Knowledge in the Situation Calculus

Adrian Pearce

joint work with Ryan Kelly

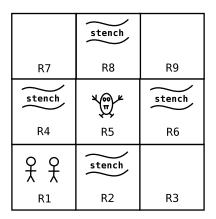
Department of Computing and Information Systems The University of Melbourne

22 September 2014

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A (slightly adapted) well-known illustrative example



Ann and Bob can only observe each other's actions if they are in the same room. They can hear each other's actions from adjacent rooms.

They have no other means of synchronisation!



Introduction

A (slightly adapted) well-known illustrative example



Ann and Bob can only observe each other's actions if they are in the same root. They can hear each other's actions from adjacent rooms.

They have no other means of synchronisation!

Ann and Bob are hunting a Wumpus in a dungeon with many interconnecting rooms. They can fully observe each other's actions if they are in the same room, can hear each other's actions from adjacent rooms, and have no other means of synchronisation.

Like any Wumpus, this one does not move, causes a stench in all adjacent rooms, and if shot will emit a piercing scream that can be heard anywhere in the dungeon.

Can Ann and Bob coordinate their knowledge and actions in order to find and shoot the Wumpus?

(Multi-agent) Hunt The Wumpus in the Situation Calculus

Action description axioms \mathcal{D}_{ad} :

$$\begin{aligned} &Poss(move(agt, r), s) \equiv Adjacent(r, Loc(agt, s)) \\ &Poss(shoot(agt, r), s) \equiv Adjacent(r, Loc(agt, s)) \\ &Poss(alert(agt), s) \equiv Stench(Loc(agt, s), s) \end{aligned}$$

Successor state axioms \mathcal{D}_{ssa} :

 $Wumpus(S_0) = R5$

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Knowledge in the Situation Calculus

Reasoning tasks

 $\mathsf{E}\mathsf{x}\mathsf{tensions}$ to the Situation Calculus for representing and reasoning about knowledge

- Epistemic reasoning [HalpernMoses92] (not this work) Epistemic reasoning about unrestricted forms of nested knowledge in K_n – Reasoning about group-level knowledge modalities
- Synchronous Knowledge [ScherlLevesque03] (builds on this work) Knowledge, action and the frame problem
- Asynchronous Knowledge [KellyPearce07] (our work) Asynchronous knowledge in the situation calculus — reasoning about knowledge with hidden actions

Two aspects to knowledge

- Incomplete information (through action can learn)
- a lack of synchronisation (don't know how many actions have occurred)

Asynchronous Knowledge in the Situation Calculus?

Explanation closure assumes complete knowledge of \mathcal{D}_{ssa}

- Golog assumes complete knowledge of \mathcal{D}_{ad} and \mathcal{D}_{una} in S_0
- What if incomplete knowledge: $\mathbf{Knows}(\phi, s)$?

Limitation: Synchronicity This works well, but it depends on two assumptions:

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- Complete knowledge (linear plan, no sensing)
- Synchronous domain (agents proceed in lock-step)

Nearly universal in the literature: "assume all actions are public".

Challenge: Regression depends intimately on Synchronicity

Axiomatizing Observations

First, we must represent asynchronicity.

We reify the *observations* made by each agent, by adding the following action description function of the following form to D_{ad} :

Obs(agt, c, s) = o

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If $Obs(agt, c, s) = \emptyset$ then the actions are completely hidden.

$$\begin{split} &View(agt,S_0) = \epsilon \\ &Obs(agt,c,s) = \emptyset \rightarrow View(agt,do(c,s)) = View(agt,s) \\ &Obs(agt,c,s) \neq \emptyset \rightarrow View(agt,do(c,s)) = Obs(agt,c,s) \cdot View(agt,s) \end{split}$$



-Axiomatizing Observations

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- Reify (make concrete) the local perspective of each agent by explicitly talking abo what it has observed
- when actions c are performed in situation s, agent agt will perceive observations c
- Each agent makes a set of *observations*, each situation then corresponds to a loca *view* for that agent.
- Allowing the set of observations to be *empty* lets us model truly asynchronous domains.
- Definitions at bottom are added to foundational actions (since they do not change from domain to domain).

