{f}}_w = V\Sigma_2^{-1}U_1^T(y - C\hat{x})$$

(65)

### 3.4.2 Estimation of input fault signals

From (64) we have:

$$\dot{\hat{f}} = \bar{Q}_x\dot{\hat{x}} + \bar{Q}_y\dot{y}$$

(66)

Substituting (61) into (66) gives:

$$\dot{\hat{x}} = \bar{Q}_x[(A - L\bar{C})\hat{x}_2 + \bar{B}_2u + (\bar{G}_2 + \bar{L}\bar{H})\hat{y}] + \bar{Q}_y\dot{y}$$

(67)

From (34) and (67), the estimate of the fault signal $\hat{f}$ may be obtained, after replacing $x$ by its estimate, $\hat{x}$, as:

$$\hat{f} = R\Sigma_1^{-1}Q_1^T(\Psi_1\hat{x}_2 + \Psi_2\hat{y} + \Psi_3\dot{y} + \Psi_4u)$$

(68)

where $\Psi_1 = \bar{Q}_x(A - L\bar{C}) - A\bar{Q}_x$, $\Psi_2 = \bar{Q}_x(\bar{G}_2 + \bar{L}\bar{H}) - A\bar{Q}_y$, $\Psi_3 = \bar{Q}_y$, and $\Psi_4 = \bar{Q}_y\bar{B}_2 - B$.

### 4 Design and simulation results

In this Section we design the unknown input observer described by (61) and proceed to design the fault reconstruction filters described by (65) and (68). Then the resulting fault detection scheme is tested through computer simulation involving five distinct case studies involving linear and nonlinear models of the power system. It will be shown that the designed observer-based fault detection scheme is capable of dealing with all possible fault scenarios and provide exact information about the fault conditions.

#### 4.1 Observer design

The procedure outlined in Section 3 is carried out on the power system model outlined in Section 2. This basically involves the derivation of the fault-free system described by (60a) and (60b) and then the design of the unknown input observer described by (61). The observer design involves the determination of the observer gain matrix $L$ to stabilise $A - L\bar{C}$.

The actual design parameters are not listed here due to space constraints, but may be obtained, together with the power system model and data, by contacting the authors directly. The reason for not providing these design parameters is that they may only make sense if accompanied by detailed models of each of the power system components and their respective data. To do so would require allocation of disproportionate space, which the authors deem not to be necessary.

#### 4.2 Case studies and simulation results

In this Section we report on the performance of the observer-based fault detection scheme outlined in Section 3 and designed in Section 4. The performance of the scheme is tested through the Simulink™ program for the following five case studies are considered.

#### 4.2.1 Case study 1

This initial study tests the performance of the designed observer. In doing so, we arbitrarily set all initial state values for the power system to the value 0.5, while maintaining all initial observer states at zero value. Then at time $t = 10s$, we apply a 5% step increase in $V_{\text{REF}}$. The response of a sample of the output variables, including responses of the load angle, acceleration and output power, are shown in Fig. 2. Figure 2a demonstrates the convergence property of the observer, as the observer outputs converge to the true outputs after around 4s. Once the convergence takes place the observer emulates exactly the behaviour of the power system, as expected. Figure 2b also shows that after the application of the step change in $V_{\text{REF}}$ at time $t = 10s$, the load angle slips back, allowing for the excitation system to increase the resultant airgap flux, which in turn boosts the terminal voltage up to the required level. Once this is achieved, the load angle settles to a new steady state, as expected. Finally, Fig. 2c shows that as the net change in the power is zero, the output power, after experiencing a small initial transient response, remains unchanged. This is again in line with the physical behaviour of power system under study.

#### 4.2.2 Case study 2

In this study, we assume that the power system is in equilibrium operating under a normal (fault-free) condition. We also assume that the observer was switched on for long enough for it to track the states of the power system. Then the excitation system reference command, $V_{\text{REF}}$, is stepped 5% at time $t = 5s$. This is followed by a solid three-phase line-to-ground fault occurring half-way along line 2 at time $t = 10s$. This fault is simulated by setting the fault parameters as: $f_1 = 1$ and $\gamma_1 = 0.5$ (which corresponds to $\rho = 2$). The purpose of this study is first to detect the occurrence of the fault, second to identify the faulty line, and third to determine the exact location of the fault. This simulation study has been performed using the Simulink™ model.

