Student Number: 

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CSSE Login: 

The University of Melbourne  
Mid Semester Test 2010 - Sample Solutions  

Department of Computer Science and Software Engineering  
433-324 Graphics and Interaction  

Handed Out 14 September 2010.  
Writing Time 50 Minutes during the Lecture on 16 September 2010  

This paper has 10 pages including this cover page.  

Identical Examination Papers: None.  
Common Content Papers: None.  

Authorised Materials:  
Open Book: Any materials are allowed into the Mid Semester Test  

Instructions:  
Students must write all of their answers on this examination paper. Students may not remove any part of the examination paper from the lecture.  

Instructions to Students:  
This mid semester test counts for 10% of your final grade. ANSWER ONLY TWO questions in the indicated answer boxes in THE PAPER PROVIDED IN THE LECTURE (do not write in the paper handed out two days earlier). Only material written inside the boxes will be marked. Note that if you answer more than two questions you will only be marked for the FIRST TWO questions answered. If you need to make rough notes, or prepare draft answers, you may do so on the reverse of any page.  

Paper to be held by Baillieu Library: No.  

Examiners use only:  

| Q1: | Q2: | Q3: |
Question 1  
(5 Marks)

A form of the Phong illumination model is shown below, in terms of an equation and a diagram that shows how angles, $\theta$ and $\alpha$, from the equation relate to the (normalised) vectors $\overline{L}$ for the light source, $\overline{N}$ for the objects surface normal, $\overline{R}$ for the direction of reflection and $\overline{V}$ for the viewpoint direction.

\[
I_\lambda = I_{\lambda x}k_xO_{y\lambda} + f_{att}I_{t\lambda}[k_yO_{y\lambda}\cos\theta + k_z\cos^n\alpha]
\]

(a). Write down an expression for the intensity of ambient reflection, in the direction of the viewpoint $\overline{V}$, using terms from the equation above.

Example solution:

\[
I_{x\lambda}k_zO_{y\lambda}
\]

(b). Write down an expression for the intensity of Lambertian reflection, in the direction of the viewpoint $\overline{V}$, using terms from the equation above.

Example solution:

\[
f_{att}I_{t\lambda}k_yO_{y\lambda}\cos\theta
\]

(c). Write down an expression for the intensity of Specular reflection, in the direction of the viewpoint $\overline{V}$, using terms from the equation above.

Example solution:

\[
f_{att}I_{t\lambda}k_z\cos^n\alpha
\]
(d). In practice, implementing the Phong illumination model using the equation in the form shown above is expensive in terms of computation. Write down a new form of the equation above that reduces the computation cost of evaluation. State (in one sentence) why your new equation is more efficient.

Example solution:

\[ I_\lambda = I_x \lambda k_x O_{g\lambda} + f_{att}I_t \lambda [k_y O_{g\lambda}(\vec{N}.\vec{L}) + k_a(\vec{R}.\vec{V})^n] \]

Replacing trigonometric functions with vectors is more efficient, since computing dot products is significantly faster than computing trigonometric functions.
(e). Draw a diagram in the box below that explains how *Lambertian reflection* is related to the orientation of the viewer, \( \vec{V} \) in the figure, and how it depends on angles \( \theta \) and/or \( \alpha \). Be sure to explain why.

Example solution:

Independence of surface orientation:

For a given small surface patch, the amount of light radiated towards the viewer is greatest when the surface normal is pointing straight at the viewer, and falls off according to a cosine law as the surface slants away from the viewer, according to \( \theta \).

However, at the same time, for a given visual angle subtended at the viewer, more of the surface is seen within that angle as the surface slants away from the viewer, again according to a cosine law.

These two effects exactly compensate, so, overall, Lambertian reflection is independent of surface orientation with respect to the viewer.
Question 2  

This question concerns transformation geometry and matrices. The figure below shows the transformation of a polygon ABCDE (on the left) into new position A’B’C’D’E’ (on the right).

The \((x, y)\) coordinates of each point of the polygon are indicated on each of the vertices as follows.

- Note that the figure is not drawn accurately to scale (although accurate coordinates are provided).
- Note that coordinates \(X\) and \(Y\) of point \(D'\) have not been specified.

(a). Write down, in words, a sequence of transformations that transforms object ABCDE into the object A’B’C’D’E’, as shown above. For example, an incorrect sequence would be “rotation 90 degrees anticlockwise; translation 10 in \(x\) dimension and \(-5\) in \(y\) dimension; etc.”.

Example solution:

(i). Translate \(A\) to origin \((-2\) in \(x\) and \(-2\) in \(y\)).
(ii). Scale by reflecting about the \(y\) axis \((1\) in \(y\) and \(-1\) in \(x\)).
(iii). Scale \(\sqrt{2}\) in \(x\) and \(\sqrt{2}\) in \(y\).
(iv). Rotate by \(\theta = 45\) degrees (clockwise).
(v). Translate \((3\) in \(x\) and \(3\) in \(y\)).
(b). Explain the main advantage of using homogeneous coordinates for transformations using matrices.