Observations

In synchronous domains, everyone observes every action:

$$a \in Obs(agt, c, s) \equiv a \in c$$

Sensing results can be easily included as action#sensing pairs:

$$a \# r \in Obs(agt, c, s) \equiv a \in c \land SR(a, s) = r$$

And observability can be axiomatised explicitly

$$a \in Obs(agt, c, s) \equiv a \in c \land CanObs(agt, a, s)$$

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- Sensing actions can be considered as private actions (but remember will have to handle situation that arbitrary many unobservable actions have occurred).
- CanObs is introduced as a new action description predicate.

Axiomatizing observations

Multi-agent Hunt the Wumpus

OBSERVATION axioms (added to \mathcal{D}_{ad}):

$$\begin{split} move(agt1,r1) &\in Obs(agt,c,s) \equiv move(agt1,r1) \in c \\ &\wedge [Loc(agt,s) = Loc(agt1,s) \lor Loc(agt,s) = r1] \\ shoot(agt1,r1) &\in Obs(agt,c,s) \equiv shoot(agt1,r1) \in c \\ &\wedge Loc(agt,s) = Loc(agt1,s) \\ alert(agt1) &\in Obs(agt,c,s) \equiv alert(agt1) \in c \\ &\wedge Loc(agt,s) = Loc(agt1,s) \end{split}$$

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Axiomatizing observations

Multi-agent Hunt the Wumpus

OBSERVATION axioms:

$$\begin{split} \textit{footsteps} &\in \textit{Obs}(agt, c, s) \equiv \exists agt1, r1: move(agt1, r1) \in c \\ &\wedge [\textit{Adjacent}(\textit{Loc}(agt, s), r1) \lor \\ &\textit{Adjacent}(\textit{Loc}(agt, s), \textit{Loc}(agt1, s))] \\ \textit{alert} \in \textit{Obs}(agt, c, s) \equiv \exists agt1: \textit{alert}(agt1) \in c \\ &\wedge \textit{Adjacent}(\textit{Loc}(agt, s), \textit{Loc}(agt1, s)) \\ \textit{stench} \in \textit{Obs}(agt, c, s) \equiv \exists r1: move(agt, r1) \in c \\ &\wedge \textit{Stench}(r1, s) \\ \textit{scream} \in \textit{Obs}(agt, c, s) \equiv \exists agt1, r1: \textit{shoot}(agt1, r1) \in c \\ &\wedge r1 = \textit{Wumpus}(s) \land \neg \textit{Killed}(s) \end{split}$$

◆□ → < 部 → < 差 → < 差 → 差 < う へ ペ 12/43 Action: global event changing the state of the world Observation: local event changing an agent's knowledge

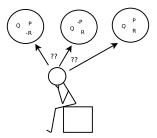
Situation: global history of actions giving current world state View: local history of observations giving current knowledge

How can we let agents reason using only their local view?

Knowledge

If an agent is unsure about the state of the world, there must be several different states of the world that it considers possible.

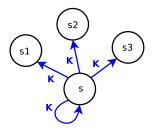
The agent knows ϕ iff ϕ is true in all possible worlds.



 $\mathbf{Knows}(Q) \land \neg \mathbf{Knows}(P) \land \neg \mathbf{Knows}(R) \land \mathbf{Knows}(P \lor R)$

Knowledge

Introduce a possible-worlds fluent K(agt, s', s):



We can then define knowledge as a simple macro:

$$\mathbf{Knows}(agt, \phi, s) \stackrel{\text{\tiny def}}{=} \forall s' \left[K(agt, s', s) \to \phi(s') \right]$$

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Knowledge follows Observation

Halpern & Moses, 1990:

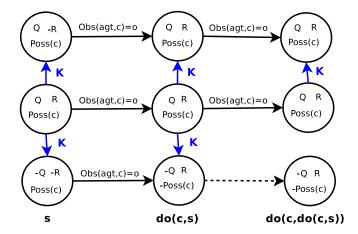
"an agent's knowledge at a given time must depend only on its local history: the information that it started out with combined with the events it has observed since then"

Clearly, we require:

$$K(agt, s', s) \equiv View(agt, s') = View(agt, s)$$

We must enforce this in the successor state axiom for K.