The simulation results are shown in Fig. 3. Figure 3a shows the command signal $V_{\text{REF}}$ being applied at time $t = 5s$. Figures 3b–3e show the estimates of the two fault signals and their locations. From Figs. 3b–3e it can be concluded that the fault detection scheme is insensitive to external disturbances such as $V_{\text{REF}}$ and $P_e$. This is demonstrated by the fact that whereas the system itself undergoes a transient period after the application of the step change in $V_{\text{REF}}$, as shown in Fig. 4, the fault detection filter does not respond. Figures 3 and 4 also demonstrate the fact that the fault detection filter is able to instantaneously detect around 4s. Once the convergence takes place the observer emulates exactly the behaviour of the power system, as expected. Figure 2b also shows that after the application of the step change in $V_{\text{REF}}$ at time $t = 10s$, the load angle slips back, allowing for the excitation system to increase the resultant airgap flux, which in turn boosts the terminal voltage up to the required level. Once this is achieved, the load angle settles to a new steady state, as expected. Finally, Fig. 2c shows that as the net change in the power is zero, the output power, after experiencing a small initial transient response, remains unchanged. This is again in line with the physical behaviour of power system under study.
4.2.3 Case study 3: In this study, we run the power system in equilibrium condition, i.e., disturbance free and fault free, for 1 s. Then the reference command, $V_{REF}$, is stepped 5% at time $t = 1$ s. This is followed by solid three-phase line-to-ground faults on both lines. The faults on lines 1 and 2 occur at locations $\gamma_1 = 0.2$ and $\gamma_1 = 0.8$ away from the generator terminal, respectively. The fault on line 1 takes place at time $t = 6$ s, whereas the fault on line 2 takes place at time $t = 11$ s. The purpose of this study is first to detect the occurrence of the two faults, and then to determine their exact locations. Therefore, for this case study four parameters need to be estimated on line and in real-time. They are $f_1$, $\gamma_1$, $f_2$, and $\gamma_2$.

The simulation results are shown in Fig. 5. Figure 5a shows the command signal $V_{REF}$ being applied at time $t = 1$ s. Figures 5b–5e show the estimates of the two fault signals and their locations. From Figs. 5b–5e it can be easily seen that the fault detection scheme proposed in this study is insensitive to known external disturbances such as $V_{REF}$ and $P_r$. This is borne out by the fact that whereas the system itself experiences some transient behaviour after the introduction of the change in $V_{REF}$, as shown in Fig. 6, the fault detection filter remains unaffected. Figures 5 and 6 also demonstrate the successful detection of the two faults, at the precise moment of their occurrence, and the successful detection of their exact locations.
fault was cleared the filter detected this again and switched back to no-fault mode. The same is demonstrated by Figs. 7c and 7f shown in Fig. 8, where the scheme was able to detect the occurrence of fault on line 2 and its exact location. The voltage response of this study is here again, as for the case studies 1–3, Figs. 7 and 8 demonstrate the ability of the fault detection scheme to deal with all possible scenarios and provide exact results.

4.2.5 Case study 5: In this study the same fault detection filter is tested on the original nonlinear system using the nonlinear simulator reported in [22]. The test involves the application of a series of disturbances to the nonlinear power system model, followed by a fault. The performance of the fault detection filter is then examined. The specific disturbances are pulses to the excitation system and turbine governor system setpoints, \( v_{\text{REF}} \) and \( P_{\text{REF}} \), of amplitude 0.01 pu. The duration of these disturbances is 1 s, applied at \( t = 1 \text{s} \) and \( t = 5 \text{s} \), as shown in Figs. 9a and 9b. Then at \( t = 8 \text{s} \), a solid three-phase short-circuit is applied, at a location midway along line 1 (i.e. \( g_1 = 0.5 \)), as shown in Figs. 9c and 9e. The processed estimated fault signals, \( f_{e1} \) and \( f_{e2} = \gamma_2 f_{e1} \), are shown in the Figs. 9d and 9f. From which it can be seen that the fault detection filter is: (i) insensitive to the disturbances; (ii) able to clearly detect the onset of the fault at \( t = 8 \text{s} \); and (iii) able
to estimate the location of the fault as, \( \tilde{\gamma}_2 = \tilde{f}_{e2}/\tilde{f}_{e1} = 0.786/1.338 = 0.587 \) (which is close to the actual value of 0.5).

It is to be noted that, in general, fault detection filters designed for linearised models of power systems are not expected to perform optimally when used on the original nonlinear models. This is basically because the presence of the severe faults (large disturbances) can shift the operating condition drastically to a point where the linearisation is not valid. In such cases some simple signal processing may need to be employed. In generating the responses of Figs. 9a and 9f, a simple processing of the original signals, shown in Fig. 10, was necessary because of the severe nature of the applied fault (line-to-ground fault). The processing of the fault signal involves capturing the fault signal immediately after its occurrence. In hardware this can be implemented by using a relay where the threshold can be adjusted appropriately.

5 Conclusions

A fault detection scheme for power systems has been proposed. The scheme has been shown to be able to provide exact information about any fault or a combination of faults when and where they occur. One of the main advantages of the proposed approach is that only one observer design is required to detect any number of faults, provided that the output measurement contains enough information about the state of the system.

The proposed scheme has been tested on a power system consisting of a generating unit connected to an infinite busbar through a double-circuit transmission line. Various fault scenarios have been studied. Simulation results on linear and nonlinear models of the power system have shown that in all of the studied cases, the fault detection scheme was not only able to accurately detect the occurrence of faults, but also their exact locations.

6 References

4. Mori, H., et al.: ‘A hybrid intelligent system for fault detection in power systems’, Presented at the 2002 Joint Int. Conf. on Neural Networks, (IJCNN’02), 2002