**Example solution:**

The additional (redundant) dimension $w$ allows translation to become linear and allows translation to be written as a matrix product rather than a sum, in homogeneous coordinates (with $w = 1$) as

$$
\begin{pmatrix}
x' \\
y' \\
1
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & x_t \\
0 & 1 & y_t \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
1
\end{pmatrix}
\Rightarrow
\begin{pmatrix}
x + x_t \\
y + y_t \\
1
\end{pmatrix}
$$

Any transformation (or combination of transformations) can consequently be expressed in homogeneous coordinates by means of the matrix equation:

$$
\begin{pmatrix}
x' \\
y' \\
1
\end{pmatrix} = 
\begin{pmatrix}
m_{00} & m_{01} & m_{02} \\
m_{10} & m_{11} & m_{12} \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
1
\end{pmatrix}
$$

(c). Write down each of the transformation matrices named in your answer to the question about the sequence of transformations, in *homogeneous form*, giving the numerical values of their coefficients. Be sure to identify the transformations by name.

**Example solution:**

Translate A to origin

$$
T_1 = \begin{pmatrix}
1 & -2 \\
1 & -2 \\
1 & 1
\end{pmatrix}
$$
Scale. Use \( AB \) and \( A'B' \) to determine scale in \( x \) and \( AG \) and \( A'G' \) to determine scale in \( y \).

\[
S_x = \frac{\sqrt{2^2 + 2^2}}{2^2 + 0^2} = \sqrt{2}
\]

\[
S_y = \frac{\sqrt{1^2 + 1^2}}{0^2 + 1^2} = \sqrt{2}
\]

Scale. Reflect about the \( x \) axis.

\[
S_x = 1
\]

\[
S_y = -1
\]

\[
T_4 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}
\]

Rotate by \( \theta = 45 \) deg

\[
T_2 = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}
\]

\[
= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}
\]

Translate to \( A' \).

\[
T_5 = \begin{bmatrix} 1 & 2 \\ 1 & 4 \\ 1 \end{bmatrix}
\]

Translate \( A \) to origin: \( T_1 = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \)

Scale by reflecting about the \( y \) axis \( S_x = -1 \) and \( S_y = 1 \) followed by enlargement scaling. Use \( AE \) and \( A'E' \) to determine scale in \( y \) and \( AB \) and \( A'B' \) to determine scale in \( x \), where

\[
S_x = \frac{\sqrt{2^2 + 2^2}}{2^2 + 0^2} = \sqrt{2} \quad \text{and} \quad S_y = \frac{\sqrt{1^2 + 1^2}}{0^2 + 1^2} = \sqrt{2}.
\]

\[
S_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} 
\quad \quad \quad S_2 = \begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

Rotate by \( \theta = 45 \) deg clockwise

\[
R = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]
(d). Derive an overall transformation matrix \( M \), in homogeneous form, that performs the above transformation and use it to derive coordinates \( X \) and \( Y \) of point \( D' \). Be sure to show all the steps involved in the derivation of \( M \) below.

**Example solution:**

- Solution \( M = T_2 \cdot R \cdot S_2 \cdot S_1 \cdot T_1 \).
- Note that order *does* matter, so need to multiply in following order \( ... [3][2][1] \).
- Note that in general, matrix multiplication is *not* commutative (\( T_1 \cdot T_2 \neq T_2 \cdot T_1 \))

\[
S_1 \cdot T_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} 
\]

\[
S_2 \cdot S_1 \cdot T_1 = \begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\sqrt{2} & 0 & 2\sqrt{2} \\ 0 & \sqrt{2} & -2\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix} 
\]

\[
R \cdot S_2 \cdot S_1 \cdot T_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\sqrt{2} & 0 & 2\sqrt{2} \\ 0 & \sqrt{2} & -2\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} 
\]

\[
M = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 1 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 3 \\ -1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} 
\]

\[
\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3.5 \\ 4.5 \\ 1 \end{bmatrix} 
\]
Question 3 (5 Marks)

This question concerns the scan-line polygon filling algorithm. Consider the polygon below, assume $y$ is the vertical axis and $x$ is the horizontal axis for each point $(x, y)$.

(a). Draw in the box below the state of the edge-table for the above polygon for *all* scan lines from $y = 0$ to $y = 5$. Be sure to include and describe the relevant parameters that are contained in your edge table.

Example solution:

Edge table:
5
4
3 → [5, 3, −1] → [5, 3, 0]
2
1 → [5, 1, 0] → [5, 5, 0]
0
where entries comprise $[ymax, x(ymín) \frac{1}{m}, ]$

Notes:
- Empty at $y = 0, 2, 4, 5$ as there are no new edges.
- Horizontal edges are not included.

(b). Draw in the box below the state of the *active* edge-table for the above polygon at $y = 2$, and at $y = 4$.

Example solution:

Active edge table:
$y = 2 \rightarrow [5, 1, 0] \rightarrow [5, 5, 0]$

$y = 4 \rightarrow [5, 1, 0] \rightarrow [5, 2, −1] \rightarrow [5, 3, 0] \rightarrow [5, 5, 0]$
(c). Explain why filling a polygon under \textit{perspective projection}, using equal steps in image dimension $x$ along a scan line, does \textit{not necessarily} lead to equal steps in depth, $z$, along the polygonal face. You can use a diagram to explain your answer if you like by drawing in the box below.

\textbf{Example solution:}

- Because of perspective foreshortening
- Either a suitable diagram that shows how this occurs or a description of that elucidates it.