Knowledge: The Synchronous Case



Knowledge: The Synchronous Case

In the synchronous case, K_0 has a simple successor state axiom:

$$K_0(agt, s'', do(c, s)) \equiv \exists s', c' : s'' = do(c', s') \land K_0(agt, s', s)$$

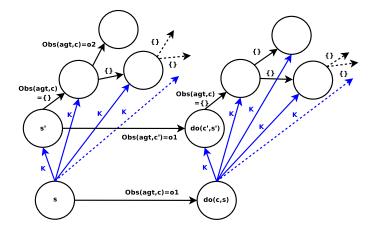
$$\land Poss(c', s') \land Obs(agt, c, s) = Obs(agt, c', s')$$

And a correspondingly simple regression rule:

$$\mathcal{R}(\mathbf{Knows_0}(agt, \phi, do(c, s)) \stackrel{\text{\tiny def}}{=} \exists o : Obs(agt, c, s) = o$$
$$\land \forall c' : \mathbf{Knows_0}(agt, Poss(c') \land Obs(agt, c') = o \rightarrow \mathcal{R}(\phi, c'), s)$$

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Knowledge: The Asynchronous Case



Knowledge: The Asynchronous Case

First, some notation:

$$s <_{\alpha} do(c, s') \equiv s \leq_{\alpha} s' \land \alpha(c, s')$$

$$PbU(agt,c,s) \stackrel{\text{\tiny def}}{=} Poss(c,s) \wedge Obs(agt,c,s) = \{\}$$

Then the intended dynamics of knowledge update are:

$$\begin{split} K(agt, s'', do(c, s)) &\equiv \exists o : Obs(agt, c, s) = o \\ & \wedge \left[o = \emptyset \to K(agt, s'', s) \right] \\ & \wedge \left[o \neq \emptyset \to \exists c', s' : K(agt, s', s) \right] \\ & \wedge Obs(agt, c', s') = o \land Poss(c', s') \land do(c', s') \leq_{PbU(agt)} s'' \end{split}$$

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Sync vs Async

We've gone from this:

$$K_0(agt, s'', do(c, s)) \equiv \exists s', c' : s'' = do(c', s') \land K_0(agt, s', s) \\ \land Poss(c', s') \land Obs(agt, c, s) = Obs(agt, c', s')$$

To this:

$$\begin{split} K(agt, s'', do(c, s)) &\equiv \exists o : Obs(agt, c, s) = o \\ & \wedge \left[o = \emptyset \to K(agt, s'', s) \right] \\ & \wedge \left[o \neq \emptyset \to \exists c', s' : K(agt, s', s) \right] \\ & \wedge Obs(agt, c', s') = o \land Poss(c', s') \land do(c', s') \leq_{PbU(agt)} s'' \end{split}$$

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It's messier, but it's also hiding a much bigger problem...

Our new SSA uses $\leq_{PbU(agt)}$ to quantify over all future situations. Regression cannot be applied to such an expression.

An asynchronous account of knowledge cannot be approached using the standard regression operator.

In fact, this quantification requires a second-order induction axiom. Must we abandon hope of an effective reasoning procedure?

Property Persistence [KellyPearce2010]

Property persistence facilitates "factoring out" the quantification, this allows us to get on with the business of doing regression.

The *persistence condition* $\mathcal{P}[\phi, \alpha]$ of a formula ϕ and action preconditions α to mean: assuming all future actions satisfy α , ϕ will remain true.

$$\mathcal{P}[\phi, \alpha](s) \equiv \forall s' : s \leq_{\alpha} s' \to \phi(s')$$

Like \mathcal{R} , the idea is to transform a query into a form that is easier to deal with.

Property Persistence [KellyPearce2010]

The persistence condition can be calculated using *fixpoint approximates* [CousotCousot79,Tarski55]

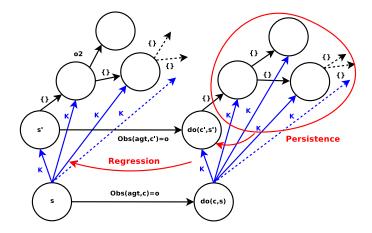
$$\begin{aligned} \mathcal{P}^{1}(\phi, \alpha)[s] &\stackrel{\text{\tiny def}}{=} \phi[s] \land \forall c : \alpha(c, s) \to \mathcal{R}(\phi, c)[s] \\ \\ \mathcal{P}^{n}(\phi, \alpha) &\stackrel{\text{\tiny def}}{=} \mathcal{P}^{1}(\mathcal{P}^{n-1}(\phi, \alpha), \alpha) \end{aligned}$$

$$\left[\mathcal{P}^{n}(\phi,\alpha) \to \mathcal{P}^{n+1}(\phi,\alpha)\right] \Rightarrow \left[\mathcal{P}^{n}(\phi,\alpha) \equiv \mathcal{P}(\phi,\alpha)\right]$$

Corresponds to the greatest fixpoint – sound but not complete (might have to go beyond ω in case of infinite ground actions, as SO)

- This calculation provably terminates over complete, finite lattices (e.g. context-free case, STRIPS)
- Can be computed *offline* using *static domain reasoning* for non-disjunctive queries [DemolombePozosParra00]

Regressing Knowledge



4 ロ ト (部 ト (重 ト (重 ト) 重 の Q () 25 / 43 It becomes possible to define the regression of our Knows macro:

$$\mathcal{R}[\mathbf{Knows}(agt, \phi, do(c, s))] = \\ [Obs(agt, c, s) = \{\} \rightarrow \mathbf{Knows}(agt, \phi, s)] \\ \wedge [\exists o : Obs(agt, c, s) = o \land o \neq \{\} \rightarrow \\ \mathbf{Knows}(agt, \forall c' : Obs(agt, c') = o \rightarrow \\ \mathcal{R}[\mathcal{P}[\phi, PbU(agt)](do(c', s'))], s)] \end{cases}$$

The regression operator can be modified to act over observation histories, instead of over situations:

$$\begin{aligned} \mathcal{R}[\mathbf{Knows}(agt, \phi, o \cdot h)] = \\ \mathbf{Knows}(agt, \forall c' : Obs(agt, c', s') = o \rightarrow \\ \mathcal{R}[\mathcal{P}[\phi, PbU(agt)](do(c', s'))], h) \end{aligned}$$

We can equip agents with a situation calculus model of their own environment.

Multi-agent Hunt the Wumpus

	stench	
R7	R8	R9
stench	L L L	stench
R4	R5	R6
£ £	stench	
R1	R2	R3

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Initially, Ann does not know where the wumpus is:

$$\mathcal{D} \cup \mathcal{D}_K^{obs} \models \neg \exists r : \mathbf{Knows}(A, Wumpus() = r, S_0)$$

Regression:

$$\mathcal{R}(\neg \exists r : \mathbf{Knows}(A, Wumpus() = r, S_0)) \Rightarrow \\ \neg \exists r : \mathbf{Knows}_0(A, \mathcal{R}((\mathcal{P}(Wumpus() = r, PbU(A))[S_0])^{-1}, S_0))$$

Fixpoint search terminates after single iteration:

$$\mathcal{P}(\mathit{Wumpus}() = r, \mathit{PbU}(A)) \Rightarrow \mathit{Wumpus}() = r$$

Gives: $\mathcal{R}(\neg \exists r : \mathbf{Knows}(A, Wumpus() = r, S_0)) \Rightarrow$ $\neg \exists r : \mathbf{Knows}_0(A, \mathcal{R}((Wumpus() = r)[S_0])^{-1}, S_0)$



Knowledge in the Situation Calculus

2014-09-22

No sequence of hidden actions could result in Ann learning Wumpus location:

- The notation $\phi[s']$ represents a uniform formula in which all fluents have their situation argument replaced with the particular situation term s'
- ϕ^{-1} represents a uniform formula with the situation argument removed from all its fluents.

Bob knows that the wumpus is not in an adjacent room, since he knows there is no stench in room R1.

 $\mathcal{D} \cup \mathcal{D}_{K}^{obs} \models \mathbf{Knows}(B, Wumpus() \neq R2 \land Wumpus() \neq R4), S_{0})$

All initial situations have $\neg Stench(R1)$ and the background theory has Adjacent(R2, R1), Adjacent(R4, R1), and the axiom relating a stench to an adjacent wumpus.

All initial situations thus have no Wumpus in rooms adjacent to R1, so the regressed query is entailed by the domain.

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Knowledge in the Situation Calculus

Introduction

2014-09-22

Example 2

Bob knows that the wumpus is not in an adjacent room, since he knows there is no stench in room R1.

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All initial situations thus have no Wampus in rooms adjacent to R1, so the regressed query is entailed by the domain.

It is therefore safe for him to move to an adjacent room.

Regressing as in the previous example gives the equivalent query:

After Bob moves into room R2, he knows that it has a stench.

 $\mathcal{D} \cup \mathcal{D}_{K}^{obs} \models \mathbf{Knows}(B, Stench(R2), do(\{move(B, R2)\}, S_{0}))$

$$\mathcal{R}(\mathbf{Knows}(B, Stench(R2), do(\{move(B, R2)\}, S_0)) \Rightarrow \\ \exists o: Obs(B, \{move(B, R2)\}, S_0) = o \\ \land [o = \emptyset \to \mathbf{Knows}(B, Stench(R2), s)] \\ \land [o \neq \emptyset \to \mathbf{Knows}(B, \forall c': Obs(B, c') = o \\ \land Poss(c') \to \mathcal{R}(\mathcal{P}(Stench(R2), PbU(B)), c'), s)] \end{cases}$$

Expanding " $\exists o$ " clause into a finite disjunction – only two possible values for $Obs(B, \{move(B, R2)\}, s)$, corresponding to room having or not having a stench:

- $Stench(R2, s) \equiv Obs(B, \{move(B, R2)\}, s) = \{move(B, R2), stench\}$
- $\neg Stench(R2, s) \equiv Obs(B, \{move(B, R2)\}, s) = \{move(B, R2)\}$

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Expanding and replacing each observation with its preconditions yields:

 $\mathcal{R}(\mathbf{Knows}(B, Stench(R2), do(\{move(B, R2)\}, S_0)) \Rightarrow (Stench(R2, S_0) \land [\mathbf{Knows}(B, \dots)]) \\ \lor (\neg Stench(R2, S_0) \land [\mathbf{Knows}(B, \dots)])$

The domain entails $Stench(R2, S_0)$ so we can simplify the other option away, leaving:

$$\mathcal{R}(\mathbf{Knows}(B, Stench(R2), do(\{move(B, R2)\}, S_0))) \Rightarrow$$
$$\mathbf{Knows}(B, \forall c' : Poss(c') \land Obs(B, c') = \{move(B, R2), stench\} \rightarrow$$
$$\mathcal{R}(\mathcal{P}(Stench(R2), PbU(B)), c'), S_0)$$

The persistence condition calculation again terminates after one iteration:

 $\mathcal{P}(Stench(R2), PbU(B)) \Rightarrow Stench(R2)$ $\mathcal{R}(\mathcal{P}(Stench(R2), PbU(B)), c') \Rightarrow Stench(R2)$

So the query further simplifies to:

$$\mathbf{Knows}(B, \forall c': Poss(c') \land Obs(B, c') = \{move(B, R2), stench(R2)\} \rightarrow Stench(R2), S_0\}$$

Since domain has finite number of possible actions, we can expand the " $\forall c'$ " clause into a finite conjunction – indeed, the only value of c' that can produce those observations is $\{move(B, R2)\}$.

Substituting it and its action description predicates *Poss* and *Obs* gives:

Knows $(B, Adjacent(R2, Loc(B)) \land Stench(R2) \rightarrow Stench(R2), S_0)$

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This tautology is clearly entailed by the domain.

Ann learns that Bob is in room R2 by observing Bob's move.

 $\mathcal{D} \cup \mathcal{D}_{K}^{obs} \models \mathbf{Knows}(A, Loc(B) = R2, do(\{move(B, R2)\}, S_{0}))$

Example 5

After Bob alerts that there is a stench, Ann knows there is a stench in room R2, since Ann knows that Bob is in room R2.

 $\mathcal{D} \cup \mathcal{D}_{K}^{obs} \models \mathbf{Knows}(A, Stench(R2), do([\{move(B, R2)\}, \{alert(B)\}], S_{0}))$

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After moving to room R4 Ann observes a stench, knows that there is a stench in both R2 and R4:

$$\begin{split} \mathcal{D} \cup \mathcal{D}_{K}^{obs} \models \mathbf{Knows}(A, Stench(R2) \land Stench(R4), \\ do([\{move(B, R2)\}, \{alert(B, R2)\}, \{move(A, R4)\}], S_0)) \end{split}$$

And hence knows that the wumpus is in room R5:

 $\begin{aligned} \mathcal{D} \cup \mathcal{D}_{K}^{obs} \models \mathbf{Knows}(A, Wumpus() = R5, \\ do([\{move(B, R2)\}, \{alert(B, R2)\}, \{move(A, R4)\}], S_{0})) \end{aligned}$

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Ann doesn't know where Bob is, since she can't observe his footsteps from room R4.

$$\mathcal{D} \cup \mathcal{D}_{K}^{obs} \models \neg \exists r : \mathbf{Knows}(A, Loc(B) = r, \\ do([\{move(B, R2)\}, \{alert(B, R2)\}, \{move(A, R4)\}], S_0))$$

Example 8

Ann shoots the wumpus, observes the scream and knows the wumpus is dead.

 $\mathcal{D} \cup \mathcal{D}_{K}^{obs} \models \mathbf{Knows}(A, Killed, do([\{move(B, R2)\}, \{alert(B)\}, \{move(A, R3)\}, \{shoot(A, R4)\}], S_{0}))$

Ann knows that Bob knows the wumpus is dead, as he will have heard the scream regardless of this location.

$$\begin{split} \mathcal{D} \cup \mathcal{D}_{K}^{obs} &\models \mathbf{Knows}(A, \mathbf{Knows}(B, Killed), \\ do([\{move(B, R2)\}, \{alert(B)\}, \{move(A, R4)\}, \{shoot(A, R5)\}], S_0)) \end{split}$$

- At this point the hunters will have common knowledge that the Wumpus is dead.
- However, the current formalism is not rich enough to reason directly about common knowledge
- Although we have preliminary work on this topic in [KellyPearce08] that could be integrated with the approach taken here.

Applications of Knowledge in the Situation Calculus

What kind of applications?

- Public actions
- Private actions
- Guarded sensing actions
- Speech acts
- Explicit observability axioms
- Observability interaction
- Observing the effects of actions
- Delayed communication

Summary

A robust account of knowledge based on observations, allowing for arbitrarily-long sequences of hidden actions, in asynchronous settings

Allows agents to reason about their own knowledge using only their local information

- Main point: Facilitates first-order theorem proving for reasoning about hidden actions Regression rule avoids SO logic by utilizing persistence condition
- Subsumes existing accounts of knowledge equivalent to [ScherlLevesque03] in synchronous case
- Elaboration tolerant compatible with existing techniques
- Preliminary Prolog implementation using modal variant of the LeanT^AP theorem prover [BernhardBeckertPosegga95,Fitting98], verified its operation on some simple examples

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